# ZITTERBEWEGUNG
# AND THE WHEELER-FEYNMAN TIME-SYMMETRIC THEORY

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**Abstract:** A discussion of Zitterbewegung for the W.F.E.M.F.V.P.T.S.T. model, suggesting Zitterbewegung should not exist for real on-shell matter and might exist for the off-shell virtual particles–giving a test for the W.F.E.M.F.V.P.T.S.T..

## §1 W.F.E.M.F.V.P.T.S.T.

§1.1 In a previous paper *A Dynamical Theory of the Electromagnetic Potential*[1][2] I suggested fermions can be modelled as longitudinal electromagnetic scalar potential waves. This required constructing the scalar potential across the Wheeler-Feynman time-symmetric theory[3][4] with energy and momentum constraints leading to an electromagnetic wave that evolves longitudinally along the axis of time, this method is similar to Maxwell’s transverse electromagnetic wave for light[5][6]. It results in a particle model that has an exact value for the electron’s reduced Compton wavelength that matches the wavelength of an electron in $\mathbb{R}^{1,3}$, also a perfectly spherical charged electron in $\mathbb{R}^3$, travels at the classical velocity for electrons, and has an *on-shell mass*.

This was possible by rewriting the Wheeler-Feynman time-symmetric model from its classical **E** notation with its retarded (ret) and advanced (adv) fields,

$$\mathbf{E}_{total} + \mathbf{E}_{free} = \sum_n \tfrac{1}{2}\left(\mathbf{E}_n^{ret} + \mathbf{E}_n^{adv}\right) + \sum_n \tfrac{1}{2}\left(\mathbf{E}_n^{ret} - \mathbf{E}_n^{adv}\right) = \sum_n \mathbf{E}_n^{ret} \qquad (1)$$

to the electromagnetic four-vector form $A_\mu^{total}$ with its ret and adv potentials (note the $A_\mu^{adv}$ and $A_\mu^{ret}$ are *virtual* potentials arising from the quantum vacuum),

$$A_{\mu}^{total} = \sum_n \frac{1}{2}\left(A_{\mu,n}^{ret} + A_{\mu,n}^{adv}\right)_{total} + \sum_n \frac{1}{2}\left(A_{\mu,n}^{ret} - A_{\mu,n}^{adv}\right)_{free} \tag{2}$$

We can re-express this as a wavefunction with epoch angles $\varepsilon$, and where the ½ appears as a result of splitting the $A_\mu$ into two parts along the axis of Time into the anterograde direction and the retrograde direction.

$$\psi = \exp(-\tfrac{iq}{2\hbar}\int A_\mu dx^\mu + \varepsilon) \tag{3}$$

This turns the Wheeler-Feynman summation into a total wavefunction

$$\psi_{total} = \exp(-\tfrac{iq}{2\hbar}\int A_\mu^{\mathrm{ret}}\mathrm{dx}^\mu - \varepsilon_{\mathrm{ret}}) * \exp(-\tfrac{iq}{2\hbar}\int A_\mu^{\mathrm{adv}}\mathrm{dx}^\mu + \varepsilon_{\mathrm{adv}}) \tag{4}$$

To ensure constant phase the difference of the epoch angles $\varepsilon$ is zero and is referred to as the interference terms,

$$\Delta\varepsilon = \varepsilon_{adv} - \varepsilon_{ret} = 0 \tag{5}$$

As a *mathematical trick* we can without loss of generality equate the interference terms to the sum of the advanced and retarded potentials of the Wheeler-Feynman free terms since they also sum to zero,

$$\varepsilon_{\mathrm{ret}} = -\tfrac{iq}{2\hbar}\int A_\mu^{\mathrm{adv}}\mathrm{dx}^\mu \quad \textit{and} \quad \varepsilon_{\mathrm{adv}} = -\tfrac{iq}{2\hbar}\int A_\mu^{\mathrm{ret}}\mathrm{dx}^\mu \tag{6}$$

The total wavefunction now becomes,

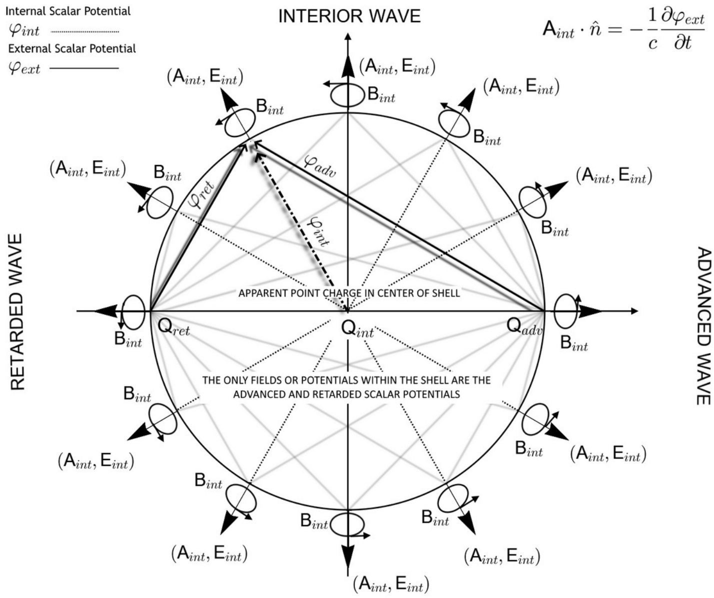
$$\psi_{total} = \exp(-\tfrac{iq}{2\hbar}\int A_\mu^{\mathrm{ret}} - A_\mu^{\mathrm{adv}}\mathrm{dx}^\mu) * \exp(-\tfrac{iq}{2\hbar}\int A_\mu^{\mathrm{adv}} + A_\mu^{\mathrm{ret}}\,\mathrm{d\,x}^\mu) \tag{7}$$

