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VALUE OF THE COSMOLOGICAL CONSTANT Λ FROM THE COSMOLOGICAL EQUATIONS OF THE UNIVERSE. CONNECTION OF THE COSMOLOGICAL CONSTANT Λ WITH FUNDAMENTAL PHYSICAL CONSTANTS.

Abstract. *A mathematical method for obtaining the value of the cosmological constant Λ from the cosmological equations of the Universe has been found. The method is based on the revealed connection of the cosmological constant Λ with fundamental physical constants. The new large scale numbers 10^140, 10^160 and 10^180 obtained from the scaling law allowed us to obtain cosmological equations linking the cosmological constant Λ with the fine structure constant "alpha", Planck's constant, the speed of light and the electron constants. The approximate Eddington equation Λ≈[(me/αħ)^4][(2Gmp/π)^2] is refined to an exact equation. A large number of new cosmological equations are derived, which include the cosmological constant Λ. The value of the constant Λ is obtained by different methods: from the finalized Eddington equations; from the coincidence of large numbers; from the cosmological equations of the universe and the speed of light; from the cosmological equations of the universe and Planck's constant; from the experimental value of the Pioneer anomaly; from the Kepler relation for the universe. All methods give the same value of the cosmological constant Λ (Λ = 1.36285...x 10^(-52) m^(-2)). The theory based on the law of scaling of large numbers predicts a value of the constant Λ close to the experimental one. The accuracy of the calculated value of Λ is close to the accuracy of the Newtonian constant of gravitation G. The reason for the large number of equivalent equations that include the cosmological constant Λ remains a mystery.*

Keywords: large numbers, cosmological equations, cosmological constant Λ, parameters of the observable Universe, fine-tuning of the Universe.

1. Introduction

Elucidating the physics of the cosmological constant Λ remains one of the most important challenges of science. The value of the constant Λ obtained in quantum field theory is 120 orders of magnitude higher than the experimental value. As Lee Smolin [1] notes: "it must just qualify as the worst prediction ever made by a scientific theory". A new approach is needed to elucidate the essence of the cosmological constant Λ.

Many coincidences of large numbers of order 10^{120}, 10^{140}, 10^{160}, 10^{180} are associated with the constant Λ. Therefore, it is necessary to solve the problem of the cosmological constant within the framework of the problem of coincidence of large numbers. We consider the problem of the cosmological constant as a part of the problem of coincidence of large numbers.

The problem of coincidence of large numbers has arisen more than 100 years ago at attempts to establish a connection between parameters of the universe and fundamental physical constants. Weyl was the first to draw attention to the appearance of a large number of order 10^{40} in the relationship between the radius of the universe and the radius of the electron ($R_U/r_e \approx 10^{40}$) [2].

In 1925 James Rice [3] attempted to relate the constant alpha to the radius of the Universe. He proposed an approximate formula:

$$\frac{4\pi}{\alpha} \approx \frac{r_e^2 c^2}{6R_U G m_e} \qquad (1)$$

Stewart J. O. in 1931 [4, 5] proposed a cosmological equation that relates the constant “alpha» to the Newtonian constant of gravitation G and to the Hubble constant:

$$\frac{G m_e^2}{\hbar H r_e} = \alpha^2 \qquad (2)$$

Eddington tried to find a relationship between the cosmological constant Λ and the fundamental physical constants.He proposed an approximate equation of the form [6, 7, 8]:

$$\boldsymbol{\Lambda \approx (m_e/\alpha\hbar)^4 (2Gm_p/\pi)^2} \qquad (3)$$

Approximate equations and unknown exact values of large numbers did not give the possibility to obtain mathematically the value of the cosmological constant Λ and the value of other parameters of the Universe.The law of scaling of large numbers [9] and new large numbers on scales 10^{140}, 10^{160} and 10^{180} allow us to solve the problem of the cosmological constant and obtain its value from the cosmological equations of the Universe.

2. The law of scaling of large numbers.

The values of large numbers, which follow from the relations of dimensional parameters of the Universe, allowed us to derive the law of scaling of large numbers [9]. The law of scaling of large numbers includes two dimensionless constants: fine structure constant "alpha" and Weyl number.

