Maxwell's Equations and Three Fundamental Axioms of Electromagnetism

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[Abstract] Based on the axiomatization thought of science proposed by Euclid, this study summarizes hundreds of electromagnetic theorems and formulas in the field of technical application into three fundamental axioms of electromagnetism: Coulomb's law, Lorentz's law of magnetic field generation and Lorentz's law of magnetic field force. Through the comparative analysis of the above three axioms of electromagnetism and Maxwell's equations, and it is revealed that the four equations contained in the Maxwell's equations are all wrong.

1. Introduction

Based on first principles, that is, classical Newtonian physics, Iron Man Musk broke through the existing rocket theory and technology, and achieved the recyclable rocket SpaceX for the first time in human history, which was a great success in technology and commerce.

In 300 B.C., the ancient Greek mathematician Euclid proposed the axiomatization thought: Select as few as possible a set of axioms that are self-evident without proof and logically unfalsifiable, and take this as a starting point to establish a complete theoretical deduction system by using the rules of pure logical reasoning. Based on five major postulates and five axioms, 《Euclid's Elements》deduced more than 460 propositions or theorems from simple to complex, and established the first complete axiom deduction system in human history. 《Euclid's Elements》is the source of mathematics as well as science. "Here, the emperor has no privileges". In the field of science, the only thing that needs to be followed is the axiomatization of science.

Physics is essentially an experimental science, a fundamental science that is the closest to technological applications and industrial development. The axioms of physics must be verified by rigorous experiments and observations, or the fundamental laws of nature that cannot be logically falsified.

Since the development of electromagnetism, hundreds of theorems and formulas have been formed. In theoretical physics, Maxwell's equations are the cornerstone of electromagnetism, and the four equations contained in it can be regarded as the four axioms of electromagnetism. Below are Maxwell's equations for differential and integral forms.

$\nabla \cdot E = \frac{\rho}{\varepsilon_0}$	(1-1A)	$\oiint_S E \cdot ds = \frac{Q}{\varepsilon_0}$	(1-1B)
$\nabla \cdot B = 0$	(1-2A)	$\oiint_S B \cdot ds = 0$	(1-2B)
$\nabla \times E = -\frac{\partial B}{\partial t}$	(1-3A)	$\oint_L E \cdot d\ell = -\frac{d\Phi_B}{dt}$	(1-3B)
$\nabla \times B = \mu_0 (J + \varepsilon_0 \frac{\partial E}{\partial t})$	(1-4A)	$\oint_L B \cdot d\ell = \mu_0 I + \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}$	(1-4B)
Differential form		Integral form	

Maxwell's equations of differential form have a set of systematic mathematical algorithms for point product, cross product, divergence and curl, which are convenient for the derivation of theoretical formulas. Theoretical physicists prefer to use the differential form of Maxwell's equations. The integral form of Maxwell's equations has a more intuitive physical concept, which is illustrated below by the integral form of Maxwell's equations.

Eq. (1-1B) is Gauss's law for an electric field. It states that the electric flux passing through a certain closed surface is proportional to the amount of charge Q enclosed by the closed surface, and the coefficient is $1/\varepsilon_0$, where the vacuum dielectric constant ε_0 = 8.854 x 10-12 F/m. The electric field is the active field.

Eq. (1-2B) is Gauss's law for a magnetic field. It states that the magnetic flux passing through a certain closed surface must be equal to zero. Since there is no magnetic monopole in nature, the N pole and the S pole cannot be separated; that is, the magnetic field is a passive field.

Eq. (1-3B) is Faraday’s law of electromagnetic induction. The law states that a magnetic field induces an electric field; that is, the induced electromotive force in a closed coil is proportional to the change rate of the magnetic flux passing through the cross-section of the coil, and the coefficient is −1.

Eq. (1-4B) is the Ampere-Maxwell law. According to Ampere’s circuital law, the line integral of the magnetic induction intensity B along a closed curve L is equal to μ_0 multiplied by the current passing through the closed curve L, where the vacuum permeability μ_0 =1.257 x 10^{-6} H/m.

