SOLUTION OF EQUATIONS OF THE GALAXY GRAVITATIONAL FIELD

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ABSTRACT. The general solution of the equations of the gravitational field of the galaxy with an additional variable parameter *n* is found. The additional variable parameter n determines in GR the distribution of the average mass density mainly in the friable galactic nucleus. The velocity of the orbital motion of stars is close to Kepler only for $n>2^{25}$. At $n<2^{15}$, it is slightly less than the highest possible velocity even at the edge of the galaxy. The maximum allowable value of the average mass density of a substance outside the friable galactic nucleus negligibly weakly depends on the parameter *n* in GR. If the energy-momentum tensor is formed not on the basis of external thermodynamic parameters, but on the basis of intranuclear gravithermodynamic parameters of the substance, then the dependence of the average mass of the substance on the value of the parameter *n* becomes very significant. The permissible value of the average mass density of matter outside the friable galactic nucleus is determined by the value of the parameter, which is responsible for the curvature of space. And it can be arbitrarily small. Therefore, in relativistic gravithermodynamics, in contrast to GR, there can be no shortage of baryonic mass.

Key words: General Relativity, Kepler's law, logarithmic gravitational potential, non-baryonic dark matter, orbital velocities, Relativistic Gravithermodynamics.

1. Logarithmic gravitational potential

Physical laws are based only on increments of metrical distances and not on increments of coordinates. Therefore, gravitational field strength k is determined via its gravitational potential φ in the following way:

$$k=-grad(\varphi)=-\frac{1}{\sqrt{a}}\frac{\partial\varphi}{\partial r}=-\sqrt{1-\frac{r_g}{r}-\frac{\Lambda r^2}{3}}\frac{\partial\varphi}{\partial r},$$

where: *a* is square of the ratio between increment of metrical segment and increment of radial coordinate *r*, and r_g is gravitational radius of astronomical body, from where observation takes place.

Nowadays, the following gravitational potential is used in GR and in practical calculations:

$$\varphi = cv_{cj} = c^2\sqrt{1-r_g/r}$$

When Λ=0 that potential forms the same spatial distribution of gravitational field strength as in classical physics:

$$k=-c^2 r_g/2r^2=-GMr^{-2} \qquad (r_g=2GMc^{-2}).$$

However, it does not correspond to Einstein's opinion that free fall of bodies in gravitational field is inertial motion. According to this potential the kinetic energy of falling body is less that the difference between rest energies of the body in the starting point of the falling and in the point of its instantaneous disposition. Wrong opinion that gravitational field has own energy corresponds to that gravitational potential (Logunov & Mestvirishvili, 1989).

In contrast to this potential, the potential that is in a form of logarithm of the rest energy E_0 of matter corresponds to inertial motion of freely falling body (Danylchenko, 2004) with the conservation of its total energy (Hamiltonian):

$$\varphi_j=c^2\ln(E_{0j}/E_{00})=c^2\ln(v_{cj}/c) \qquad (1)$$

Such representation of potential is based on the possibility of proportional synchronization of all quantum clocks and on proportionality of pseudo-forces of inertia and gravitation to the Hamiltonian of matter. This is in good correspondence with the principle of mass and energy equivalence. Such representation also makes the proof of equivalence of inert and gravitational masses redundant. Logarithmic gravitational potential forms the following spatial distribution of gravitational field strength:

$$k=-grad(c^2\ln E_0)=-grad(c^2\ln v_c)=$$

$$=-\frac{c^2}{r^2}\frac{(r_g/2-\Lambda r^3/3)}{\sqrt{1-r_g/r-\Lambda r^2/3}} \qquad (r_g=2GMc^{-2})$$

Effective value of gravitational constant:

$$G_{eff} = G/\sqrt{1-2GM/c^2r-\Lambda r^2/3} \qquad (2)$$

tends to infinity while approaching the Schwarzschild sphere and is continuously decreasing while distancing from the gravity center. And, of course, this should successfully prevent the false conclusions about the deficit of baryonic matter in the centers of the galaxies.