The law of scaling of large numbers has the form (Fig. 1):

$$D_i = (\sqrt{\alpha D_0})^i$$

$$i = 0, \pm 1, \pm 2, \pm 3, \pm 4, \pm 5, \pm 6, \pm 7, \pm 8, \pm 9.$$

Fig. 1. The scaling law of large numbers. D_0 is a large Weyl number (D_0 = 4.16561...x 10^{42}), α - fine structure constant.

The scaling law (Fig. 1) provides a new method for calculating the values of large numbers from dimensionless constants. The scaling law generates large numbers up to scale 10^{180} with high accuracy. The large numbers obtained from the scaling law are close to the accuracy of the Newtonian constant of gravitation G. The values of the large numbers and the formulas for their calculation are given in Fig. 2.

$$
\begin{aligned}
&(\sqrt{\alpha D_0})^0 = 1\\
D_{20} &= (\sqrt{\alpha D_0})^1 = 1.74349...\bullet 10^{20}\\
D_{40} &= (\sqrt{\alpha D_0})^2 = 3.03979...\bullet 10^{40}\\
D_{60} &= (\sqrt{\alpha D_0})^3 = 5.29987...\bullet 10^{60}\\
D_{80} &= (\sqrt{\alpha D_0})^4 = 9.24033...\bullet 10^{80}\\
D_{100} &= (\sqrt{\alpha D_0})^5 = 16.1105...\bullet 10^{100}\\
D_{120} &= (\sqrt{\alpha D_0})^6 = 28.088...\bullet 10^{120}\\
D_{140} &= (\sqrt{\alpha D_0})^7 = 48.972...\bullet 10^{140}\\
D_{160} &= (\sqrt{\alpha D_0})^8 = 85.383...\bullet 10^{160}\\
D_{180} &= (\sqrt{\alpha D_0})^9 = 148.86...\bullet 10^{180}
\end{aligned}
$$

Fig. 2. Large numbers and formulas for their calculation.

The scaling law gives new large numbers on scales 10^{140}, 10^{160} and 10^{180} and leads to the solution of the cosmological constant problem.

3. The set of coincidences of large numbers associated with the constant Λ.

The table in Fig. 3 summarizes the relations of dimensional quantities containing the constant Λ. These relations yield large numbers. The many large-number fits involving the cosmological constant Λ make it possible to derive exact cosmological equations for various combinations of cosmological parameters and fundamental physical constants.

