**Andrey V. Voron**

# A method for calculating Euler parallelepipeds based on the values of Pythagorean triples

**Annotation.** A non-formulaic method has been found for calculating Euler parallelepipeds of the second family based on the values of Pythagorean triples of Euler parallelepipeds of the first family, the largest common divisors. To do this, three triangles with integer values of the sides are allocated in the figure. Next, Pythagorean triples are determined from the obtained triangles by selecting the values of their greatest common divisors. These triples are entered in the table. By using a cross-arrangement in the table of two values (out of three) of Pythagorean triples (using the described algorithm of mathematical operations), the values of the three sides of the "derivative" Euler parallelepiped are calculated.

**Keywords:** a method for obtaining Euler parallelepipeds, Pythagorean triple, Euler parallelepiped of the second family, the greatest common divisor.

**Introduction.** It is known that a rectangular parallelepiped with integer values of edges (a, b, c) and diagonals of faces (d, e, f) is called an Euler parallelepiped (Figure).

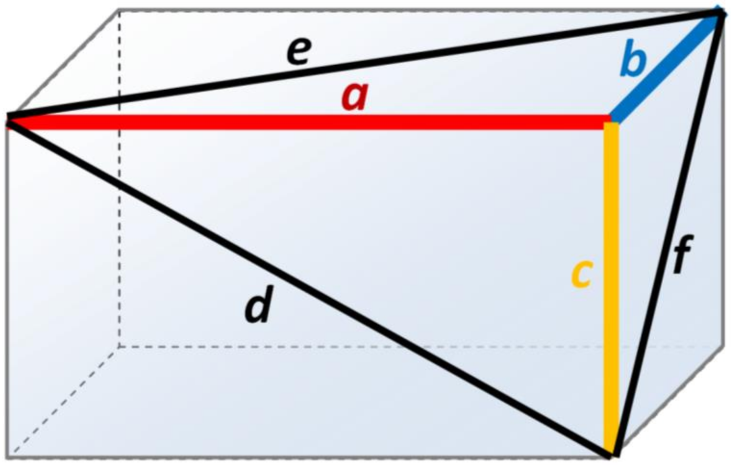


Figure – Euler's Parallelepiped

It is known that Euler described two families of parallelepipeds, which are given by formulas similar to those for finding the values of Pythagorean triangles. One of the families of Euler parallelepipeds (the first one) is given by the formulas (for n>3):

$$a=n^6-15n^4+15n^2-1 \tag{1}$$

$$b=6n^5-20n^3+6n \tag{2}$$

$$c=8n^5-8n \tag{3}$$

In any rectangular parallelepiped, three edges can be distinguished that define the linear dimensions of a three-dimensional figure. In the case of the Euler parallelepiped, three different right-angled triangles with integer values of the edges and diagonals of the figure can also be distinguished. Further, from the values of the linear dimensions of the obtained right triangles, it is possible to identify their greatest common divisors and obtain, in this way, three primitive Pythagorean triples. Thus, any Eulerian parallelepiped can be represented as a geometric object based on three primitive Pythagorean triples. In this regard, we have put forward a hypothesis: it is assumed that there are Euler parallelepipeds of the second family and a method for calculating them based on the values of Pythagorean triples of other Euler parallelepipeds of the first family.

**The main part.** We calculated the same number of Euler parallelepipeds of the second family from eight Euler parallelepipeds of the first family using the original method (table). The first family "Ⅰ" can be defined as the "parent", and the second "Ⅱ" as the "derivative". The "Ⅰ" family is made up of the already known values of four Euler parallelepipeds up to the edge values of no more than 1000 [1] and four more from the obtained values given by the formulas (1), (2), (3). The other eight Euler parallelepipeds of the "Ⅱ" family (called "derivatives" by us) are derived from the first eight "parent" Euler parallelepipeds of the "Ⅰ" family.

Table – Values of primitive Pythagorean triples, the largest common divisors of three edges of Euler parallelepipeds of two families and their values