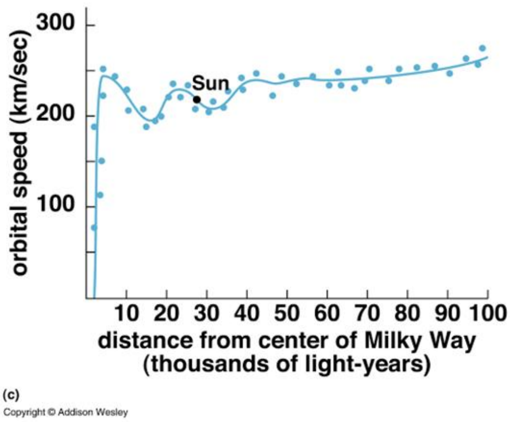
Usage of logarithmic gravitational potential does not require the adjustment of the values of mass of the Sun and the planets. If gravitational radius of Sun is 2.95 *km* then its mass should be decreased on just two millionth parts of it. It is 35 times less than the determination error of Sun mass. On the Mercury orbit the strength of Sun gravitational field should be decreased on just 20 billionth parts of it. The Earth itself has very small gravitational radius 0,887 *cm*. Due to this fact Earth mass should be decreased on just one billionth part of it. At the same time, Earth mass determination error is 100000 bigger.

Unlike for the Solar System, the usage of logarithmic gravitational potential can be very essential for the far galaxies.

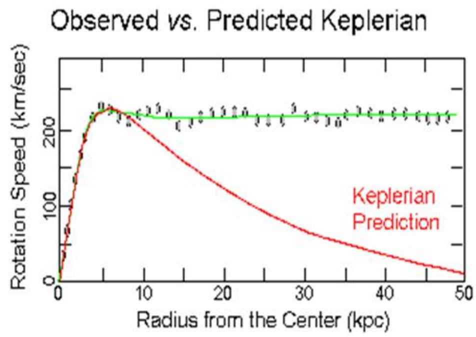
2. The inconsistency of the motion of galaxies with Kepler's laws

Laws of motion of single astronomical objects, found by Kepler, are based on gravitational influence of mainly central massive body. According to those laws, the velocity of rotation of galactic objects should decrease in inverse ratio to the square root of the distance to galaxy center. However, observations reveal the different picture: this velocity remains quasi constant on quite far distance from galaxy center for many galaxies, including ours [Bennett et al., 2012].

When single objects and their aggregates form big collection (cluster) their total mass can essentially exceed the mass of central astronomical body (supermassive neutron star or quasar). The attraction of astronomical objects of the internal spherical layers of the galaxy can be much stronger than the attraction to the central body of the galaxy. Then, their collective gravitational influence can essentially distort the correspondence of the motion of peripheral astronomical objects to Kepler's laws. And, therefore, according to astronomical observations the velocities of rotation of galaxy's peripheral astronomical objects required for prevention of joint collapse of all matter of the galaxy are much higher than the velocities of rotation of the separate peripheral astronomical objects required for prevention of the independent fall of those objects onto the central astronomical body.



a)



b)

Figure: Dependencies of velocity of rotation of astronomical objects on the distance to gravity center: **a**) our Milky Way galaxy (Bennett et al., 2012; Rieke, 2016), **b**) comparing to prognosed Keplerian velocities (Thompson, 2011)

The quite close dependency to the observed one is the following dependence of galactic velocity of rotation v_g of astronomical objects on the distance to the galaxy center. It is determined by the common galactic clock when the radial distribution of the average relativistic density of corrected relativistic mass of matter in the galaxy is the following:

$$\mu_r = \frac{\mu + p v_z^2 c^{-4}}{1 - v_z^2 c^{-2}} = \frac{\mu_0 r_e^2}{r^2}\left[1 - \left(1 - \frac{r}{r_e}\right)\exp\left(-\frac{r}{r_e}\right)\right] +$$

$$+ \frac{\sigma \mu_0 r_m}{r}\left[\frac{r_m}{r}\sin\left(2\pi\frac{r}{r_m}\right) + 2\pi\cos\left(2\pi\frac{r}{r_m}\right)\right] = \frac{\eta + \chi_0 r}{\kappa c^2 r^2}\ , \quad (3)$$

