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FROM MODIFIED NEWTONIAN DYNAMICS (MOND) TO AUGMENTED NEWTONIAN DYNAMICS (AuND)

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Abstract. *MOND assumes that Newtonian dynamics is not accurate. Therefore, it needs modification. AuND assumes that Newtonian dynamics is accurate but not complete. For more than 300 years, Newtonian dynamics has remained an incomplete mathematical model. The shortcomings of the incomplete Newtonian dynamics have given reason to consider the classical model of gravity to be flawed and limited. Newton's formula F_N = GMm/r^2 gives the force of gravitational interaction between two bodies. Accordingly, Newton's law formula gives only a part of the force of universal gravitation and does not apply to the universe. In addition to the forces described by Newtonian dynamics, there is a gravitational force in the Universe, which is observed in experiments, but does not follow from Newton's law of gravitation. Newtonian dynamics "does not see" the additional cosmological force of gravitational interaction of bodies with the mass of the Universe. The additional cosmological force is represented by a new law of gravitation, different from Newton's law. The law of cosmological force is presented using the cosmological constant Λ: F_{Cos} = (mc^2)√Λ. The cosmological force has a linear dependence on the mass of a body and does not obey the law of inverse squares. On small scales, the additional cosmological force is much smaller than the Newtonian force. On the scale of the universe, the cosmological force exceeds the Newtonian force and has a theoretical limit equal to the Planck force F_P = c^4/G = 1.21027-10^44 N. This large force is not represented in Newton's law of universal gravitation.A new mathematical formula for the law of universal gravitation is given. The law of universal gravitation is represented by two equations, Newton's law F_N = GMm/r^2 and the cosmological force law F_{Cos} = (mc^2)√Λ. Augmented Newtonian dynamics (AuND) reanimates the classical model of gravitation and provides a solution to the problems of cosmology without involving the hypothesis of "dark matter"*

Keywords*: The law of universal gravitation; The law of cosmological force; Quantum theory of gravitation; Large numbers; Cosmological equations; Parameters of the observable universe; Galaxy rotation curves; Pioneer anomaly.*

1. Introduction

The dominant force in the universe is gravity. Newton's law of gravity was a real breakthrough in science. Newton's law of gravitation (1) impresses with its simplicity and mathematical perfection:

$$F_N = G\frac{Mm}{r^2} \qquad (1)$$

Gravitational interaction has become the fourth fundamental interaction. Newton's law of gravitation makes it possible to explain and predict with great accuracy the motions of celestial bodies.

The attractive thing about Newton's law of gravitation is the simple dependence of force on the parameters of interacting bodies.

At the same time, simple and perfect in mathematical representation, Newton's law of gravitation has limitations and limits of applicability. Newton's law of gravitation describes the interaction of two point masses. But it does not account for the gravitational interaction of bodies with the universe. It does not answer: "with what force does any mass interact with the mass of the Universe distributed in space?", "on what parameters of the Universe does the cosmological force depend?".

For a long time it was thought that Newtonian dynamics was a complete description of all types of motion occurring in the Universe. We now know that this is not the case.It was found that Newtonian dynamics gives predictions that do not agree with experimental observations [1, 2]. Against this background, the view of Newtonian dynamics changed. The prevailing view was that Newtonian dynamics is only an approximation that is valid under certain conditions.

The impossibility of obtaining an analytical solution to the problem of three and more bodies, the impossibility of extending Newton's law of gravitation to the Universe, the deviation of the velocity of stars from the predictions of Newton's law gave rise to distrust of the classical theory and even gave reason to question the correctness of Newton's law of gravitation. Numerous attempts were made to modify Newtonian dynamics.

In 1745 Alexis Clairaut [3] proposed a modification of Newton's law in which the law of inverse squares was changed. In 1894 Hall [4] proposed the replacement of the square of distance by a slightly greater degree. Hugo von Seeliger and Carl Gottfried Neumann proposed a modification of the law with a faster than Newton's law of gravitation decreasing with distance [5]. Attempts to modify Newton's law of gravitation [1] and attempts to question the law of inverse squares do not stop.

In the MOND theory [1] it was shown that for small accelerations of the order of $2 \cdot 10^{-10}$ m/s² Newton's law of gravitation may not work.

