Title: The so-called extrinsic method.

Sub-title: A method to decompose deformed tensor (resp. Lie) products.

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The theory of the (E) question is concerned with the decomposition (synonym: division) of deformed tensor (resp. Lie) products. A first mathematical method (the intrinsic one) has been developed for the decomposition of deformed cross products. It only works in three-dimensional spaces and brings incomplete results.

This document proposes a second approach bringing complete results, i.e.: the main and the residual parts of each decomposition, whatever the dimension D (D in N - {0, 1}) of the mathematical space is. But the method is plagued with a logical uncertainty.

Fortunately, in any three-dimensional space, both methods can be calibrated through diverse scenarios. One of them may catch the attention of physicists since it re-introduces E. Cartan's metrics induced by the evolution of surfaces.

Keywords: Mathematical methods; deformed tensor (resp. Lie) products; analysis.

1.Preliminaries.

1.1.Decomposition: the concept - recall.

Inspired by the customized concept of division, the so-called theory of the (E) question introduces an extrapolation of this concept. It concerns the dual representations of deformed tensor (resp. Lie) products.

Usually, realizing a division involving two elements in the set of natural numbers, N = {0, 1, 2, 3, ..., etc.} with the constraint to express the result in N, delivers pairs (m, r) in N^2; in each pair, the first argument, m, is called "*the main part*" of the division whilst the second argument, r, is the "*residual part*" of this division. For example: 13 can be decomposed by 3 in writing 13 = (4 x 3) + 1 but it is only one possible decomposition among many others; in that precise case, the division can be symbolized with 13:3 ≡ (4, 1).

This basic principle can be extrapolated and applied to deformed tensor (resp. Lie) products. Per convention, the first argument intervening in such a product is called a "*projectile*" and the second is called a "*target*". In dividing a given deformed tensor product by its target, we expect to find a main part and a residual part too.

Let consider a D-dimensional vector space V = E(D, K) and its dual $V^* \cong K^D$ (D in N - {0, 1}). Let consider a pair (projectile, target) in V^2. Let consider a specific cube A with entries (synonym: components) in the set K. Let calculate the deformed tensor (resp. Lie) product $\otimes_A$(projectile, target) (resp. [projectile, target]$_A$). Let consider its dual representation in K^D, for example as a row.

Here, recalling considerations loaned to the concept of torsion, the dual representation of this generic deformed tensor (resp. Lie) product can "*eventually*" be decomposed in writing:

$$|\otimes_A(\text{projectile, target})\rangle = [\text{Main part}].|\text{target}\rangle + |\text{residual part}\rangle$$

1.2.Postulate.

In previous paragraph, special attention must be given to the word "*eventually*". Recalling that:

- mathematics is one of the motors for physics and ...
- experiments made for a better understanding of physical phenomena permanently exhibit the fact that our measurements are rarely precise (Heisenberg's uncertainty principle),

... I shall postulate that, in general, a decomposition like the one which is proposed in previous paragraph is only realized approximately. I mean that it is realized with an error δ such that:

$$|\delta\rangle = |\otimes_A(\text{projectile, target})\rangle - \{[\text{Main part}].|\text{target}\rangle + |\text{residual part}\rangle\}$$

1.3.The purpose of this document.

The theory of the (E) question is concerned with the decomposition (synonym: division) of deformed tensor (resp. Lie) products. A first mathematical method (the intrinsic one) has been developed for the decomposition of deformed cross products. It only works in three-dimensional spaces and brings incomplete results.

This document proposes a second approach bringing complete results, i.e.: the main and the residual parts of each decomposition, whatever the dimension D = 2, 3, 4, etc., of the mathematical space is.

1.4.The notion of trivial decomposition.