$$\psi_{total} = \exp(-\tfrac{iq}{2\hbar}\int A_\mu^{\mathrm{ret}} - A_\mu^{\mathrm{adv}} + A_\mu^{\mathrm{adv}} + A_\mu^{\mathrm{ret}}\,\mathrm{dx}^\mu) \tag{8}$$

After simplifying the ½ vanishes yielding a unitary total wavefunction $\Psi_{total}$

$$\Psi_{total} = \exp(-\tfrac{iq}{\hbar}\int A_\mu^{\mathrm{ret}}\,\mathrm{dx}^\mu) \tag{9}$$

This can be shown[1] to model longitudinal electromagnetic scalar potential waves resulting in a charged hollow spherical particle with combined wavefunction and particle, reduced Compton wavelength and was labelled as the Wheeler-Feynman Electromagnetic Four-Vector Potential Time-Symmetric Theory or the W.F.E.M.F.V.P.T.S.T..




IN THIS MODEL AN APPARENT POINT charge $Q_{INT}$ APPEARS IN THE CENTER OF THE SHELL

§1.2 Furthermore in another previous paper "The Arrow of Time and the Wheeler-Feynman Time-Symmetric Theory"(7) *that as the TCP theorem reverses the direction of Time it also inverts the components of the four-potentials,*(8)(9)

$$A_\mu^{total} = \tfrac{1}{2}[(-A_\mu^{\text{ret}} + A_\mu^{\text{adv}}) + (-A_\mu^{\text{adv}} - A_\mu^{\text{ret}})] = -A_\mu^{\text{ret}} \qquad (10)$$

*"This result is also consistent with the positron model as being the negative energy mode of an electron moving forward in Time, note the absence of a negative sign in the exponential requires this as antimatter,"*

$$\Psi_{total}(antimatter) = \exp\left(\tfrac{iq}{\hbar}\int A_\mu^{\text{ret}}\,\mathrm{d}x^\mu\right) \qquad (11)$$

We can simplify the above process by reiterating the identities from the previous paper [1],

$$
\begin{aligned}
|\psi_{\text{ret}}| &= |\psi_{\text{adv}}| \\
\psi_{\text{ret}} &= \psi^*_{\text{adv}} \\
\psi_{\text{adv}} &= \psi^*_{\text{ret}} \\
\psi_{\text{ret}} \cdot \psi^*_{\text{ret}} &= I \\
\psi_{\text{total}} &= \psi_{\text{ret}} \cdot \psi_{\text{adv}} \\
\psi_{\text{interference}} &= \psi_{\text{ret}} \cdot \psi^*_{\text{adv}} = I
\end{aligned}
\tag{12}
$$

This is the W.F.E.M.F.V.P.T.S.T. in its most elegant form,

$$
\begin{aligned}
\Psi_{\text{total}} &= \psi_{\text{total}} \cdot \psi_{\text{interference}} \\
&= \psi_{\text{ret}} \cdot \psi_{\text{adv}} \cdot \psi_{\text{ret}} \cdot \psi^*_{\text{adv}} \\
&= \psi_{\text{ret}} \cdot \psi^*_{\text{ret}} \cdot \psi_{\text{ret}} \cdot \psi^{**}_{\text{ret}} \\
&= \psi_{\text{ret}} \cdot \psi^*_{\text{ret}} \cdot \psi_{\text{ret}} \cdot \psi_{\text{ret}} \\
&= \psi_{\text{ret}} \cdot \psi_{\text{ret}} \\
&= \Psi_{\text{ret}}
\end{aligned}
\tag{13}
$$

This is only possible if and only if $\psi_{ret} = \psi^*_{adv}$, and if and only if the magnitudes $|\psi_{\text{ret}}| = |\psi_{\text{adv}}|$ are identical, then and only then does this formulation work for wavefunctions. If $\psi_{\text{ret}} \cdot \psi^*_{\text{adv}} \neq I$ then not only do the equations not balance but the resultant particles are virtual and intrinsically off-shell.

The crucial result is that the $\Psi_{\text{adv}}$ explicitly vanishes even for antimatter, as reversing the direction of Time also inverts both the charge and the parity, so regardless of the species of particle the model must conform to the Wheeler-Feynman summation of excluding the advanced potentials.