2. The second formula for the gravitational interaction of two bodies.

In the framework of Newtonian dynamics we can propose the second formula for the gravitational interaction of two bodies.

$$F_K = \frac{R^3 m}{T^2 r^2} \qquad (2)$$

where: F_K is the force represented by the Kepler relation, m is the mass of the body, R and T are orbit parameters, r is the distance.

This is not a modified Newton's formula. Formula (2) is equivalent to formula (1). Formula (2), like formula (1), includes the law of inverse squares.

Newton's law of gravitation (1) is not the only formula for the force of gravitational interaction between two bodies. The law of gravitational interaction between two bodies can be represented by a formula that does not include the gravitational constant G and gravitational mass M.

3. Our ideas about gravity need to be revised.

After it was discovered that the velocities of stars around the centers of galaxies have a very large deviation from Newton's predictions, it became clear that our ideas about gravity need revision. Different hypotheses were proposed to explain the rotation curves of galaxies. Some hypotheses retained Newton's law of gravitation, while others questioned the correctness of Newton's law [1, 6, 7].

A hypothesis was proposed that preserves the law of gravitation as Newton proposed it, but introduces a new entity - "dark matter". The MOND theory (modified Newtonian dynamics) was proposed, which explains the deviation at the cost of correction of Newton's law of gravitation.

The limits of applicability of Newton's law of gravitation were especially evident in the study of spiral galaxies. The curves of spiral galaxies (Fig. 1) showed a significant discrepancy between the observed experimental curves (B) and the curve obtained theoretically (A).

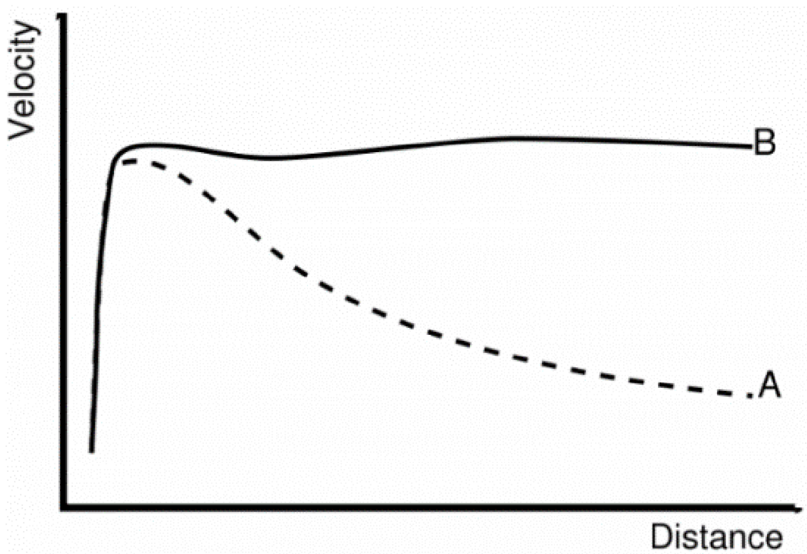

Fig. 1. Rotation curve of a typical spiral galaxy: predicted (A) and observed (B) [1, 8].

The reason of such a significant discrepancy between theory and observations has no convincing explanation. Is there a solution to this problem within the framework of classical gravitation? Below it is shown that there is a solution and this solution is not connected with the hypothesis of dark matter. The cause of the problem was hidden in the incompleteness of the classical theory of gravitation.

Newton's formula (1) gives the force of gravitational interaction between two bodies. Accordingly, Newton's law formula gives only a part of the force of universal gravitation and does not apply to the universe.

Formula (2), as well as formula (1), gives the same dependence of the form (A), which shows a significant divergence from the curve (B). Newton's law of gravitation is clearly not sufficient to explain the dependence of the form (B). Formulas (1) and (2) “fail to see” a significant fraction of the gravitational force on the scale of the universe. This large difference indicates that for the missing gravitational force there is an unknown law of gravity different from Newton's law. The discovery of this unknown law of gravitation as an addition to Newton's law will make the classical theory of gravitation complete.

No refinements and revisions of Newton's law of gravitation have made it applicable in cosmology. The simple and mathematically perfect formula of Newton's law was not applicable in cosmology. The law of inverse squares and point masses are the main limiting factors in extending

Newton's law to the Universe.Newton's law has limits of applicability. Beyond these limits it is necessary to search for another law of gravitation free from idealized point masses and the law of inverse squares.