where:

$$\eta = \frac{\kappa c^2}{r}\int_0^r \mu_r r^2 dr = \kappa c^2 \mu_0 \left\{ r_e^2 \left[1 - \exp\left(-\frac{r}{r_e}\right)\right] + \sigma r_m^2 \sin\left(\frac{2\pi r}{r_m}\right)\right\}$$

$$\chi_0 = \kappa \mu_0 c^2 [r_e \exp(-r/r_e) + 2\pi\sigma r_m \cos(2\pi\ r/r_m)],$$

$v_z = v_g b^{-1/2}$ is zonal velocity of rotation (motion intensity) of astronomical objects by the clock of the outer space that surrounds them and is not dragged by the motion of astronomical objects themselves, μ_0, r_e, $r_{m,}$ σ is constants.

In this case on the large distances to the central astronomical body with the radius r_e ($r >> r_e$) the parameter η is only weakly sinusoidally modulated. And, also, the square of velocity of orbital rotation of astronomical objects of the galaxy, that can be found from the condition of equality of centrifugal pseudo force of inertion $F_i=\mathrm{H}v_g^2/c^2ba^{1/2}r$ and pseudo force of gravity $F_g=(\mathrm{H}/c^2a^{1/2})d(\ln{}^u v_{cg}/c)/dr$:

$$\frac{[v_z^2]_{GR}}{c^2}=\frac{v_g^2}{c^2b}=r\frac{d\ln({}^u v_{cg}/c)}{dr}=\frac{rb'}{2b}=$$

$$=\frac{a}{2}[1-1/a+(\kappa p-\Lambda)r^2]=\frac{[\eta+(\kappa p-2\Lambda/3)r^2]}{2(1-\eta-\Lambda r^2/3)} \qquad (4)$$

very slightly depends on $r>>r_e$ due to the smallness of $\exp(-r/r_e)$, pressure p in the outer space of the galaxy and cosmological constant Λ. And its value can only slightly increase together with increasing of r due to the gradual increasing of the parameter η.

Here “galactic” value of coordinate velocity of light ${}^u v_{cg}=cb^{1/2}$, Hamiltonian $\mathrm{H}=m_{cr}c^2b^{1/2}(1-v_g^2/bc^2)^{-1/2}=$ $=mc^2(1-v_z^2c^{-2})^{-1/2}$ and increment of the metric radial distance $d\check{r}=a^{1/2}dr$ are determined by the parameters b and $a=1/(1-\eta-\Lambda r^2/3)$ of the equations of GR gravitational field:

$$b'/abr-r^{-2}(1-1/a)+\Lambda=\kappa p\,,$$

$$\frac{a'}{a^2r}+\frac{1}{r^2}\left(1-\frac{1}{a}\right)-\Lambda=\kappa\frac{\mu c^4+pv_z^2}{c^2-v_z^2}=\kappa\mu_r c^2\,.$$

As we can see, exactly the logarithmic potential of gravitational field and the spatial distribution of gravitational strength defined by it in the extremely filled by stellar substance space of the galaxy correspond to these astronomical observations. The quite significant decreasing of the average density of matter when distancing from the center of the galaxy towards the periphery also corresponds to these astronomical observations. Together with the deepening into cosmological past ($\tau_p<\tau_e$) the average density of matter in the gravithermodynamic frame of reference of spatial coordinates and time (GT-FR) of the galaxy is decreasing on its periphery proportionally to the square of radial coordinate r_p. In the picture plane of astronomical observation this radial decreasing of the density of matter is even more significant:

$$[\mu_{rp}]_{obs}=\mu_p(r_p/[r_p]_{obs})^3=\mu_{rp}\exp[-3H(\tau_e-\tau_p)]=$$

$$=\mu_0 r_e^2 r_p^{-2}\exp\left[-\sqrt{3\Lambda}(r_p-r_e)\right],$$

since, in contrast to GT-FR of the central astronomical object of the observed galaxy, in GT-FR of terrestrial observer all other astronomical objects of this galaxy belong to the same moment of cosmological time $\tau_p=\tau_e$.