Ratios of dimensional constants	Scale
$\frac{Gm_e^2}{r_e\alpha^2\hbar\sqrt{\Lambda}c} = \frac{G\hbar}{r_e^3\sqrt{\Lambda}c^3} = \frac{Gm_e}{r_e^2\alpha\sqrt{\Lambda}c^2} = \frac{Gm_e^3}{\alpha^3\hbar^2\sqrt{\Lambda}} = \frac{c^2}{M_U R_U G\Lambda} = \frac{c^2}{M_U G\sqrt{\Lambda}} = \frac{c^2 R_U}{M_U G} = \frac{l_{Pl}^4}{\Lambda r_e^6} = \frac{R_U}{M_U G T_U^2\Lambda} = \frac{c^3 T_U^3\Lambda}{R_U} = . \frac{c r_e^3 A_0}{G\hbar} = \frac{c^2}{M_U R_U \Lambda G} = \frac{M_U R_U\sqrt{\Lambda}A_0 G}{c^4} = 1$	10^0
$D_{20} = \frac{r_e}{l_{Pl}} = \frac{t_0}{t_{Pl}} = \frac{\alpha\, m_{Pl}}{m_e} = \frac{l_{Pl}}{r_e^2\sqrt{\Lambda}} = \frac{l_{Pl}R_U}{r_e^2} = \frac{c^2 l_{Pl}}{r_e^2 A_0} = \sqrt{\alpha D_0}$	10^{20}
$D_{40} = \frac{T_U}{t_0} = \frac{R_U}{r_e} = \frac{m_e c}{\alpha\hbar\sqrt{\Lambda}} = \frac{r_e\alpha c^2}{Gm_e} = \frac{1}{t_0 c\sqrt{\Lambda}} = \frac{r_e^2}{l_{Pl}^2} = \frac{t_0^2}{t_{Pl}^2} = \frac{\alpha^2 m_{Pl}^2}{m_e^2} = \frac{l_{Pl}^2}{r_e^4\Lambda} = \frac{c^2}{r_e A_0} = \frac{1}{r_e\sqrt{\Lambda}} = (\sqrt{\alpha D_0})^2$	10^{40}
$D_{60} = \frac{T_U}{t_{Pl}} = \frac{R_U}{l_{Pl}} = \frac{M_U}{m_{Pl}} = \frac{1}{l_{Pl}\sqrt{\Lambda}} = \frac{r_e^3}{l_{Pl}^3} = \frac{t_0^3}{t_{Pl}^3} = \frac{c^2}{Gm_{Pl}\sqrt{\Lambda}} = \frac{c^2}{l_{Pl}A_0} = (\sqrt{\alpha D_0})^3$	10^{60}
$D_{80} = \frac{R_U^2}{r_e^2} = \frac{\sqrt{\Lambda}M_U^2\alpha G}{c^2 m_e} = \frac{cT_U}{r_e^2\sqrt{\Lambda}} = \frac{r_e}{\sqrt{\Lambda}l_{Pl}^2} = \frac{1}{r_e^2\Lambda} = (\sqrt{\alpha D_0})^4$	10^{80}
$D_{100} = \frac{m_e c}{l_{Pl}\alpha\hbar\Lambda} = \frac{r_e\alpha M_U}{l_{Pl}m_e} = \frac{\sqrt{\Lambda}M_U^2\alpha G r_e}{c^2 m_e l_{Pl}} = \frac{R_U^2}{r_e l_{Pl}} = \frac{1}{r_e l_{Pl}\Lambda} = (\sqrt{\alpha D_0})^5$	10^{100}
$D_{120} = \frac{T_U^2}{t_{Pl}^2} = \frac{R_U^2}{l_{Pl}^2} = \frac{M_U^2}{m_{Pl}^2} = \frac{R_U}{r_e^3\Lambda} = \frac{M_U c}{\hbar\sqrt{\Lambda}} = \frac{GM_U^2}{\hbar c} = \frac{c^3}{G\hbar\Lambda} = \frac{1}{l_{Pl}^2\Lambda} = (\sqrt{\alpha D_0})^6$	10^{120}
$D_{140} = \frac{r_e^2 m_e c}{l_{Pl}^3\alpha\hbar\Lambda} = \frac{r_e^3\alpha M_U}{l_{Pl}^3 m_e} = \frac{R_U^3}{l_{Pl}r_e^2} = \frac{1}{c^3 t_{Pl}t_0^2\sqrt{\Lambda}\Lambda} = \frac{1}{l_{Pl}r_e^2\Lambda\sqrt{\Lambda}} = \frac{c^2}{l_{Pl}r_e^2 A_0\Lambda} = (\sqrt{\alpha D_0})^7$	10^{140}
$D_{160} = \frac{M_U c^2\alpha^2}{Gm_e^2\sqrt{\Lambda}} = \frac{M_U^2 G\alpha}{c^2 r_e^2 m_e\sqrt{\Lambda}} = \frac{1}{r_e^4\Lambda^2} = \frac{r_e^2}{l_{Pl}^4\Lambda} = (\sqrt{\alpha D_0})^8$	10^{160}
$D_{180} = \frac{r_e^4 m_e c}{l_{Pl}^5\alpha\hbar\Lambda} = \frac{r_e^5\alpha M_U}{l_{Pl}^5 m_e} = \frac{R_U}{l_{Pl}^3\Lambda} = \frac{1}{l_{Pl}^3\Lambda\sqrt{\Lambda}} = \frac{c^2}{l_{Pl}^3 A_0\Lambda} = \frac{GM_U T_U^2 l_{Pl}}{\Lambda r_e^6} = \frac{1}{r_e^3\Lambda^2 l_{Pl}} = (\sqrt{\alpha D_0})^9$	10^{180}

Fig. 3. Relations of dimensional quantities containing the constant Λ. M_U is the mass of the observable Universe, α is the fine structure constant, ħ is Planck's constant, G is the Newtonian gravitational constant, Λ is the cosmological constant, R_U is the radius of the observable Universe, T_U is the time of the Universe, A_0 is the cosmological acceleration, r_e is the classical radius of the electron; c - speed of light in vacuum; $t_0 = r_e/c$, m_e - electron mass, D_0 - large Weyl number, t_{pl} - Planck time, l_{pl} - Planck length, m_{pl} - Planck mass.