The law of inverse squares, was formulated in 1645 by Ismail Bullialdus [5]. The law of inverse squares proved to be very productive for solving the two-body problem. This was shown by Newton's law of gravitation. The same law of inverse squares became an insurmountable obstacle in extending Newton's law of gravitation to the Universe. Obviously, with respect to the Universe there is another yet undiscovered law of gravitation, different from Newton's law of gravitation. To find a new law of gravitational interaction, we use new cosmological equations obtained by means of the law of scaling of large numbers.

4. Systems of algebraic equations of the Universe

Several systems of equations can be formed from the cosmological equations. In Fig . 2 shows four systems of cosmological equations.

$$\begin{cases} G\hbar/r_e^3A_0 = c^1 \\ 1/T_U^2\Lambda = c^2 \\ M_UA_0G = c^4 \\ M_UR_UA_0G/T_U = c^5 \\ M_UR_UA_0^2G = c^6 \end{cases}$$

a)

$$\begin{cases} M_U\Lambda G = A_0 \\ c^5r_e^3/M_UG^2 = \hbar \\ M_UGT_u^2 = R_u^3 \\ GM_U = c^2R_U \\ M_U\Lambda GT_u^2 = R_U \end{cases}$$

b)

$$\begin{cases} \dfrac{c^5r_e^3}{M_UG^2} = \hbar \\ M_U\Lambda cr_e^3 = \hbar \\ \dfrac{R_U\Lambda c^3r_e^3}{G} = \hbar \\ \dfrac{cr_e^3A_0}{G} = \hbar \\ \dfrac{M_Ur_e^3}{R_UT_U} = \hbar \end{cases}$$

c)

$$\begin{cases} \dfrac{M_UG^2m_e}{c^4r_e^2} = \alpha, \\ \dfrac{m_e}{M_U\Lambda r_e^2} = \alpha, \\ \dfrac{Gm_e}{R_U\Lambda c^2r_e^2} = \alpha, \\ \dfrac{Gm_e}{r_e^2A_0} = \alpha, \\ \dfrac{Gm_eT_U}{r_e^2c} = \alpha. \end{cases}$$

d)

Fig. 2. Systems of cosmological equations for calculating the parameters of the observed Universe. Where : α - fine-structure constant, $\hbar$ - Planck constant, M_U - mass of the observable Universe, G - Newtonian constant of gravitation, Λ - cosmological constant, R_U - radius of the observable Universe, A_0 - cosmological acceleration, r_e - classical electron radius; c - speed of light in vacuum; m_e - electron mass, D_0 - large number.

5. Parameters of the Universe

In these systems of cosmological equations (Fig. 2) only the fundamental physical constants **ħ, r_e, G, c, α, m_e** are known quantities. The unknown cosmological parameters **M_U, R_U, Λ, A_0, T_U** are easily found by solving the system of equations. All the given systems of cosmological equations (Fig. 2) give the same values of the Universe parameters (Fig. 3).

$$M_U = 1.15348... \bullet 10^{53}\, kg$$
$$R_U = 0.856594... \bullet 10^{26}\, m$$
$$T_U = 2.85729\, ... \bullet 10^{17}\, s$$
$$\Lambda = 1.36285\, ... \bullet 10^{-52}\, m^{-2}$$
$$A_0 = 10.4922\, ... \bullet 10^{-10}\, m/s^2$$

Fig. 3. Identical values of the Universe parameters obtained from different systems of algebraic equations of the Universe.

The values of the Universe parameters and formulas for their calculation are given in Fig. 4.

$$M_U = m_e \alpha D_0^2 = 1.15348... \bullet 10^{53}\, kg$$
$$R_U = r_e \alpha D_0 = 0.856594... \bullet 10^{26}\, m$$
$$T_U = \frac{r_e \alpha D_0}{c} = 2.85729\, ... \bullet 10^{17}\, s$$
$$\Lambda = \frac{1}{r_e^2 \alpha^2 D_0^2} = 1.36285... \bullet 10^{-52} m^{-2}$$
$$G = \frac{r_e^3}{t_0^2 m_e D_0} = 6.67430\, ... \bullet 10^{-11} kg^{-1} m^3 s^{-2}$$
$$A_0 = \frac{c^2}{r_e \alpha D_0} = 10.4922\, ... \bullet 10^{-10}\, m/s^2$$

Fig. 4. Values of the Universe parameters and formulas for their calculation.