And, therefore, the quantity of baryonic matter currently present in galaxies can be quite enough for examined here justification for observed velocities of astronomical objects of galaxies. The one more contributing fact is that having the same quantity of matter ($m_{crp}=m_{cre}$) its gravitational mass $m=m_{cr}b^{1/2}$ on the galaxy periphery is bigger than in its center since $b_p>b_e$.

The GR gravitational field equations de facto correspond to spatially inhomogeneous thermodynamic states of only utterly cooled down matter. The similar to them equations of relativistic gravithermodynamics (RGTD) correspond to spatially inhomogeneous thermodynamic states of gradually cooling down matter. That is why in RGTD the four-momentum is formed not by enthalpy but by intranuclear Gibbs free energy. That is why in the RGTD the four-momentum is formed not by enthalpy but by the ordinary internal energy of matter (multiplicative component of its total energy). According to this, in the tensor of energy-momentum of the RGTD not only intranuclear pressure p_N but also intranuclear temperature T_N is taken into account:

$$\frac{b'}{abr}-\frac{1}{r^2}\left(1-\frac{1}{a}\right)+\Lambda=\frac{\kappa}{V}(T_NS_N-p_NV_N)=$$

$$=\kappa\mu_{cr}c^2[1/\sqrt{b}-\sqrt{b}]=\kappa\mu c^2(1/b-1)\,, \qquad (5)$$

$$\frac{a'}{a^2r}+\frac{1}{r^2}\left(1-\frac{1}{a}\right)-\Lambda=\kappa\mu c^2\left[1+\frac{v_z^2}{b(c^2-v_z^2)}\right]=$$

$$=\frac{\kappa[\mu_{cr}c^4b^{3/2}V+(T_NS_N-p_NV_N)v_g^2]}{V(bc^2-v_g^2)}= \qquad (6)$$

The defined by the same spatial distribution (3) average relativistic density of corrected relativistic mass of galaxy matter in RGTD has the following form:

$$\mu_r=\mu_{cr}\sqrt{b}[1+v_z^2/b(c^2-v_z^2)],$$

where:

$$\sqrt{b}=\frac{{}^u v_{\lg}}{c}=\frac{1}{\sqrt{a}}\left(1+\frac{\kappa c^2}{2}\int_{r_e}^{r}\frac{m_{cr}a^{3/2}rdr}{V(1-v_z^2c^{-2})}\right),$$

$\mu_{cr}=m_{cr}/V$, V is volume of matter, $m_{cr}=b^{-1/2}m$ is intrinsic value of the mass of matter that corresponds to “critical” equilibrium value of the ordinary internal energy of matter (b=1), and ${}^u v_{\lg}\equiv{}^u v_{cg}$ is maximum possible (extreme) value of velocity of matter in the outer space of the galaxy (Danylchenko, 2009; 2020].

According to this we find the square of the rotation velocity of astronomical object relatively to the galaxy center according to the equations of gravitational field of RGTD:

$$[v_z^2]_{RGTD}=c^2r\frac{d\ln({}^u v_{\lg}/c)}{dr}=$$

$$=\frac{c^2a}{2}\left\{\eta+\left[\frac{\kappa(T_NS_N-p_NV_N)}{V}-\frac{2}{3}\Lambda\right]r^2\right\}=$$

$$=\frac{c^2a}{2}\left[\frac{\eta+\chi r}{b+(1-b)v_z^2c^{-2}}-\frac{2}{3}\Lambda r^2\right]>>[v_z^2]_{GR}\,, \qquad (7)$$

where: $$\chi=(1-b)(1-v_g^2/bc^2)\chi_0\,.$$

As we can see, at the same radial destribution of the average density of the mass μ_r of baryonic matter the circular velocities of rotation of astronomical objects relatively to the galaxy center are much bigger in RGTD than in GR. And this is, of course, related to the fact that:

$$(T_NS_N-p_NV_N)/V=\mu c^2(1/b-1)>>p\,.$$

Therefore, we can get rid of the imaginary necessity of dark non-baryonic matter in galaxies that follows from the equations of GR gravitational field if we analyze the motion of their astronomical objects using the equations of gravitational field of RGTD.