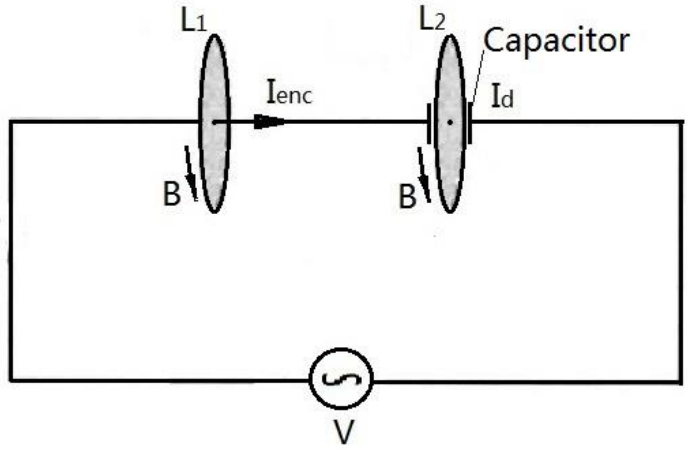

Figure 1.1 Schematic representation of the Ampere-Maxwell law

Figure 1.1 shows a simple circuit containing a capacitor. A conducting current $\mathbf{I}_{enc}$ flows through the cross-section of the closed curve L_1; hence,

$$\oint_{L1} B \cdot d\ell = \mu_0 \mathrm{I}_{enc}$$

The cross-section of the closed curve L_2 is between two plates of the capacitor, and there is no conducting current I_{enc} passing through, but there is an electric field and electric flux between the two plates. Therefore, Maxwell introduced the "displacement current" hypothesis in 1865 and defined the "displacement current" I_d as

$$\mathrm{I}_d = \varepsilon_0 \frac{d\Phi_E}{dt}$$

There is a "displacement current" I_d passing through the cross-section of the closed curve L_2. Then,

$$\oint_{L2} B \cdot d\ell = \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}$$

Maxwell introduced the "displacement current" hypothesis and extended Ampere's circuital law to the full current law, that is, Ampere-Maxwell law.

$$\oint_{L} B \cdot d\ell = \mu_0 \mathrm{I}_{enc} + \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt} \qquad (1\text{-}4B)$$

Ampere-Maxwell law reveals that a magnetic field can be induced by a "displacement current," that is, by a changing electric field.

According to the Ampere-Maxwell law in Equation (1-4B), a changing electric field induces a magnetic field. According to Faraday's law of electromagnetic induction in Equation (1-3B), a changing magnetic field induces an electric field. The electric field and the magnetic field are closely linked and induce each other to form unified electromagnetic waves, as shown in Figure 1.2.

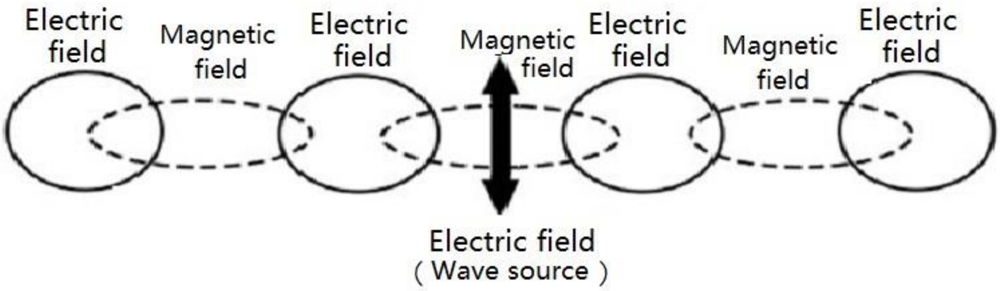

Figure 1.2 Schematic of electromagnetic waves

2. Three fundamental axioms of electromagnetism

Theoretical physicists like to use Maxwell's equations to study electromagnetism. Technical application specialists from the field of electromagnetism are more likely to use engineering approximate formulas, and hundreds of electromagnetism theorems and formulas can be summarized into three fundamental axioms of electromagnetism.

2.1 Coulomb's law

Coulomb's law is the law of the interaction forces between stationary point charges. Due to the superposition of the point charge forces, Coulomb's law can be extended to the point charges with varying amounts of charge. Consider a point charge $q(t_1)$ in space, and take a point A at a distance of $r(t_1)$ from the charge $q(t_1)$. At point A, the electric field intensity $\mathbf{E}_A(t)$ is:

$$\mathbf{E}_A(t) = k\frac{q_{(t_1)}}{r_{(t_1)}{}^2}\mathbf{r} \qquad (2\text{-}1)$$

Eq. (2-1) is Coulomb's law in the form of an electric field, where k is called the electrostatic force constant, with $k=9.0\times10^9$ Nm^2/c^2 in a vacuum, and **r** is the unit vector of $r(t_1)$. The speed of the electric field is equal to the light speed c, so $t_1 = t - r(t_1) / c$.