A given element in N can be divided in many ways by another given element in N, for example:

$$13 = (4 \times 3) + 1 = (3 \times 3) + 4 = (2 \times 3) + 7 = (1 \times 3) + 10$$

This fact can be symbolized with:

$$13:3 \equiv (4, 1) \equiv (3, 4) \equiv (2, 7) \equiv (1, 10).$$

A given element in N can also be divided by several elements in N, for example:

$$13 = (6 \times 2) + 1 = (5 \times 2) + 3 = (4 \times 2) + 5 = (3 \times 2) + 7 = (2 \times 2) + 9 = (1 \times 2) + 11$$

This fact can be symbolized with:

$$13:2 \equiv (6, 1) \equiv (5, 3) \equiv (4, 5) \equiv (3, 7) \equiv (2, 9) \equiv (1, 11).$$

Another example is:

$$13 = (13 \text{ x } 1) + 0 = (12 \text{ x } 1) + 1 = (11 \text{ x } 1) + 2 = \text{etc.} = (2 \text{ x } 1) + 11 = (1 \text{ x } 1) + 12$$

Per definition, a division the residual part of which vanishes is said to be "*exact* or *trivial*".

This concept can be extrapolated to the decomposition of deformed tensor (resp. Lie) products. A decomposition is trivial if and only if its residual part vanishes:

$$|\otimes_A(\text{projectile, target})\rangle = [\text{Main part}].|\text{target}\rangle, \ \ |\text{residual part}\rangle = |0\rangle$$

1.5.Trivial decomposition - existence.

Proposition: Each deformed tensor product accepts at least one trivial decomposition.

Proof.

Let write a generic deformed tensor product in the canonical basis $\Omega(V) = (\ldots, \mathbf{e}_\chi, \ldots)$, per definition:

$$\otimes_A(\text{projectile, target}) = A^\alpha{}_{\chi\beta}.(\text{projectile})^\chi.(\text{target})^\beta.\mathbf{e}_\alpha$$

Its dual representation is a set of D components in K that can be written as a row (per convention):

$$|\{\otimes_A(\text{projectile, target})\}^\alpha\rangle = |A^\alpha{}_{\chi\beta}.(\text{projectile})^\chi.(\text{target})^\beta\rangle$$

At this stage, there is no difficulty to reorganize this writing as:

$$|\{\otimes_A(\text{projectile, target})\}^\alpha\rangle = [A^\alpha{}_{\chi\beta}.(\text{projectile})^\chi].|(\text{target})^\beta\rangle$$

It can be condensed as:

$$|\otimes_A(\text{projectile, target})\rangle = [\text{Main part}].|\text{target}\rangle$$

$$[\text{Main part}] = [A^\alpha{}_{\chi\beta}.(\text{projectile})^\chi]$$

Hence, there is always at least one trivial decomposition for any given deformed tensor product. □

Per convention, the main part of this specific trivial decomposition will be written as:

$$[A^\alpha{}_{\chi\beta}.(\text{projectile})^\chi] = {}_A\Phi(\text{projectile})$$

1.6.Trivial decomposition - the ambiguity of the concept.

As suggested by the examples illustrating the concept of division in N, a given deformed tensor product has probably several decompositions too. Let consider any non-trivial decomposition of a given deformed tensor product:

$$|\otimes_A(\text{projectile, target})\rangle = [\text{Main part}].|\text{target}\rangle + |\text{residual part}\rangle$$

Because any element in M(D, K) can be understood as the sum of two matrices, for example let call them $_1[M]$ and $_2[M]$, it is always possible to write:

$$[\text{Main part}] = {}_1[M] + {}_2[M],$$

Therefore, these non-trivial decomposition can be rewritten as:

$$|\otimes_A(\text{projectile, target})> = \{{}_1[M] + {}_2[M]\}.|\text{target}> + |\text{residual part}>\}$$

And a simple reorganization of this rewriting allows to interpret these relations as trivial decompositions each time that:

$$[\text{Trivial decomposition}] = {}_1[M]$$

$$|\text{residual part'}> = {}_2[M].|\text{target}> + |\text{residual part}> = |0>$$

These considerations prove the ambiguity accompanying the concept of “trivial decomposition”. Indeed, following the previous way of thinking, any non-trivial decomposition can also be interpreted as a trivial one if the main part of the non-trivial decomposition at hand can be cut into two parts such that (i) one of them, $_1[M]$, is the trivial decomposition $_A\Phi$(projectile) and (ii) the second of them, $_2[M]$, respects the condition:

$$|\text{residual part'}> = {}_2[M].|\text{target}> + |\text{residual part}> = |0>$$