If we do not take into account local peculiarities of distribution of average density of the mass in galaxies and examine only the general tendency of typical dependence of the orbital velocity of their objects on radial distance to the galaxy center, then the following dependence of this velocity on parameter *b* and, thus on radial distance *r*, can be matched with the graphs on Fig.:

$$v_z = v_{z\max}\{[(b/b_e)^n + (b_e/b)^n]/2\}^{-1/2} = \\ = c\{[2n\ln(r/r_e)]^2 + (c/v_{z\max})^4\}^{-1/4}, \qquad (8)$$

Where according to (4):

$$b = b_e\left[(v_{z\max}/v_z)^2 \pm \sqrt{(v_{z\max}/v_z)^4 - 1}\right]^{1/n} = \\ = b_e\left[\pm\frac{2nv_{z\max}^2}{c^2}\ln\left(\frac{r}{r_e}\right) + \sqrt{1 + \left[\frac{2nv_{z\max}^2}{c^2}\ln\left(\frac{r}{r_e}\right)\right]^2}\right]^{1/n}, \quad (9)$$

$$r = r_e\exp\left[\pm(c^2/2n)\sqrt{v_z^{-4} - v_{z\max}^{-4}}\right] = \\ = r_e\exp\left\{\pm(c^2v_{z\max}^{-2}/4n)\left[(b/b_e)^n - (b_e/b)^n\right]\right\},$$

and: r_e is radius of the conventional friable galactic nucleus, on the surface of which the orbital velocity of objects can take its maximum possible value $v_{ze}(b_e) = v_{z\max}$.

The smaller value b corresponds to the larger value n of the index of density of friable galactic nucleus on the same big radial distances. However, only when values are extremely large $n > 2^{25}$ the significantly lesser average density of matter beyond the friable galactic nucleus takes place and that is why the dependence of orbital velocities of galactic objects on radial distances can be close to Keplerian. When the parameter values are $n < 2^{15}$ the orbital velocities of extra-nuclear objects are, according to (8), quite close to their maximum values $v_{z\max} < 225$ [*km/s*] (Fig.b) on quite big radial distances $r/r_e < 20$: $\Delta v_z = v_{z\max} - v_z < 0{,}683$ [*km/s*].

This, of course, is related to the fact that big gradients of gravitational field on the periphery of such galaxies are formed not by their nuclei but by all large set of their objects.

Then, taking into account the negligible smallness of cosmological constant and of the pressure in the outer space of the galaxy, the following typical radial distribution of average density of mass of matter in the galaxy will take place in GR:

$$[\mu]_{GR} \approx \frac{2v_z^2(c^2 - v_z^2)[c^4 + 2v_z^2c^2 - 4n^2v_z^4\ln(r/r_e)]}{\kappa c^{10}r^2(1 + 2v_z^2c^{-2})^2} = \\ = \frac{1}{\kappa c^2}\left[\frac{a'}{a^2r} + \frac{1}{r^2}\left(1 - \frac{1}{a}\right)\right]\left(1 - \frac{v_g^2}{bc^2}\right) \approx \frac{v_z^2}{4\pi Gr^2},$$

where: $G = \kappa c^4/8\pi$ is Newton's gravitational constant, and according to (4):

$$a \approx 1 + 2\frac{v_z^2}{c^2} = 1 + 2\left\{\left[2n\ln\left(\frac{r}{r_e}\right)\right]^2 + \left(\frac{c}{v_{z\max}}\right)^4\right\}^{-\frac{1}{2}}. \qquad (10)$$

Thus, according to GR, the bigger the index n and the lesser the value of parameter b_e, the lesser is maximum possible value of average density of mass of the matter on the edge of the galaxy. However, when $v_{z\max} = 225$ [*km/s*], $r_e = 5$ *kpc*, $r_{\lim}/r_e = 20$, $n = 2^{15}$ ($v_{z\lim} = 224{,}317294$ [*km/s*]) and $b_e = 0{,}99999551225433188$ ($b_{\lim} = 0{,}99999988026921702$): $[\mu_{\lim}]_{GR} = 6{,}276\ 10^{-24}$ [kg/m^3] is only 0,4% smaller than its approximate value. And, therefore, due to v<<c it quite weakly depends on the index n of the density of friable galactic nucleus.