4. Refinement of the approximate Eddington equation to the exact equation

The **GΛ**-equation of Eddington [6, 7, 8] is an approximate equation. At a scale of 10^{40}, the exact equations that relates the constants **G** and **Λ** are obtained (Fig. 4). From the exact equations the value of the constant is obtained: $\Lambda = 1.36285... \times 10^{-52}$ m^{-2}.

Approximate Eddington equation	Exact equation	Value of Λ
$\Lambda \approx (m_e/\alpha\hbar)^4(2Gm_p/\pi)^2$	$\Lambda = G^2m_e^4/\alpha^4\hbar^2r^2c^2$	$\Lambda = 1.36285. \bullet 10^{-52} m^{-2}$
	$\Lambda = G^2m_e^2/\alpha^2r^4c^4$	$\Lambda = 1.36285. \bullet 10^{-52} m^{-2}$

Fig. 4. Eddington's equation refined to exact equivalent equations.

5. The constant Λ in cosmological equations

Below are new cosmological equations, which contain in one equation the cosmological constant Λ and parameters of the Universe in different combinations - 2 parameters, 3 parameters, 4 parameters, 5 parameters, 6 parameters. The table in Fig. 5 shows cosmological equations that include the cosmological constant Λ and one parameter of the Universe:

Name	Parameters Universe	Formulas	Note	Value
GΛ-equations	G, Λ	$\frac{c^3 r_e^3\sqrt{\Lambda}}{G} = \hbar$ $G\hbar/r_e^3\sqrt{\Lambda} = c^3$ $G\hbar\alpha^3D_0^3\Lambda = c^3$ $c^3/G\hbar\Lambda=(\alpha m_{Pl}/m_e)^6$ $\Lambda= G^2m_e^2/\alpha^2r^4c^4$ $\Lambda = G^2m_e^4/\alpha^4\hbar^2r^2c^2$ $\frac{Gm_e}{\sqrt{\Lambda}r_e^2c^2} = \alpha$ $\Lambda = G^2m_e^6/\alpha^6\hbar^4$	[10] [11], [12]	$\Lambda = 1.36285 \bullet 10^{-52} m^{-2}$
MΛ-equations	M_U, Λ	$M_U\Lambda c r_e^3 = \hbar$ $\frac{m_e}{M_U\Lambda r_e^2} = \alpha$ $M\Lambda\alpha r_e^2 = m_e$ $\hbar\alpha^3D_0^3 = \frac{M_U c}{\sqrt{\Lambda}}$		$\Lambda = 1.36285. \bullet 10^{-52} m^{-2}$ $M_U = 1.15348... \bullet 10^{53} kg$
TΛ-equations	T_U, Λ	$\Lambda T_U^2c^2 = 1$		$\Lambda = 1.36285 \bullet 10^{-52} m^{-2}$ $T_U = 2.85729 ... \bullet 10^{17} s$
A_0Λ-equations	A_0, Λ	$A_0^2 = \Lambda c^4$		$\Lambda = 1.36285.. \bullet 10^{-52} m^{-2}$ $A_0 = 10.4922.. \bullet 10^{-10} m/s$

Fig. 5. Cosmological equations that include the cosmological constant Λ and one parameter of the Universe. Where : α - fine-structure constant, ħ - Planck constant, M_U - mass of the observable Universe, G - Newtonian constant of gravitation, Λ - cosmological constant, R_U - radius of the observable Universe, A_0 - cosmological acceleration, r_e - classical electron radius; c - speed of light in vacuum; m_e - electron mass, D_0 - large number, r_e - classical electron radius; c - speed of light in vacuum; m_e - electron mass.

The table in Fig. 6 shows cosmological equations, which include the cosmological constant Λ and 2 parameters of the Universe:

Name	Parameters Universe	Formulas	Note	Value
GMΛ-equations	G, M_U, Λ	$M_U^2G^2\Lambda = c^4$ $M_UG\sqrt{\Lambda} = c^2$	[12]	$M_U = 1.15348... \bullet 10^{53} kg$ $\Lambda = 1.36285... \bullet 10^{-52} m^{-2}$
$GA_0\Lambda$-equations	G, A_0, Λ	$\frac{c^5 r_e^3 \Lambda}{GA_0} = \hbar$, $\frac{Gm_e A_0}{\Lambda c^4 r_e^2} = \alpha$		$\Lambda = 1.36285 ... \bullet 10^{-52} m^{-2}$ $A_0 = 10.4922 ... \bullet 10^{-10} m/s$
GRΛ-equations	G, R_U, Λ	$\frac{R_U \Lambda c^3 r_e^3}{G} = \hbar$, $\frac{Gm_e}{R_U \Lambda c^2 r_e^2} = \alpha$		$R_U = 0.856594... \bullet 10^{26} m$ $\Lambda = 1.36285 ... \bullet 10^{-52} m^{-2}$
ΛRT-equations	Λ, R_U, T_U	$\Lambda c R_U T_U = 1$ $R_U/T_U^3\Lambda = c^3$		$R_U = 0.856594... \bullet 10^{26} m$ $T_U = 2.85729 ... \bullet 10^{17} s$ $\Lambda = 1.36285... \bullet 10^{-52} m^{-2}$
MRΛ-equations	M_U, R_U, Λ	$M_U R_U \Lambda c r_e^3 \sqrt{\Lambda} = \hbar$ $M_U R_U \Lambda r_e^2 \alpha \sqrt{\Lambda} = m_e$		$M_U = 1.15348... \bullet 10^{53} kg$ $R_U = 0.856594 ... \bullet 10^{26} m$ $\Lambda = 1.36285 ... \bullet 10^{-52} m^{-2}$
$A_0\Lambda T$-equations	A_0, Λ, T_U	$A_0/\Lambda^2 T_U^3 = c^5$		$A_0 = 10.4922... \bullet 10^{-10} m/s$ $\Lambda = 1.36285 ... \bullet 10^{-52} m^{-2}$ $T_U = 2.85729... \bullet 10^{17} s$

Fig. 6. Cosmological equations that include the cosmological constant Λ and 2 parameters of the Universe.

The table in Fig. 7 shows cosmological equations that include the cosmological constant Λ and 3 parameters of the Universe:

Name	Parameters Universe	Formulas	Value
$GA_0R\Lambda$-equations	**G, A_0, R_U, Λ**	**$A_0c^2r_e^2R_U\Lambda = Gm_e$**	$R_U = 0.856594... \bullet 10^{26}\, m$ $\Lambda = 1.36285... \bullet 10^{-52}\, m^{-2}$ $A_0 = 10.4922... \bullet 10^{-10}\, m/s$
$GMR\Lambda$-equations	**G, M_U, R_U, Λ**	**$M_UR_U\Lambda G = c^2$** **$R_U^2c^2\sqrt{\Lambda} = GM_U$**	$M_U = 1.15348 ... \bullet 10^{53}\, kg$ $R_U = 0.856594... \bullet 10^{26}\, m$ $\Lambda = 1.36285 ... \bullet 10^{-52}\, m^{-2}$
$GMT\Lambda$-equations	**G, M_U, T_U, Λ**	**$M_UGT_U\Lambda = c$**	$M_U = 1.15348. \bullet 10^{53}\, kg$ $T_U = 2.85729... \bullet 10^{17}\, s$ $\Lambda = 1.36285 ... \bullet 10^{-52}\, m^{-2}$
$GM\Lambda A_0$-equations	**G, M_U, Λ, A_0**	**$M_U\Lambda G = A_0$** **$M_UA_0^2G/\sqrt{\Lambda} = c^6$**	$M_U = 1.15348.. \bullet 10^{53}\, kg$ $\Lambda = 1.36285 ... \bullet 10^{-52}\, m^{-2}$ $A_0 = 10.4922.. \bullet 10^{-10}\, m/s$
$M\Lambda TA_0$-equations	**M_U, Λ, T_U, A_0**	**$M_U\Lambda T_UA_0r_e^2\alpha = m_ec$**	$M_U = 1.15348. \bullet 10^{53}\, kg$ $\Lambda = 1.36285 ... \bullet 10^{-52}\, m^{-2}$ $A_0 = 10.4922 ... \bullet 10^{-10}\, m/s$ $T_U = 2.85729.. \bullet 10^{17}\, s$
$A_0\Lambda RT$-equations	**A_0, Λ, R_U, T_U**	**$A_0/\Lambda^2R_UT_U^3\sqrt{\Lambda} = c^5$**	$R_U = 0.856594.. \bullet 10^{26}\, m$ $T_U = 2.85729... \bullet 10^{17}\, s$ $\Lambda = 1.36285 ... \bullet 10^{-52}\, m^{-2}$ $A_0 = 10.4922... \bullet 10^{-10}\, m/s$

Fig. 7. Cosmological equations that include the cosmological constant Λ and 3 parameters of the Universe.