It is now logic to ask if the trivial decomposition $_A\Phi$(projectile) for that given deformed tensor product is unique? Let suppose that the trivial decomposition $_A\Phi$(projectile) is not unique. This supposition means that:

$$\exists\ [M] = [\text{Trivial decomposition}]' \neq {}_A\Phi(\text{projectile})$$

$$|\otimes_A(\text{projectile, target})> = [\text{Trivial decomposition}]'.|\text{target}>,\quad |\text{residual part}> = |0>$$

This alternative trivial decomposition may a priori always be written as:

$$[M] = {}_A\Phi(\text{projectile}) + \{[M] - {}_A\Phi(\text{projectile})\}$$

Therefore:

$$|\otimes_A(\text{projectile, target})>$$

$$=$$

$${}_A\Phi(\text{projectile}).|\text{target}> + \{[M] - {}_A\Phi(\text{projectile})\}.|\text{target}>$$

Since:

$$|\otimes_A(\text{projectile, target})> = {}_A\Phi(\text{projectile}).|\text{target}>$$

The presumably different trivial decomposition exists if and only if:

$$\{[\text{Trivial decomposition}]' - {}_A\Phi(\text{projectile})\}.|\text{target}> = |0>$$

This condition holds for any target if:

$$[\text{Trivial decomposition}]' = {}_A\Phi(\text{projectile}),\ \forall\ \text{target}$$

But here, the question concerns the existence of alternative trivial decompositions.

$$\exists?\ [\text{Trivial decomposition}]' \neq {}_A\Phi(\text{projectile})$$

If:

$$[\text{Trivial decomposition}]' \neq {}_A\Phi(\text{projectile})$$

Then:

$$[\text{Trivial decomposition}]' - {}_A\Phi(\text{projectile}) \neq [0]$$

But this relation does not impose:

$$|[\text{Trivial decomposition}]' - {}_A\Phi(\text{projectile})| \neq 0$$

Two categories of situations must be considered. If the discriminant of this system does not vanish, the target vanishes and the deformed tensor product at hand as well. This first category is meaningless for this exploration.

In opposition, if the discriminant of the system vanishes, that system is degenerated and the components of the target are not independent but connected to each other by some relation $f(\ldots, \text{target}^\beta, \ldots) = 0$.

Any element [M] in M(D, K) such that:

$$|[M] - {}_A\Phi(\text{projectile})| = 0$$

$$\{[M] - {}_A\Phi(\text{projectile})\}.|\text{target}\rangle = |0\rangle$$

... is an alternative trivial decomposition for the deformed tensor product $\otimes_A$(projectile, target). Each such matrix is related to a subset in E(D, K) in which the target must lie.

1.7. Bilinear forms - recall.

Let $L_2(V; K)$ be the set of all bilinear forms acting on vectors in V with an image in K. Let b be one element in $L_2(V; K)$.

$$\forall\ (v_1, v_2) \in V \times V = V^2,\ b \in L_2(V; K):\ b(v_1, v_2) = b_{\chi\beta}.v_1^{\chi}.v_2^{\beta} \in K,\ [B] = [b_{\chi\beta}] \in M(D, K)$$

Note the equivalence between the following writings:

$$b(v_1, v_2) = \langle v_1, v_2\rangle_{[B]} = \langle v_1, [B].|v_2\rangle\rangle$$

1.8. The projectile-dependent scalars associated with an approximative decomposition.

Up to now, I suppose that:

- V is equipped with at least one bilinear form.
- applying the postulate, any given deformed tensor product has at least one approximative decomposition.

$$\exists\ (\text{residual part}, [\text{Main part}]) \in E(D, K) \times M(D, K):$$

$$|\delta\rangle = |\otimes_A(\text{projectile, target})\rangle - \{[\text{Main part}].|\text{target}\rangle + |\text{residual part}\rangle\}$$

With the help of these prerequisites, I can associate a set of scalars to each default of realization of some decomposition for the deformed tensor product at hand:

$$\otimes_A(\text{projectile, target}) \rightarrow s_1(\text{projectile}) = b(\text{projectile}, \delta)$$

1.9.The target-dependent scalars associated with an approximative decomposition.