In RGTD (taking into account the negligible smallness of only cosmological constant) the completely different typical radial distribution of the average density of mass of the matter in the galaxy takes place:

$$[\mu]_{RGTD} \approx \frac{b\delta}{\kappa c^2r^2a(1-b)} = \frac{b[2v_z^2c^{-2} - (a-1)]}{\kappa c^2r^2a(1-b)} = \\ = \frac{b[4v_{z\max}^2c^{-2}b_e^nb^n - (a-1)(b^{2n} + b_e^{2n})]}{\kappa c^2r^2a(1-b)(b^{2n} + b_e^{2n})}, \qquad (11)$$

according to which in the case of fulfillment of condition (10) it becomes infinitely small. The tendency to 1 of not only parameter *a*, but also parameter *b*, prevents the limitless decrease to zero of average density of mass of matter on the edge of the galaxy. That is why in RGTD, in contrast to GR, there cannot be in principle any shortage of baryonic mass not only in the center, but also on the edge of the galaxy.

Taking into account that in the outer space on the periphery of the galaxy $a_{\lim} - 1 \approx 1 - b_{\lim}$ and, thus, $a_{\lim} = 1{,}00000111973203677$ (when $2v_{z\lim}{}^2c^{-2} = 1{,}11973203777\ 10^{-6}$), having the same initial data we can find the acceptable value of the average density of mass of matter on the edge of the galaxy: $[\mu_{\lim}]_{RGTD} = 5\ 10^{-26}$ [kg/m^3]. However, of course, when we have value b_e, that guarantees $\delta_{\lim} < 10^{-15}$, the significantly smaller average density of mass of the matter on the edge of the galaxy can take place in RGTD. When $n = 1$ ($v_{z\lim} = 224{,}99999999936$ [*km/s*]) and the same value $\delta_{\lim} = 10^{-15}$ ($b_e = 0{,}99999606363264543$, $b_{\lim} = 0{,}999999436721227408$) $[\mu_{\lim}]_{RGTD} = 1{,}4\ 10^{-27}$ [kg/m^3].

As we can see in RGTD, in contrast to GR, index $n = 2^{15}$ quite significantly (almost 36 times) increases the acceptable average value of density of mass of matter on the edge of the galaxy. However, due to mutual dependence of variable parameters *n*, b_e and a_e, that is defined by the principles of expedience and by corresponding to them negative feedbacks, the increasing of $[\mu_{\lim}]_{RGTD}$ will be indeed significantly smaller. The increasing of $[\mu_{\lim}]_{RGTD}$ on the galaxy periphery due to $n = 2^{15}$ can be partially compensated by its decreasing due to decreasing of the value $\delta_{\lim.}$

As a result of evolutional decreasing of average density of matter in the Universe and gradual cooling down of the galaxy nuclei their parameters *n*, b_e ($b_{\lim}$) и a_e ($a_{\lim}$) are gradually changing. It is manifested in a gradual distancing of astronomical objects from the galaxy center. The speeds of gradual change of these parameters are not equal for different galaxies that may result in the non-equality of galactic values of Hubble constant. However, the difference of galactic values from the global value of Hubble constant, which corresponds only to evolutional expansion of the Universe, is negligibly small in modern time. But in far cosmological past it could be more significant due to the big values of average density of matter in the Universe and, thus, due to the smaller values of parameter b (and, consequently, of defined by them values of coordinate velocity of light) in the outer space of the Universe. Nowadays it is more significant only in non-rigid FRs (Danylchenko, 1994) of cooling-down astronomical bodies.