| Parallelepiped number, family number | Pythagorean triples of parallelepiped edges | | | The largest common divisor of the edges of a parallelepiped | The value of the edges of the Euler parallelepiped "I" Family | Parallelepiped number, family number | Pythagorean triples of parallelepiped edges | | | The largest common divisor of the edges of a parallelepiped | The value of the edges of the Euler parallelepiped "II" Family |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | 11 | 60 | 61 | *4 | 240 | | 11 | 60 | 61 | *39 | 2340 |
| 0-I | 39 | 80 | 89 | *3 | 117 | 0-II | 39 | 80 | 89 | *11 | 880 |
| | 44 | 117 | 125 | *1 | 44 | | 44 | 117 | 125 | *20 | 429 |
| | 11 | 60 | 61 | *12 | 720 | | 11 | 60 | 61 | *17 | 1584 |
| 1-I | 17 | 144 | 145 | *5 | 132 | 1-II | 17 | 144 | 145 | *11 | 1020 |
| | 85 | 132 | 157 | *1 | 85 | | 85 | 132 | 157 | *12 | 187 |
| | 20 | 21 | 29 | *12 | 275 | | 20 | 21 | 29 | *55 | 1155 |
| 2-I | 48 | 55 | 73 | *5 | 252 | 2-II | 48 | 55 | 73 | *21 | 1100 |
| | 252 | 275 | 373 | *1 | 240 | | 252 | 275 | 373 | *4 | 1008 |
| | 7 | 24 | 25 | *33 | 792 | | 7 | 24 | 25 | *20 | 693 |
| 3-I | 20 | 99 | 101 | *8 | 231 | 3-II | 20 | 99 | 101 | *7 | 480 |
| | 160 | 231 | 281 | *1 | 160 | | 160 | 231 | 281 | *3 | 140 |
| | 69 | 260 | 269 | *12 | 3120 | | 69 | 260 | 269 | *407 | 105820 |
| 4-I | 407 | 624 | 745 | *5 | 2035 | 4-II | 407 | 624 | 745 | *69 | 43056 |
| | 828 | 2035 | 2197 | *1 | 828 | | 828 | 2035 | 2197 | *52 | 28083 |
| | 611 | 1020 | 1189 | *8 | 8160 | | 611 | 1020 | 1189 | *33 | 332384 |
| 5-I | 33 | 544 | 545 | *15 | 4888 | 5-II | 33 | 544 | 545 | *611 | 33660 |
| | 495 | 4888 | 4913 | *1 | 495 | | 495 | 4888 | 4913 | *68 | 20163 |
| | 793 | 1776 | 1945 | *35 | 62160 | | 793 | 1776 | 1945 | *3531 | 6271056 |
| 6-I | 3531 | 5180 | 6269 | *12 | 42372 | 6-II | 3531 | 5180 | 6269 | *793 | 4107740 |
| | 27755 | 42372 | 50653 | *1 | 27755 | | 27755 | 42372 | 50653 | *148 | 2800083 |
| | 429 | 700 | 821 | *192 | 134400 | | 429 | 700 | 821 | *1679 | 1175300 |
| 7-I | 1679 | 2400 | 2 929 | *56 | 94024 | 7-II | 1679 | 2400 | 2 929 | *429 | 1029600 |
| | 10296 | 11753 | 15 625 | *8 | 82368 | | 10296 | 11753 | 15 625 | *100 | 720291 |
| | 3201 | 4160 | 5 249 | *63 | 262080 | | 3201 | 4160 | 5 249 | *11651 | 52432380 |
| 8-I | 11651 | 16380 | 20 101 | *16 | 201663 | 8-II | 11651 | 16380 | 20 101 | *3201 | 48468160 |
| | 186416 | 201663 | 274 625 | *1 | 186416 | | 186416 | 201663 | 274 625 | *260 | 37294851 |

Euler parallelepipeds of the " Ⅱ " family are obtained from Euler parallelepipeds of the " Ⅰ " family by means of a method whose algorithm consists of the following mathematical operations:

– calculation of the values of the edges of the Euler parallelepiped of the family " Ⅰ " according to Euler's formulas (1), (2), (3);

– determination of the three largest common divisors of the edges of a parallelepiped (comparing the values of the divisors of three pairs of edges);

– identification by dividing the values of the edges of the parallelepiped of the family " Ⅰ " by the obtained values of the largest common divisors of the edges of primitive Pythagorean triples;

– tabular compilation of three positions of three Pythagorean triples in three rows and two columns. In this case, the first column is duplicated with the values of three primitive Pythagorean triples of the parallelepiped family " Ⅰ " (table);

– entry in the column "The greatest common divisor of the edges of the parallelepiped" Ⅱ of the family of values of a pair of numbers of primitive Pythagorean triples with their smallest sum. In this case, two values are cross–arranged in a table of three Pythagorean triples - one of the smallest values of one Pythagorean triple will be a multiplier for the other triple and vice versa (table) and, accordingly, also for the other Pythagorean triple;

– multiplication of the values of the table of two Pythagorean triples (values of the larger and smaller legs) by the values of two multipliers – the value of the smallest of the edges of the parallelepiped of the family is calculated (table);

– multiplication of the second numbers of each primitive Pythagorean triple of the parallelepiped family (larger cathet) for the values of two multipliers (the obtained values are the lengths of the three edges of the parallelepiped of the family);

– finding the multiplier of the third Pythagorean triple by dividing the obtained value of the edge of the Euler parallelepiped (average ranking) by the smallest value of the largest sum of the three primitive Pythagorean triples.

**Conclusion**.

1. A non-formulaic method has been found for calculating Euler parallelepipeds of the second family based on the values of Pythagorean triples of Euler parallelepipeds of the first family, the largest common divisors.

2. The algorithm of the calculation method consists of a number of mathematical operations: three triangles with integer values of the sides are allocated in the figure. Next, Pythagorean triples are determined from the obtained triangles by selecting the values of their greatest common divisors. These triples are entered in the table. By using a cross-arrangement in the table of two values (out of three) of Pythagorean triples (using the described algorithm of mathematical operations), the values of the three sides of the "derivative" Euler parallelepiped are calculated.

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