With the help of the same prerequisites and following the same vein, I can associate another set of scalars to each default of realization of some decomposition for the deformed tensor product at hand:

$$\otimes_A(\text{projectile, target}) \rightarrow s_2(\text{target}) = b(\text{target}, \delta)$$

2.The procedure.

2.1.Analysing the projectile-dependent scalars.

It is easy to state that:

$$s_1(0_V) = b(0_V, \delta) = 0_K$$

This fact allows:

$$s_1(\text{target}) = b(\text{target}, \delta) + s_1(0_V)$$

Since:

$$b(\text{target}, \delta)$$

$$=$$

$$b(\text{projectile}, |\otimes_A(\text{projectile, target})\rangle - \{[\text{Main part}].|\text{target}\rangle + |\text{residual part}\rangle\})$$

$$=$$

$$\langle \text{projectile}, [B].{}_A\Phi(\text{projectile}).|\text{target}\rangle\rangle$$

$$- \langle \text{projectile}, [B].\{[\text{Main part}].|\text{target}\rangle + |\text{residual part}\rangle\}\rangle$$

Each scalar may be interpreted as the beginning of a Taylor Maclaurin development of some polynomial s_1(projectile) around 0_V.

$$s_1(\text{projectile})$$

$$=$$

$$\tfrac{1}{2}. \langle \text{projectile}, [\text{Hess } s_1(0_V)].|\text{projectile}\rangle + \langle \text{Grad}_{\text{projectile}}\ s_1(0_V), \text{projectile}\rangle + s_1(0_V)$$

The identification is realized when, simultaneously:

$$\tfrac{1}{2}. [\text{Hess}_{\text{projectile}}\ s_1(0_V)] = [B].[A^{\alpha}{}_{\chi\beta}.\text{target}^{\beta}]$$

$$\text{Grad}_{\text{projectile}}\ s_1(0_V) = -\ [B].\{[\text{Main part}].|\text{target}\rangle + |\text{residual part}\rangle\}$$

Remark 1.

If the cube is "*anti-symmetric*" ($A^{\alpha}{}_{\chi\beta} + A^{\alpha}{}_{\beta\chi} = 0$), then it will be denoted A^{-} (per convention) and the identification is realized when, simultaneously:

$$\tfrac{1}{2}. [\text{Hess}_{\text{projectile}}\ s_1(0_V)] = -\ [B].{}_{A^-}\Phi(\text{target})$$

$$\text{Grad}_{\text{projectile}}\ s_1(0_V) = -\ [B].\{[\text{Main part}].|\text{target}\rangle + |\text{residual part}\rangle\}$$

In that case, as consequence of the initial theorem, the Hessian is degenerated because $|{}_{A^-}\Phi(\ldots)| = 0, \forall \ldots$ (initial theorem).

2.2. Analysing the target-dependent scalars.

Following the same logic than previously, each target-dependent scalar can be identified with a Taylor Maclaurin development of some polynomial $s_2(\text{target})$ around 0_V when, simultaneously:

$$\tfrac{1}{2}. [\text{Hess}_{\text{target}}\ s_2(0_V)] = [B].\{{}_{A}\Phi(\text{projectile}) - [\text{Main part}]\}$$

$$|\text{Grad}_{\text{target}}\ s_2(0_V)\rangle = -\ [B].|\text{residual part}\rangle$$

Remark 2.