The solution of the systems of algebraic equations of the Universe gives two important parameters: the cosmological constant Λ ($\Lambda = 1.36285... \bullet 10^{-52}$ m^{-2}) and the cosmological acceleration A_0 ($A_0 = 10.4922... \bullet 10^{-10}$ ms^{-2}). The values of these constants are obtained with an accuracy close to that of the Newtonian constant of gravitation G.

6. Cosmological acceleration constant.

One of the solutions of the systems of algebraic equations of the universe is the value of the cosmological acceleration constant as one of the parameters of the universe:

$$\mathbf{A_0 = 10.4922... \bullet 10^{-10}\ ms^{-2}} \qquad (3)$$

This parameter of the universe is unknown in Newton's law of gravitation. For this reason, Newton's theory of gravitation has limitations and does not apply to cosmology. The value of the cosmological acceleration $A_0 = 10.4922... \bullet 10^{-10}$ ms^{-2} is obtained with an accuracy close to the accuracy of the Newtonian constant of gravitation G. The value of the cosmological acceleration $A_0 = 10.4922... \bullet 10^{-10}$ ms^{-2} is close to the critical acceleration proposed in MOND [1].

Using Newton's second law, we obtain the value of the cosmological force F_{cos}, which acts on all bodies in the universe and causes the acceleration A_0.

$$F_{Cos} = mA_0 = m \bullet 10.4922... \bullet 10^{-10}\, N \qquad (4)$$

The cosmological acceleration constant A_0 can be represented by various equivalent formulas, such as:

$$\mathbf{A_0 = GM_U\Lambda = c^2\sqrt{\Lambda} = R_U^3\Lambda/T_U^2 = c^2/r_e\alpha D_0.} \qquad (5)$$

Where: G - Newtonian constant of gravitation, M_U - mass of the observable Universe, Λ - cosmological constant, R_U is the radius of the universe, T_U is the time of the universe, α - fine-structure constant, D_0 - large Weyl number r_e - classical electron radius; c - speed of light in vacuum.

7. Law of cosmological force

The cosmological acceleration constant A_0 allows us to derive equations for the cosmological force. Fig. 5 shows three equivalent formulas for the cosmological force:

$$F_{Cos} = mc^2\sqrt{\Lambda} \qquad (6)$$

$$F_{Cos} = mGM_U\Lambda \qquad (7)$$

$$F_{Cos} = \frac{mR_U^3\Lambda}{T_U^2} \qquad (8)$$

$$F_{Cos} = \frac{mc^2}{r_e\alpha D_0} \qquad (9)$$

FIG. 5. Equivalent formulas for the law of cosmological force. Where: F_{cos} is the cosmological force, G is the Newtonian constant of gravitation, M_U is the mass of the observable Universe, Λ is the cosmological constant, R_U is the radius of the universe, T_U is the time of the universe, α is the fine-

structure constant, D_0 is the large Weyl number, r_e is the classical electron radius; c is the speed of light in vacuum, m is the mass of a body.

Formulas (7) and (8) contains three parameters of the universe. Formulas (6) and (9) includes one parameter of the Universe. Of the four equivalent formulas, equation (6) is the simplest and most mathematically perfect.

$$F_{Cos} = mc^2\sqrt{\Lambda}$$

Fig. 6. Formula of the law of cosmological force.

The law of cosmological force gives a linear dependence of the gravitational interaction force on the body mass m. The coupling constant in the new law of gravitation is the cosmological constant Λ.

The law of cosmological gravitational force of the Universe is represented by a simple formula, which is not inferior in simplicity and perfection to the formula of Newton's law of gravitation. The cosmological constant **Λ** fulfills the role of the coupling constant in the law of cosmological force. This determines the name of the law. The law shows with what force a body interacts with the mass of the Universe distributed in space. The law of cosmological force shows that any body of mass **m** is affected by the cosmological force of the Universe proportional to the mass of the body **m**. The status of the cosmological constant **Λ** in the law of cosmological force is not less significant than the status of the constant **G** in Newton's law.

Combining Newton's law of gravitation and the law of cosmological force give a new Law of universal gravitation. The resulting force of gravitational interaction is defined as a vector sum of two forces: the Newtonian force and the cosmological force.

8. The law of cosmological force law explains the Pioneer anomaly.

The cosmological force for small values of masses is much smaller than the Newtonian force. Therefore, for small values of masses it is not pronounced and can be masked by effects of a non-gravitational nature. The unknown force was first experimentally detected in the Pioneer effect [9, 10, 11]. The Pioneers effect still has no convincing explanation. An attempt has been made to explain the effect by thermal recoil [12].