The charge $q(t_1)$ and the distance $r(t_1)$ can change with time, so the electric field intensity $\mathbf{E}_A(t)$ also changes with time.

The electric field, electric field strength, electric flux, electric potential, and electric potential difference are the results of logical reasoning and deduction of Coulomb's law.

In Maxwell's equations, the Gauss's law of an electric field in Eq. (1-1B) corresponds to Coulomb's law. Considering that the propagation speed of the electric field is 2.998×10^8 m/s, the Gaussian law of the electric field in Eq. (1-1B) is not entirely correct. As shown in Figure 2.1, assuming that there is a little charge q(t) in space, select a closed surface S as a sphere with a diameter D = 2.998 x 10^8 m, and the point charge q(t) is within the sphere S and close to point A on one side of the sphere S, then the distance between point B on the other side of the sphere S and the point charge q(t) is near 2.998 x 10^8 m.

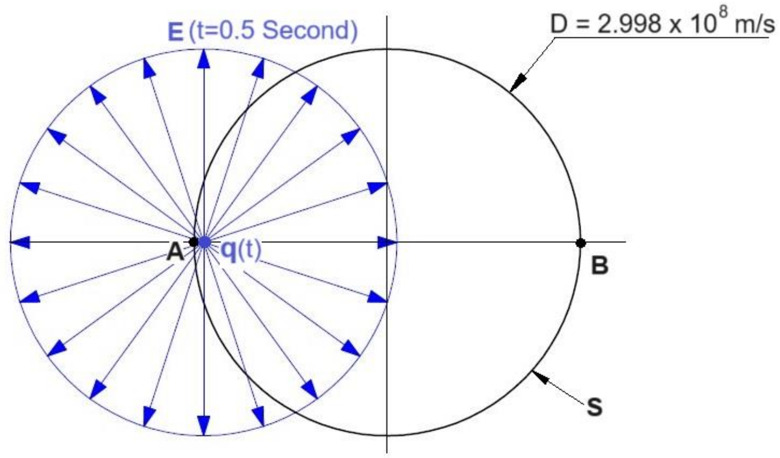

Figure 2.1 Gaussian law of an electric field

When time t < 0, let the point charge q(t) = 0; and when t > = 0, let the point charge q(t) = q_0. At time t=0, the point charge q_0 starts to generate an electric field, which can instantaneously reach point A on the side of the sphere S, and it takes near 1 second to reach point B on the other side of the sphere. As shown in Figure 2.1, at time t = 0.5 second, the electric field E generated by the point charge q_0 passes through the closed sphere S on the side of point A, but the electric field E is far from reaching the closed sphere S on the side of point B, so Gaussian law of an electric field in Eq. (1-1B) does not hold true at t = 0.5 second. When t > 1 second, the electric field E generated by the point charge q_0 reaches the closed spherical surface S on the side of point B, and the electric field E completely passes through the entire closed spherical surface S, then Gaussian law of an electric field in Eq. (1-1B) is established. From the above analysis, within a changing charge and electric field, Gaussian law of an electric field in Eq. (1-1B) does not hold true.

2.2 Lorentz's Law of Magnetic Field Generation – Moving charges produce magnetic fields

The moving charges produce magnetic fields and the moving charges are the only direct cause for the generation of magnetic fields.

In a vacuum, it is assumed that the velocity of a moving charge $q(t_1)$ is $\mathbf{v}(t_1)$, and the distance between the point A and the charge $q(t_1)$ is $r(t_1)$. The moving charge $q(t_1)$ produces a magnetic induction intensity $\mathbf{B}$(t) at point A:

$$\mathbf{B}(t) = \frac{\mu_0}{4\pi}\frac{q(t_1)\,\mathbf{v}(t_1) \times \mathbf{r}}{r(t_1)^2} \qquad (2\text{-}2)$$

Where **r** is the unit vector of $r(t_1)$, and the direction of **B**(t) is perpendicular to the plane determined by **v**(t_1) and **r**.

Lorentz's law of magnetic field generation is the microscopic physical nature of the magnetic field generation. The Ampere circuital law and the Biot-Savar law are the results of logical reasoning and deduction of Lorentz's law of magnetic field generation.

Since there is no magnetic monopole in nature, and moving charges produce magnetic fields, the N pole and the S pole must be generated at the same time and cannot be separated. In Maxwell's equations, Gauss's law of a magnetic field in Eq. (1-2B) states the above properties of the magnetic field. Similar to Gauss's law of an electric field in Eq. (1-1B), considering that the propagation speed of the magnetic field is 2.998×10^8 m/s, Gaussian law of a magnetic field in Eq. (1-2B) also does not hold true in a changing magnetic field.