Radial distribution of parameter a can be found via the solution of differential equation:

$$ra'/a+(a-1)-\kappa\mu c^2[1+v_z^2/b(c^2-v_z^2)]=$$
$$=\frac{ra'}{a}+\frac{(b^{2n}+b_e^{2n})(a-1)}{(1-b)[(b^{2n}+b_e^{2n})-2v_{z\max}^2c^{-2}b_e^nb^n]}-$$
$$-\frac{4v_{z\max}^2b^{n+1}[(b^{2n}+b_e^{2n})+2v_{z\max}^2c^{-2}b_e^nb^{n-1}(1-b)]}{b_e^{-n}(1-b)(b^{2n}+b_e^{2n})[c^2(b^{2n}+b_e^{2n})-2v_{z\max}^2b_e^nb^n]}=0$$

and, taking into account that $dr=(rc^2/2v_z^2b)db$, and $v_{z\max}<<c$, – of another equation:

$$\frac{1}{a}\frac{da}{db}-\frac{(b^{2n}+b_e^{2n})+2v_{z\max}^2c^{-2}b_e^nb^{n-1}(1-b)}{(1-b)[(b^{2n}+b_e^{2n})-2v_{z\max}^2c^{-2}b_e^nb^n]}+$$
$$+\frac{c^2v_{z\max}^{-2}b_e^{-n}(b^{2n}+b_e^{2n})^2(a-1)}{4b^{n+1}(1-b)[(b^{2n}+b_e^{2n})-2v_{z\max}^2c^{-2}b_e^nb^n]}\approx$$
$$\approx\frac{1}{a}\frac{da}{db}-\frac{1}{1-b}+\frac{c^2(b^{2n}+b_e^{2n})(a-1)}{4v_{z\max}^2b_e^nb^{n+1}(1-b)}=\frac{1}{a}\frac{da}{db}-\frac{1}{(1-b)}+$$
$$+\frac{c^2(a-1)}{2v_z^2b(1-b)}=\frac{1}{a}\frac{da}{db}-\frac{1}{1-b}+\frac{(a-1)}{(1-b)r}\frac{dr}{db}=0\,. \qquad (12)$$

3. Imaginary non-baryonic dark matter

It is obvious that not very massive bilayered shell-like quasars that have strong gravitational field only in their close neighborhood are located in the centers of many galaxies. That is possible because the effective value of gravitational constant (2) tends to infinity while approaching to median singular sphere of the quasar when logarithmic gravitational potential is used. G_{eff} depends on angular diameter α of circular orbit in the following way:

$$G_{eff}\approx G[1-4GM(1+z)^{3/2}/D_L\sin\alpha]^{-1/2}$$

when the orbital plane of astronomical body is perpendicular to the radius-vector of the galaxy center.

It is possible that imaginary deficit of baryonic matter in friable nucleus of the galaxy is indeed compensated by quite big effective value of gravitational constant for all its astronomical objects. And exactly that deficit of baryonic matter allows us to consider logarithmic gravitational potential (1) as the most effective alternative to phantom non-baryonic dark matter.

Of course, the radiation spectrum of far galaxies for sure cannot depend on the imaginary time dilation, "observed" in GT-FR in the points of instantaneous disposition of these galaxies, because the relativistic dilation of the GT-FR's intrinsic gravi-quantum time occurs only within the extended empty space of the Earth. This expanded empty space is only formally (imaginary) evolutionarily self-contracting in the comoving FR in the expanding Universe (CFREU) along with the Earth. Therefore, the time dilation is also only formally "observed" in the GT-FR. That's why, according to line element of GT-FR (Danylchenko, 2004) velocities of astronomical objects in the picture plane in intrinsic gravi-quantum time of the observer do not depend at all on the dilation of intrinsic gravi-quantum time of GT-FR in the points of instantaneous disposition of those objects.

Of course, the counting of intrinsic gravi-quantum time of the observer could be replaced by the counting of dilated gravi-quantum time in those points of GT-FR. However, then the gravi-quantum value of gravitational constant (calibrated accordingly) should be used:

$${}^jG_E=G_Ec^2v_{cj}^{-2}=G_E(1+z)^2/(1+2z).$$

Results of such imaginary "observation" of the motion in the picture plane of distant astronomical object in dilated gravi-quantum time of point j of its disposition, of course, will be changed. However, those results will correspond to the same regularities as the results of observation in standard astronomical time of observer's GT-FR.