The new force that follows from the cosmological force law surprisingly turned out to be close to the Pioneer anomaly, which casts doubt on the thermal nature of the Pioneer effect. The significance of the cosmological acceleration that follows from the cosmological force law:

$$A_0 = c^2\sqrt{\Lambda} = 10.4922... \bullet 10^{-10} m/s^2 \qquad (10)$$

The significance of the cosmological force:

$$F_{Cos} = m \bullet (10.4922 \bullet 10^{-10})N \qquad (11)$$

Value of unknown force found in the pioneer effect:

$$F_{Pioneer} = m \bullet (8.74 \pm 1.33) \bullet 10^{-10} N \quad (12)$$

In addition to the Pioneer-10 and Pioneer-11 experiment, there is anomalous acceleration data from Galileo and Ulysses [13 - 16].

The value of the unknown force for Galileo:

$$F_{Galileo} = m \bullet (8 \pm 3) \bullet 10^{-10} N \quad (13)$$

Value of unknown force for Ulysses:

$$F_{Ulysses} = m \bullet (12 \pm 3) \bullet 10^{-10} N \quad (14)$$

The value of the cosmological force F = m(10.4922... × 10^{-10})N is very close to the experimental values F = m((8.74 ± 1.33) x 10^{-10})N, F = m((8±3) x 10^{-10})N, F = m((12±3) x 10^{-10})N. The coincidence of the force values casts doubt on the explanation of the pioneer anomaly by the temperature effect. The law of cosmological force points to the gravitational nature of the Pioneer Anomaly. The gravitational nature of the Pioneer anomaly was also pointed out by Hasmukh K. Tank in a study of critical-acceleration of MOND [17].

9. The theoretical limit of the cosmological force is equal to the Planck force c^4/G.

The study of the equivalent equations of the new law of the cosmological force shows that the value of the cosmological force in the limit is equal to the Planck force:

$$\lim_{m \to M_U} F_{Cos} = \lim_{m \to M_U} mc^2\sqrt{\Lambda} = 1.21027 \bullet 10^{44} N = \frac{c^4}{G} \quad (15)$$

$$\lim_{m \to M_U} mGM_U\Lambda = 1.21027 \bullet 10^{44} N = \frac{c^4}{G} \quad (16)$$

$$\lim_{m \to M_U} \frac{mR_U^3\Lambda}{T_U^2} = 1.21027 \bullet 10^{44} N = \frac{c^4}{G} \quad (17)$$

$$\lim_{m \to M_U} \frac{mc^2}{r_e\alpha D_0} = 1.21027 \bullet 10^{44} N = \frac{c^4}{G} \quad (18)$$

The theoretical limit of the cosmological force at m → M_U reaches the enormous value c^4/G = 1.21027 x 10^{44} N.

10. The law of cosmological force explains Galaxy rotation curve

Newton's law does not reveal the presence of cosmological force of gravitational interaction of bodies with the mass of the Universe. The coupling constant in Newton's law of gravitation is the constant G. The cosmological force of gravitational interaction is an additional force to the Newtonian force of gravitation of two bodies. The additional cosmological force is represented by a different law of gravitation from Newton's law. The law of cosmological force (Fig. 6) is represented using the cosmological constant Λ. The cosmological force has a linear dependence on the mass of the body (Fig.

7) and does not obey the law of inverse squares. Fig. 7 conventionally shows the contribution of the cosmological force to the Galaxy rotation curve.