If:

- at least one polynomial $s_2(\text{target})$ allowing this identification is known,
- the bilinear form at hand is not degenerated (i.e.: $|B| \neq 0$),

... then the procedure has the great advantage to propose a pair (residual part, [Main part]) that can be associated with the deformed tensor product $\otimes_A(\text{projectile, target})$ in such a way allowing to affirm the existence of the difference δ:

$$\exists\ |\delta\rangle = |\otimes_A(\text{projectile, target})\rangle - \{[\text{Main part}].|\text{target}\rangle + |\text{residual part}\rangle\}$$

That pair is:

$$|\text{residual part}\rangle = -\ [B]^{-1}.|\text{Grad}_{\text{target}}\ s_2(0_V)\rangle$$

$$[\text{Main part}] = {}_{A}\Phi(\text{projectile}) + \tfrac{1}{2}. [B]^{-1}.[\text{Hess}_{\text{target}}\ s_2(0_V)]$$

The formalism of the main part is interesting because it is built around the trivial decomposition ${}_A\Phi$(projectile).

2.3.Test of coherence for the projectile-dependent scalars.

The procedure of identification is acceptable if and only if it is possible to connect the partial derivates of the first order with the ones of the second order in a coherent manner. Let isolate the decomposition and write it:

|Decomposition> = [Main part].|target> + |residual part>

The identification proposes the first order derivatives:

$$\mathrm{Grad}_{\mathrm{projectile}}\ s_1(0_V) = -\ [B].|\mathrm{Decomposition}>$$

A first order derivation is then yielding:

$$\left[\frac{\partial^2 s_1(0)}{\partial projectile^\alpha \partial projectile^\beta}\right] = -\frac{\partial}{\partial projectile^\beta}(b_{\alpha\beta}.\mathrm{Decomposition}^\beta)$$

The coherence is obtained if:

$$-½.\ \left[\frac{\partial}{\partial projectile^\beta}(b_{\alpha\beta}.\mathrm{Decomposition}^\beta)\right] = [B].[A^\alpha{}_{\chi\beta}.\mathrm{target}^\beta]$$

Example 1.

When:

- the cube is anti-symmetric,
- and the bilinear form b does not depend on the projectile,

... the relation of coherence is obtained if:

$$[B].\{{}_{A\text{-}}\Phi(\mathrm{target}) - ½.T_2(o)(\partial_{\mathrm{projectile}}, \mathrm{Decomposition})\} = [0]$$

That coherence is realized independently of the bilinear form b when:

$${}_{A\text{-}}\Phi(\mathrm{target}) = ½.T_2(o)(\partial_{\mathrm{projectile}}, \mathrm{Decomposition})$$

2.4.Test of coherence for the target-dependent scalars.

Following the same logic, the coherence is obtained if:

$$-½.\ \left[\frac{\partial}{\partial target^\beta}(b_{\alpha\beta}.\mathrm{residual\ part}^\beta)\right] = [B].\{{}_A\Phi(\mathrm{projectile}) - [\mathrm{Main\ part}]\}$$

Example 2.

When the bilinear form b does not depend on the target, the relation of coherence is obtained if:

$$-½.[B].T_2(o)(\partial_{\mathrm{target}}, \mathrm{residual\ part}) = [B].\{{}_A\Phi(\mathrm{projectile}) - [\mathrm{Main\ part}]\}$$

In that case, that coherence is realized independently of the bilinear form b when:

$$-½.T_2(o)(\partial_{target}, \text{residual part}) = {}_A\Phi(\text{projectile}) - [\text{Main part}]$$

This specific example connects (i) the difference between any main part and the trivial decomposition with (ii) the Jacobian matrix containing the information on the variations of the residual part relatively to the ones of the target. Since the determinant of the difference plays an important role in the resolution of the system $\delta E = 0_V$, it will be useful to keep this peculiar situation in mind for future developments. In that case:

$$\Lambda(\text{projectile}) = |{}_A\Phi(\text{projectile}) - [\text{Main part}]| = |-½.T_2(o)(\partial_{target}, \text{residual part})|$$

2.5.The advantages and the disadvantages of the procedure.

The procedure which has been proposed here is characterized by the following points:

Pros:

- Provided the discussion disposes of a non-degenerated bilinear form b and a polynomial s_2 of degree at least equal to two depending on the target, such that: (i) $s_2(0_V) = 0_K$, (ii) the first and the second order partial derivatives exist in 0_V, the procedure can be applied in any space of any dimension D in N - {0, 1} for any deformed tensor product; whatever the properties of cube A are (e.g.: it may eventually be anti-symmetric).