It is worth mentioning that analysis of the motion of astronomical objects can be done in accordance to the metrically homogeneous scale of cosmological time in CFREU using the real metrical distance ${}^rD_M=R$ to them instead of ${}^iD_M$. Such analysis would require taking into account that length standard in CFREU (at the moment of observation) is (1+z) times smaller than its size during the emission radiation. Therefore, it would be also required to use in CFREU (1+z) times bigger values of accelerations and velocities of those objects, as well as, values of the velocity of light in the points of dispositions of those objects. Furthermore, it would be required to use $(1+z)^3$ times bigger value of gravitational constant in the points of disposition of observed objects. However it is much simpler to use in CFREU not the ${}^rD_M$, but the normalized by (1+z) its value. That is because it is identical to the angular diameter distance:

$${}^rD_A=R_0=r={}^rD_M/(1+z)={}^iD_M(1+z)^{-1/2}.$$

If we follow mentioned above simpler approach, we would not need to perform all mentioned here transformations of all other characteristics and of gravitational constant. The total mutual correspondence of the motion of distant astronomical objects in picture plane in GT-FR and in CFREU denotes the possibility of mentioned above. That correspondence takes place due to invariance of angular characteristics in the case of radial transformations. Members of line elements of GT-FR and CFREU that correspond to that motion exactly match each other when performed normalization of distance ${}^rD_M=R$ (usage of the distance ${}^rD_A=R_0=r$ instead of it) is taken into account (Danylchenko, 2004).

It is obvious, that one of the possible reasons of fictive necessity of imaginary non-baryonic dark matter in the Universe is the significantly smaller density of stellar substance in CFREU and, therefore, in corresponding to it picture plane of distant observer, than in GT-FR of observed galaxy.

It is obvious, that according to results of galaxies observations in more wide spectral diapason there would be no deficit of ordinary matter (McGaugh et al., 2016) (of course when using the real value of the angular diameter distance ${}^rD_A=R_0$ in CFREU or the Schwarzschild coordinate $r=R_0$ in GT-FR). However we can totally get rid of fictive necessity of non-baryonic dark matter only when using the logarithmic gravitational potential as well as tensor of energy-momentum of RGTD. It means that, all motions of astronomical objects, observed in picture plane, can be explained without involving of phantom non-baryonic dark matter. For any arbitrary low value of density of the mass of matter on the edge of the galaxy μ_{lim} the corresponding to it values of variable parameters a_e and n can be found according to (12) (Danylchenko, 2020).

If imaginary deficit of mass occurs during some astronomical observations and when using logarithmic gravitational potential and tensor of energy-momentum of RGTD in calculations, then it can be caused by the ignoring of the possibility of self-organization of astronomical objects into cluster with extraordinary topology. That could be, for example, spiral and toroidal-like elliptical galaxies or shell-like globular clusters and spherical elliptical galaxies. These clusters and galaxies have multitude of gravity centres in the form of median line or median surface accordingly. In this case even the presence of central massive astronomical object is not required (McGaugh et al., 2016).

4. Conclusions

The general solution of the equations of the gravitational field of the galaxy with an additional variable parameter n is found. The additional variable parameter n determines in GR the distribution of the average mass density mainly in the friable galactic nucleus. The velocity of the orbital motion of stars is close to Kepler only for $n>225$. At $n<215$, it is slightly less than the highest possible velocity even at the edge of the galaxy. If the energy-momentum tensor is formed not on the basis of external thermodynamic parameters, but on the basis of intranuclear gravithermodynamic parameters of the substance, then the dependence of the average mass of the substance on the value of the parameter n becomes very significant. The permissible value of the average mass density of matter outside the friable galactic nucleus is determined by the value of the parameter, which is responsible for the curvature of space. And it can be arbitrarily small. Therefore, in relativistic gravithermodynamics, in contrast to GR, there can be no shortage of baryonic mass in principle. And, therefore, the presence of non-baryonic dark matter in the Universe is not necessary.

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