In Maxwell's equations, Eq. (1-4B) is an extension of Ampere circuital law. Maxwell introduced the "displacement current" hypothesis in 1865, predicting that a changing electric field could induce a magnetic field. On the one hand, the "displacement current" hypothesis has not been proved by any experiments so far: Hertz's experiments proved that it is the electric field waves, not the "electromagnetic waves" that achieve wireless communication. On the other hand, the "displacement current" hypothesis is not self-consistent in theory: In an alternating electric field with uniform electric field intensity, the calculated magnetic induction intensity is proportional to the size of the selected point according to the definition of "displacement current" hypothesis in Eq. (1-4B), and when the radius of the selected point approaches zero, the calculated magnetic induction intensity also approaches zero. Therefore, the "displacement current" hypothesis does not hold true, and in Maxwell's equations, Eq. (1-4B) must be wrong.

2.3 Lorentz's Law of magnetic field force – Moving charges in the magnetic field are affected by Lorentz's magnetic field force

In a vacuum, a charge q(t) with velocity **v**(t) is moving in a magnetic field **B**(t), the charge q(t) is affected by a Lorentz's magnetic field force **F**(t), and the magnetic field force is

$$\mathbf{F}(t) = q(t)\,\mathbf{v}(t) \times \mathbf{B}(t) \qquad (2\text{-}3)$$

Faraday's law of electromagnetic induction, mass spectrometers, particle accelerators, electron microscopes, and Hall devices, etc., are the results of logical reasoning and deduction of Lorentz's law of magnetic field forces.

In technical applications, Faraday's law of electromagnetic induction reveals that the electrons in the metal coil are driven by the Lorentz's magnetic field force in a changing magnetic field, and the electrons are unevenly distributed in the metal coil, generating an electric field and electromotive force. In Maxwell's equations, equation (1-3B) states that without the participation of the electronics (for example, vacuum), a changing magnetic field produces an electric field and induces an electromotive force. Furthermore, in technical applications, Faraday's law of electromagnetic induction reveals the open-circuit induced electromotive force of a metal coil (the secondary output of the transformer requires a series resistor). In Maxwell's equations, Eq. (1-3B) states the closed-loop induced electromotive force, and the left side of Eq. (1-3B) must be equal to zero. In Maxwell's equations, Eq. (1-3B) must be wrong.

3. Conclusion

Based on the axiomatization thought of science proposed by Euclid, this study summarizes hundreds of electromagnetic theorems and formulas in the field of technical applications into three fundamental axioms of electromagnetism: Coulomb's law, Lorentz's law of magnetic field generation, and Lorentz's law of magnetic field force. Theoretical physicists, on the other hand, take Maxwell's equations as the cornerstone of electromagnetism.

Looking back at the history of science, Faraday, as a great experimental physicist, discovered and revealed that the induced electromotive force in a metal circuit is proportional to the change rate of the magnetic fluxes passing through the circuit. However, Faraday had no much mathematical knowledge, so his three-volume treatise “Experiments in Electricity” did not have a single mathematical formula. On the other hand, Maxwell, as a mathematical genius, had no systematic knowledge on physics and physical experiments, so Maxwell did not have a systematic and a deep understanding of electromagnetism.

In Maxwell's equations, because the propagation speed of the electric field and the magnetic field is not considered, Gauss's law of the electric field in Eq. (1-1B) and Gauss's law of the magnetic field in Eq. (1-2B) do not hold true within a changing electric field and a changing magnetic field. In technical applications, Faraday's law of electromagnetic induction reveals the open-circuit induced electromotive force of a metal coil (the secondary output of the transformer requires a resistor). In Maxwell's equations, Eq. (1-3B) states the closed-loop induced electromotive force, and the left side of Eq. (1-3B) must be equal to zero. Therefore, Eq. (1-3B) must be wrong. Maxwell introduced the "displacement current" hypothesis in 1865, predicting that a changing electric field could induce a magnetic field. On the one hand, the "displacement current" hypothesis has not been proved by any experiments so far; On the other hand, the "displacement current" hypothesis is not self-consistent in theory. Therefore, the "displacement current" hypothesis is not true, and Eq. (1-4B) must be wrong.

The above discussion and analysis are only based on the integral form of Maxwell's equations, but the integral and differential forms of Maxwell's equations are equivalent. To sum up, the 4 equations contained in Maxwell's equations are all wrong.

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