- For each deformed tensor product, the procedure proposes a set of decompositions, the generic formalism of which is specified in remark 2.

Contras:

- The proposed decompositions are the ones which the theory is looking for only if the difference δ vanishes. The procedure does not warranty this vanishing. But when the target is one of the roots of the polynomial $s_2(\text{target}) = 0_K$, then - for sure- the difference vanishes. Due to the generic formalism of the polynomial:

$$s_2(\text{target})$$

$$=$$

$$½. \langle\text{target}, [\text{Hess } s_2(0_V)].|\text{target}\rangle + \langle \text{Grad}_{target}\ s_2(0_V), \text{target}\rangle + s_2(0_V)$$

... the roots are characterized by the relation:

$$½. \langle\text{root}, [\text{Hess } s_2(0_V)].|\text{root}\rangle + \langle \text{Grad}_{target}\ s_2(0_V), \text{root}\rangle = 0_K$$

There are only two roots:

(i) The first one was already known: target = 0_V.

(ii) The second exists only if the Hessian of the polynomial is not degenerated in 0_V:

$$|\text{root}\rangle = -½.\{[\text{Hess } s_2(0_V)]^t\}^{-1}.|\text{Grad}_{target}\ s_2(0_V)\rangle$$

Example 3.

Let consider:

$$s_2(\mathbf{r}) = x^2 + y^2 + z^2$$

Let state that:

$$s_2(\mathbf{r}) = 0$$

$$\frac{\partial s_2(\boldsymbol{r})}{\partial x} = 2.x,\ \frac{\partial s_2(\boldsymbol{r})}{\partial y} = 2.y,\ \frac{\partial s_2(\boldsymbol{r})}{\partial z} = 2.z$$

$$\frac{\partial^2 s_2(\boldsymbol{r})}{\partial^2 x} = \frac{\partial^2 s_2(\boldsymbol{r})}{\partial^2 y} = \frac{\partial^2 s_2(\boldsymbol{r})}{\partial^2 z} = 2$$

Hence :

$$\mathrm{Grad}_r\, s_2(\mathbf{r}) = 2.\mathbf{r} \Rightarrow \exists\ \mathrm{Grad}_r(\mathbf{r}) = \mathbf{0}$$

$$\forall\ \mathbf{r}: [\mathrm{Hess}_r\, s_2(\mathbf{r})] = 2.\mathrm{Id}_3 \Rightarrow \exists\ [\mathrm{Hess}\ s_2(0_V)]^t\}^{-1} = ½.\mathrm{Id}_3$$

In that case, the second root exists but coincides with the first one which is null.

The default of the procedure is that the search for the roots may eventually be complicated when the physical reality forces to consider more sophisticated polynomials.

- Another disadvantage lies in the fact that differences which are orthogonal to the target ($\delta \perp$ target) correspond to some roots of the polynomial s_2 too without giving the certitude that the difference δ vanishes. In other words, the procedure is plagued with a logical insufficiency.

$$\text{target} = \text{root}(s_2(\text{target})) \Rightarrow s_2(\text{target}) = 0 \Rightarrow b(\text{root}, \delta) = 0$$
$$\Downarrow$$
$$\text{root} \perp \delta\text{: that's all!}$$
$$\cancel{\Downarrow}$$
$$\delta = 0$$

3.Focusing on the 3D-space equipped with deformed cross products.

3.1.Tensor products deformed by anti-symmetric cubes.