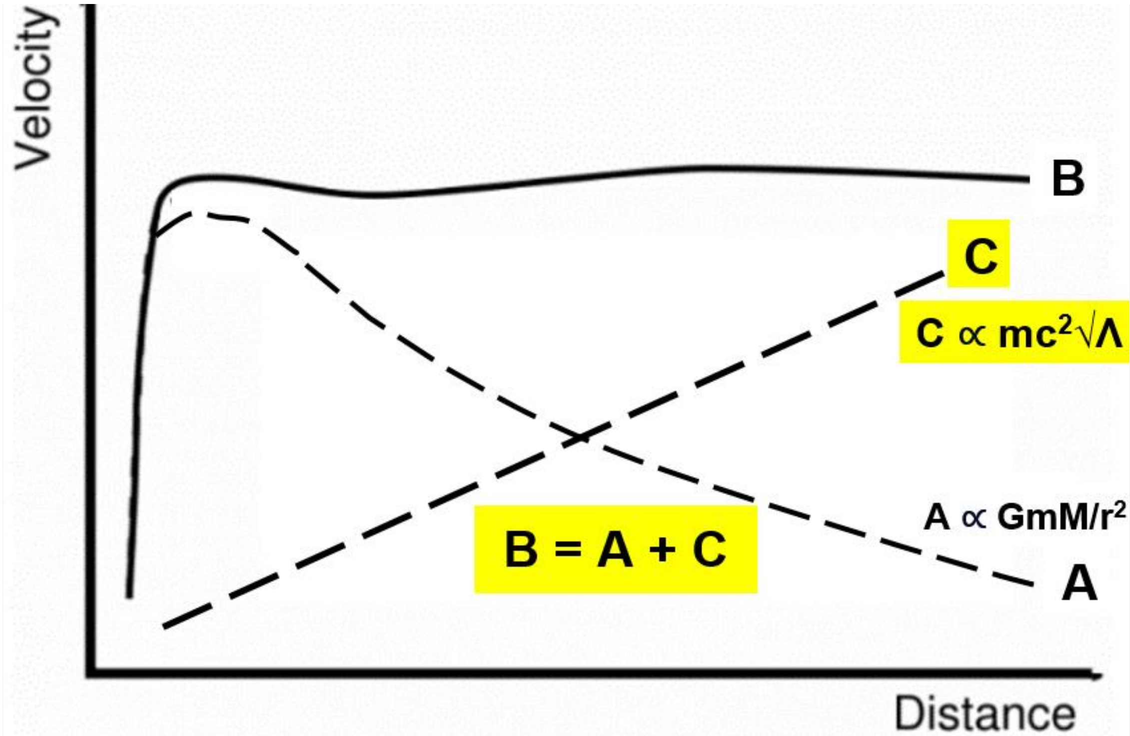

Fig. 7. Galaxy rotation curve (B) as a result of two forces: the contribution of the Newtonian force (A) and the contribution of the cosmological force (C).

On small scales, the additional cosmological force is much smaller than the Newtonian force. On the scale of the Universe, the cosmological force is enormous. For large distances it exceeds the Newtonian force and has a theoretical limit equal to the Planck force F_P = c^4/G = 1.21027•10^44 N. The total force of universal gravitation, which acts on a body, consists of two forces. This is the Newtonian force of gravitational interaction of two bodies and the additional cosmological force of gravitational interaction of a body with the mass of the Universe. Accordingly, the Galaxy rotation curve (B) (Fig. 7) is represented by the sum B = A + C. Combining the two laws of forces F_N and F_{Cos} gives the law of universal gravitation. At small distances, the main share of the force of universal gravitation is represented by the Newtonian force F_N. At large distances, the cosmological force F_{Cos} represents the major fraction of the universal gravitational force. As a result of the two forces, the velocity in the graph (Fig. 7) is represented by curve (B).

The assumption about the influence of the rest of the visible Universe on the rotation curves of galaxies was first made by Philip D. Mannheim [18].

11. A new mathematical formula for the law of universal gravitation.

The law of cosmological force allows to present the law of universal gravitation in a new form. The law of universal gravitation should include not only the Newtonian force of interaction between two bodies, but also an additional cosmological force.

Fig. 8 shows the equivalent formulas of the law of universal gravitation.

$$\begin{cases} F_N = G\dfrac{Mm}{r^2} \\ F_{Cos} = mc^2\sqrt{\Lambda} \end{cases} \quad \begin{cases} F_N = G\dfrac{Mm}{r^2} \\ F_{Cos} = mGM_U\Lambda \end{cases} \quad \begin{cases} F_K = \dfrac{mR^3}{T^2 r^2} \\ F_{Cos} = \dfrac{mR_U^3\Lambda}{T_U^2} \end{cases} \quad \begin{cases} F_N = G\dfrac{Mm}{r^2} \\ F_{Cos} = \dfrac{mc^2}{r_e \alpha D_0} \end{cases}$$

Fig. 8. Equivalent formulas of the law of universal gravitation.

The most simple and mathematically perfect is the following form of the Law of universal gravitation:

$$\begin{cases} F_N = G\dfrac{Mm}{r^2} \\ F_{Cos} = mc^2\sqrt{\Lambda} \end{cases}$$

Fig. 9. Law of universal gravitation.