*It is the zero interference of the advanced and retarded potentials that imposes the Arrow of Time on the wavefunctions regardless of the species of the particles.*

It becomes apparent that using the Wheeler-Feynman summation which is solely based on the $A^{adv}_{\mu}$ potential it is impossible to construct the wavefunction for a fermion

$$\Psi_{total} \neq \exp(\tfrac{iq}{\hbar} \int A_{\mu}^{\mathrm{adv}}\, \mathrm{dx}^{\mu}) \tag{14}$$

The resultant Wheeler-Feynman summation of the virtual potentials is always a real $A_{\mu}^{ret}$ potential and the $A_{\mu}^{adv}$ potential always vanishes, it is as if the summation swallows up the virtual potentials and spits out a real particle,

$$\Psi_{total} = \exp(-\tfrac{iq}{\hbar} \int A_{\mu}^{\mathrm{ret}}\, \mathrm{dx}^{\mu}) \tag{15}$$

Necessarily the only direction in Time that real on-mass matter can travel is always in the forward direction of Time thereby giving a possible explanation for both the Problem of the Arrow of Time and the Principle of Causality.

We can now take the predictions of §1.1 and §1.2 and apply them to the problem of Zitterbewegung.

## §2 ZITTERBEWEGUNG

As conceived by Schrödinger the Zitterbewegung(8) is the interference of the positive and negative energy states of an electron, as an oscillation term $e^{\frac{-2iHt}{\hbar}}$ in the time-dependent Dirac equation of a free particle with Hamiltonian $H$ for a time-dependent position operator $x_k(t)$,

$$\mathrm{x}_k(t) = x_k(0) + c^2 p_k H^{-1} t + \tfrac{1}{2} i\hbar c H^{-1}(\alpha_k(0) - c p_k H^{-1})(e^{\frac{-2iHt}{\hbar}} - 1) \tag{16}$$

For the W.F.E.M.F.V.P.T.S.T. model, we can identify the retarded and advanced potentials with the virtual positive and negative charged energy states of the electron, and the free term of the Wheeler-Feynman model corresponding to a zero phase difference between the negative and positive energy states, giving the Wheeler-Feynman summation as a total wavefunction as,

$$\psi_{total} = \exp(-\tfrac{iq}{2\hbar} \int A_{\mu}^{\mathrm{ret}} \mathrm{dx}^{\mu} - \varepsilon_{\mathrm{ret}}) * \exp(-\tfrac{iq}{2\hbar} \int A_{\mu}^{\mathrm{adv}} \mathrm{dx}^{\mu} + \varepsilon_{\mathrm{adv}}) \tag{17}$$

where the interference of the free terms,

$$\Delta\varepsilon = \varepsilon_{adv} - \varepsilon_{ret} = 0 \tag{18}$$

Since the interference of the negative and positive energy states is by definition zero the $e^{\frac{-2iHt}{\hbar}}$ term vanishes contradicting the argument that "*the Zitterbewegung is caused by the interference between the positive and negative energy compounds of a wave packet.*"[8], therefore there must be a zero Zitterbewegung in the W.F.E.M.F.V.P.T.S.T. model and should not exist.

(It is not suggested this model also applies to quasiparticles in macroscopic analogue states as researched by Whittaker et al[11] the absence of the Zitterbewegung is only for free relativistic particles.)

We can strengthen this argument by considering section §1.2 which suggested that *only real anterograde wavefunctions* can result under the Wheeler-Feynman summation as,

$$\Psi_{total} \neq \exp(\tfrac{iq}{\hbar} \int A_{\mu}^{\mathrm{adv}}\, \mathrm{dx}^{\mu}) \tag{19}$$

$$\Psi_{total} = \exp(-\tfrac{iq}{\hbar} \int A_{\mu}^{\mathrm{ret}}\, \mathrm{dx}^{\mu}) \tag{20}$$

Since the Zitterbewegung *vanishes* ***if*** *wave packets with exclusively positive or negative energy are considered*[12], and since *only real anterograde wavefunctions can result under the W.F.E.M.F.V.P.T.S.T.* it is predicted that for free relativistic particles the Zitterbewegung will not observed. An additional test is given by noting that conversely off-shell particles are by definition $\Delta\varepsilon = \varepsilon_{adv} - \varepsilon_{ret} \neq 0$ and necessarily should have an interference term and must have a Zitterbewegung term.

This suggests a crucial experimental test of the W.F.E.M.F.V.P.T.S.T. for if the *absence* of the Zitterbewegung could be demonstrated for on-shell matter–just as its *presence* could be demonstrated for off-shell matter–this would offer a definitive confirmation of the W.F.E.M.F.V.P.T.S.T. model.

In effect, the Wheeler-Feynman summation swallows up the virtual past and future, spits out the present as on-shell matter and the Zitterbewegung vanishes as Schrödinger's *Nebelgeist* — a ghost in the quantum mist.

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