The table in Fig. 8 shows cosmological equations that include the cosmological constant Λ and 4 parameters of the Universe:

Name	Parameters Universe	Formulas	Value
$MGT\Lambda R$ -equations	**$M_U, G, T_U, \Lambda, R_U$**	**$M_UGT_U^2\Lambda = R_U$** **$M_UR_U\Lambda^2GT_U^2 = 1$**	$M_U = 1.15348... \bullet 10^{53}\, kg$ $R_U = 0.856594 ... \bullet 10^{26}\, m$ $T_U = 2.85729 ... \bullet 10^{17}\, s$ $\Lambda = 1.36285 ... \bullet 10^{-52}\, m^{-2}$
$GMR\Lambda A_0$ -equations	**$G, M_U, R_U, \Lambda, A_0$**	**$GM_UR_UA_0\sqrt{\Lambda} = c^4$** **$M_UR_U\Lambda^2Gc^2 = A_0^2$**	$M_U = 1.15348 ... \bullet 10^{53}\, kg$ $R_U = 0.856594... \bullet 10^{26}\, m$ $\Lambda = 1.36285 ... \bullet 10^{-52}\, m^{-2}$ $A_0 = 10.4922 ... \bullet 10^{-10}\, m/s$

Fig. 8. Cosmological equations that include the cosmological constant Λ and 4 parameters of the Universe.

The table in Fig. 9 shows cosmological equations that include the cosmological constant Λ and 5 parameters of the Universe:

Name	Parameters Universe	Formulas	Value
MRΛGA₀T- equations	**M_U, R_U, Λ, G, A_0, T_U**	**$M_U R_U \Lambda^2 G A_0 T_U^3 = c$** **$M_U R_U \Lambda G A_0 T_U = c^3$** **$M_U R_U \Lambda G A_0 T_U^2 \sqrt{\Lambda} = c^2$**	$M_U = 1.15348... \bullet 10^{53}\, kg$ $R_U = 0.856594... \bullet 10^{26}\, m$ $T_U = 2.85729\ ... \bullet 10^{17}\, s$ $\Lambda = 1.36285\ ... \bullet 10^{-52}\, m^{-2}$ $A_0 = 10.4922\ ... \bullet 10^{-10}\, m/s$

Fig. 9. Cosmological equations that include the cosmological constant Λ and 5 parameters of the Universe.

6. Value of the constant Λ from the coincidence of large numbers.

The value of Λ from the coincidence of large numbers on a scale of 10^0

$$\Lambda = \frac{l_{Pl}^4}{r_e^6} = 1.36285...x\ 10^{-52}\ m^{-2} \qquad (4)$$

A similar equation is given in [13, 14].

The value of Λ from the coincidence of large numbers on a scale of 10^{20}

$$\Lambda = \frac{l_{Pl}^2}{r_e^4 \alpha D_0} = 1.36285...x\ 10^{-52}\ m^{-2} \qquad (5)$$

Value Λ from the coincidence of large numbers on the scale 10^{40}

$$\Lambda = \frac{1}{r_e^2(\sqrt{\alpha D_0})^4} = 1.36285...x\ 10^{-52}\ m^{-2} \qquad (6)$$

Value Λ from the coincidence of large numbers on the scale 10^{60}

$$\Lambda = \frac{1}{l_{Pl}^2(\sqrt{\alpha D_0})^6} = 1.36285...x\ 10^{-52}\ m^{-2} \qquad (7)$$

Value Λ from the coincidence of large numbers on the scale 10^{80}

$$\Lambda = \frac{1}{r_e^2(\sqrt{\alpha D_0})^4} = 1.36285...x\ 10^{-52}\ m^{-2} \qquad (8)$$

Value Λ from the coincidence of large numbers on the scale 10^{100}

$$\Lambda = \frac{1}{r_e l_{Pl}(\sqrt{\alpha D_0})^5} = 1.36285...x\ 10^{-52}\ m^{-2} \qquad (9)$$