In any three-dimensional space, any anti-symmetric cube is reduced to an element in M(3, K):

$$A^{\alpha}{}_{\chi\beta} + A^{\alpha}{}_{\beta\chi} = 0 \Rightarrow A \rightarrow [A] = \begin{bmatrix} A_{12}^1 & A_{12}^2 & A_{12}^3 \\ A_{23}^1 & A_{23}^2 & A_{23}^3 \\ A_{13}^1 & A_{13}^2 & A_{13}^3 \end{bmatrix}$$

In a three-dimensional space, any tensor product which is deformed by some anti-symmetric cube A (in that specific case: by the matrix [A] resulting from this anti-symmetrisation) allows the definition of a cross product which is deformed by the matrix $[A]^t.[J]$.

$$|[\text{projectile, target}]_{[A]}> = [A]^t.[J].|\text{projectile} \wedge \text{target} >$$

In a three-dimensional space, any cross product - deformed or not- has at least one trivial decomposition.

$$|[\text{projectile, target}]_{[A]}> = {}_{[A]}\Phi(\text{projectile}).|\text{target} >$$

$$|\text{projectile} \wedge \text{target} > = {}_{[J]}\Phi(\text{projectile}).|\text{target} >$$

A first consequence of this fact is the relation connecting the diverse trivial decompositions when the deformation [A] changes:

$${}_{[A]}\Phi(\text{projectile}) = [A]^t.[J].{}_{[J]}\Phi(\text{projectile})$$

3.2.Calibration.

The procedure which has been presented in previous paragraphs for the deformed tensor product can be repeated for the deformed cross products. This manoeuvre will certainly furnish similar results, with pros and cons. In the best cases, the procedure will give the pairs:

$$|\text{residual part}> = - [B]^{-1}.|\text{Grad}_{\text{target}}\ s_2(0_V)>$$

$$[\text{Main part}] = {}_{[A]}\Phi(\text{projectile}) + ½.\ [B]^{-1}.[\text{Hess}_{\text{target}}\ s_2(0_V)]$$

Recall the intrinsic method (personal work) and state that it proposes a totally different formalism for the main part although the work is supposed to decompose the same deformed cross product. Hence, a calibration is needed. In extenso, the coincidence between the main parts must be imposed in writing for the class I kerns:

$${}_{[A]}\Phi(\text{projectile}) + ½.\ [B]^{-1}.[\text{Hess}_{\text{target}}\ s_2(0_V)]$$

$$=$$

$$[A]^t.[J].\{\frac{|A|}{2}.\ [\text{Hess}_{\text{projectile}}\ \Lambda(\text{projectile})] + {}_{[J]}\Phi(\text{singular vector for } \Lambda)\},\ |A| = \pm 1$$

3.3.First kind of scenarios.

A first scenario is for example characterized by the identities:

$${}_{[A]}\Phi(\text{projectile}) = [A]^t.[J].{}_{[J]}\Phi(\text{singular vector for } \Lambda)$$

$$[A]^t.[J] = |A|.[B]^{-1}$$

$$[\text{Hess}_{\text{target}}\ s_2(0_V)] = [\text{Hess}_{\text{projectile}}\ \Lambda(\text{projectile})]$$

- The first identity imposes a strong restriction in the choice of the projectile because it must be the unique singular vector of the non-degenerated polynomial Λ (kerns of class I):

$$\text{projectile} = \text{singular vector for } \Lambda$$

- The second identity directly connects the bilinear form which has been involved in the extrinsic method to the deformation.

- The third one connects the Hessians matrices which are involved in these procedures.

This scenario must be understood as a simplified model for a whole set of similar identities because the identities can be pondered by some arbitrary scalars.

3.3.Second kind of scenarios.

A second generic scenario is for example characterized by the identities:

$$_{[A]}\Phi(\text{projectile}) = [A]^t.[J]._{[J]}\Phi(\text{singular vector for } \Lambda)$$

$$[B]^{-1} = ½.[\text{Hess}_{\text{projectile}}\, \Lambda(\text{projectile})]$$

$$|A|.[A]^t.[J] = ½.[\text{Hess}_{\text{target}}\, s_2(0_V)]$$

The first identity is the same than in the scenarios of the first kind.

The second identity is remarkable in that sense that it formally allows a link with Cartan's work [01] on geometries induced by evolutive surfaces.

The third one connects the Hessian of the polynomial s_2(target) and the deformations of the cross product.

Personal work.

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