The law of universal gravitation contains two components. The first formula in the law of universal gravitation is Newton's well-known law of gravitation, and the second formula is the new law of cosmological force. Together the two laws of gravitation give the law of universal gravitation. The coupling constants in the law of universal gravitation are two constants: the gravitational constant G and the cosmological constant Λ.

The contribution of the Newtonian force and the cosmological force to the total gravitational force depends on the distance and mass of the interacting bodies. For small masses and small distances, Newton's law of gravitation makes a significant contribution. The fraction of the cosmological force for small masses is much smaller than the Newtonian force. The reason is the small value of the cosmological constant Λ ($\Lambda = 1.36285...\text{x } 10^{-52}$ m^{-2}). As the mass of the interacting bodies increases and the distance increases, the Newton force's fraction of the law of universal gravitation decreases and the cosmological force's fraction increases. The total gravitational force acting on a body is a vector sum of two forces: the Newton force and the cosmological gravitational force of the Universe.

In the framework of Augmented Newtonian dynamics (AuND), the law of universal gravitation is presented in a new form (Fig. 10):

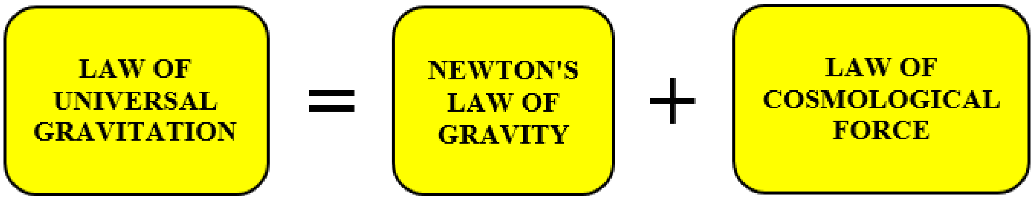

Fig. 10. Law of universal gravitation = Newton's law of gravity + law of cosmological force.

The force of universal gravitation is represented by the vector sum of two forces: the Newtonian gravitational force and the cosmological gravitational force.

$$\vec{F}_U = \vec{F}_N + \vec{F}_{Cos} \qquad (19)$$

12. Fraction of the cosmological force F_{Cos} = (mc^2)√Λ in the extended formula of the law of universal gravitation

The fraction of the cosmological force F_{Cos} in the extended formula of the law of universal gravitation depends on the distance and mass of interacting bodies. On small scales, the additional cosmological force F_{Cos} is much smaller than the Newtonian force. For example, on the Earth, F_{Cos} is 109 times smaller than the Newtonian force. At small distances, the Newtonian force F_N makes up the main part of the force of universal gravitation. With increasing mass of interacting bodies and distance, the share of Newton's force in the law of universal gravitation decreases, and the share of the cosmological force increases. On the scale of galaxies, the cosmological force Fcos becomes commensurate with the Newtonian force. On the scale of the Universe, the cosmological force is enormous. At large distances it exceeds the Newtonian force and has a theoretical limit equal to the Planck force F_P = c^4/G = 1.21027 - 10^44 N. At large distances, the cosmological force F_{Cos} makes up the bulk of the force of universal gravitation.

13. The condition of equality of the Newtonian force and the cosmological force.

At the value of M/r2 = 15.720.... kg/m2

$$\mathbf{M/r^2 = (c^2\sqrt{\Lambda})/G = 15.720...\ kg/m^2} \qquad (20)$$

the cosmological force (F_{Cos} = (mc^2)√Λ) is equal to the Newtonian force (F_N = GMm/r^2). The same constant (15.7202... kg/m2), but with much greater accuracy, is given by the following combination of electron constants:

$$\mathbf{m_e/\alpha r_e^2 = 15{,}7202729...\ kg/m^2} \qquad (21)$$

Again we see how closely related are the parameters of the universe and the electron.

14. Augmented Newtonian dynamics (AuND) extends the possibilities of the classical model of gravitation.

Augmented Newtonian dynamics (AuND) opens new possibilities for the classical model of gravitation.Augmented Newtonian dynamics (AuND) is represented by two laws of gravitational interaction force: Newton's law of gravity and the cosmological force law. As a result, Augmented Newtonian dynamics (AuND) becomes applicable to the universe. AuND provides a solution to the following problems in astrophysics and cosmology:

- gives a new mathematical formula for the law of universal gravitation;
- explains the rotation curves of galaxies;
- explains the Pioneer Anomaly;
- solves the problem of dark matter;

- solves the problem of the cosmological constant Λ ($\Lambda = 1.36285... \bullet 10^{-52}\ m^{-2}$);
- gives the value of the cosmological acceleration A_0 ($A_0 = 10.4922... \bullet 10^{-10}\ ms^{-2}$).

15. Conclusion

MOND assumes that Newtonian dynamics is not accurate. Therefore, it needs modification. AuND assumes that Newtonian dynamics is accurate but not complete. Therefore it needs a new law of gravitational force that will complete Newtonian dynamics and make it complete. This leads directly to the new law of universal gravitation. The extended law of universal gravitation adequately describes gravitational interaction on both small scales and the scale of the universe.

The correctness of Newton's formula for the gravitational interaction between two bodies is beyond doubt. At the same time, the force of gravitational interaction between two bodies is only a part of the total force of gravitational interaction. The rest of the total force is the force of gravitational interaction of bodies with the mass of the Universe. This component of the law of universal gravitation Newton's formula "does not see". The force not accounted for by Newton's law is a significant part of the gravitational force.

The discrepancy between the theoretical and observed values is clearly demonstrated by the Galaxy rotation curve.The reason for the discrepancy lies not in the imperfection of Newton's gravitational formula, but in the incompleteness of Newtonian dynamics.Accordingly, the law of universal gravitation is expressed by a more complex mathematical model.Newton's formula is only a part of the law of universal gravitation.Therefore, to call Newton's law of gravitation the law of universal gravitation is an exaggeration.The law of universal gravitation has a different mathematical formula. It includes two laws: Newton's law for the gravitational interaction of two bodies and the law of the cosmological force of the universe.Augmented Newtonian Dynamics (AuND) includes traditional Newtonian dynamics and a new mathematical formula for the law of universal gravitation.

16. Conclusions

1. For more than 300 years, Newtonian dynamics as a mathematical model of motion remained incomplete.Newton's formula of the law of gravitation gives the force of interaction between two bodies.Newton's formula "does not see" the additional cosmological force of gravitational interaction of bodies with the mass of the Universe.In addition to the forces described by Newtonian dynamics, there is a gravitational force in the Universe, which is observed in experiments, but does not follow from Newton's law of gravitation.

2. There is no reason to doubt the correctness of Newton's law of gravity. There is no reason to regard Newton's law as an approximate mathematical model. It is an exact and mathematically perfect law of Nature. The limits of its applicability extend to the gravitational interaction of two bodies. It is not applicable to the universe. For the Universe the Augmented Newtonian Dynamics (AuND) is applicable, in which Newton's law is supplemented by the law of cosmological force.

3. Instead of modification of Newton's law of gravitational interaction, the Augmented Newtonian dynamics (AuND) is proposed, in which Newton's law (F_N = GMm/r^2) is kept unchanged. Newton's law is supplemented with a new law of cosmological force (F_{Cos} = (mc^2)√Λ). The cosmological force acts on all bodies in the Universe. It depends linearly on the mass of the body and does not obey the law

of inverse squares. The cosmological force is an additional force to the Newtonian force of gravitational interaction between two bodies.

4. A new mathematical formula of the law of universal gravitation is given.The law of universal gravitation is represented by two equations - Newton's law F_N = GMm/r^2 and the law of cosmological force F_{Cos} = (mc^2)√Λ. The extended law of universal gravitation adequately describes the gravitational interaction both on small scales and on the scale of the Universe.

5. It is shown that classical gravitation using Augmented Newtonian dynamics (AuND) has a high heuristic potential. Its high heuristic potential has not been realized due to the incompleteness of Newtonian dynamics.

6. Augmented Newtonian dynamics (AuND) provides a solution to the following problems in astrophysics and cosmology:

- gives a new mathematical formula for the law of universal gravitation;- explains the rotation curves of galaxies;
- explains the Pioneer Anomaly;
- solves the problem of dark matter;
- solves the problem of the cosmological constant Λ ($\Lambda = 1.36285... \cdot 10^{-52}$ m^{-2});
- gives the value of the cosmological acceleration A_0 ($A_0 = 10.4922... \cdot 10^{-10}$ ms-2).

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