Value Λ from the coincidence of large numbers on the scale 10^{120}

$$\Lambda = \frac{1}{l_{Pl}^2(\sqrt{\alpha D_0})^6} = 1.36285...x\ 10^{-52}\ m^{-2} \qquad (10)$$

Value Λ from the coincidence of large numbers on the scale 10^{140}

$$\Lambda = \frac{r_e^2 m_e c}{l_{Pl}^3 \alpha\hbar(\sqrt{\alpha D_0})^7} = 1.36285...\text{x } 10^{-52}\ \text{m}^{-2} \qquad (11)$$

Value Λ from the coincidence of large numbers on the scale 10^{160}

$$\Lambda = \frac{r_e^2}{l_{Pl}^4 (\sqrt{\alpha D_0})^8} = 1.36285...\text{x } 10^{-52}\ \text{m}^{-2} \qquad (12)$$

Value Λ from the coincidence of large numbers on the scale 10^{180}

$$\Lambda = \sqrt{\frac{1}{r_e^3 l_{Pl} (\alpha D_0)^9}} = 1.36285...\text{x } 10^{-52}\ \text{m}^{-2} \qquad (13)$$

7. Value of the cosmological constant Λ from the Pioneer anomaly

The magnitude of the Pioneer effect is in the range of (7.41 - 10.07) × 10^{-10} m/s^2. The value of the cosmological acceleration (A_0 = 10.4922 × 10^{-10} m/s^2) follows from the system of equations (Fig. 10):

$$\begin{cases} \dfrac{c^5 r_e^3}{M_U G^2} = \hbar \\[2ex] \dfrac{m_e c^4}{M_U A_0^2 r_e^2} = \alpha \end{cases}$$

Fig. 10. System of cosmological equations, which includes the parameters **G, M, Λ**.

In addition to the Pioneer-10 and Pioneer-11 experiment, there is anomalous acceleration data from Galileo and Ulysses [15 - 18].

For Galileo:

a_0 = (8±3) x 10^{-10} m/s^2

For Ulysses:

a_0 = (12±3) x 10^{-10} m/s^2

In [9], the value of **A_0 = 10.4922... x 10^{-10} m/s^2**. From the coincidence of large numbers on a scale of 10^{40}, the following equation follows:

$$\frac{c^2}{r_e A_0} = \frac{1}{r_e \sqrt{\Lambda}} \qquad (14)$$

From equation (14), the value of the cosmological constant follows:

$$\mathbf{\Lambda = A_0^2 / c^4 = 1.36285...x\ 10^{-52}\ m^{-2}}$$

8. The value of Λ from the Kepler relation.

In [19], the value of Kepler's relation for the Universe is obtained:

$$\boldsymbol{\mu}_{\mathbf{U}} = GM_U = \frac{R_U^3}{T_U^2} = 7.69868...\text{x } 10^{42}\ \text{m}^3\text{s}^{-2} \qquad (15)$$

The Kepler ratio $\mathbf{R_U^3/T_U^2}$ can be represented using the cosmological constant by the following equivalent formulas:

$$\boldsymbol{\mu}_{\mathbf{U}} = GM_U = \frac{R_U^3}{T_U^2} = \frac{A_0}{\Lambda} = \frac{A_0^2}{c^2\Lambda\sqrt{\Lambda}} = \frac{c^2}{\sqrt{\Lambda}} = \frac{1}{T_U^2\Lambda\sqrt{\Lambda}} = 7.69868...\text{x } 10^{42}\ \text{m}^3\text{s}^{-2} \qquad (16)$$

All equivalent formulas (16) for the Kepler relation give the same value of the cosmological constant:

$$\Lambda = 1.36285...\bullet 10^{-52}\, m^{-2}$$

9. The value of the constant Λ from the cosmological equations of the universe

Among the new cosmological equations were those that contain the cosmological constant Λ and Planck's constant.

$$M_U\Lambda c r_e^3 = \hbar\,, \qquad \frac{R_U\Lambda c^3 r_e^3}{G} = \hbar\,, \qquad m_e c r_e^2\sqrt{\Lambda D_0^2} = \hbar\,,$$

$$\frac{c^5 r_e^3\Lambda}{GA_0} = \hbar\,, \qquad \frac{c^3 t_e^3\sqrt{\Lambda}}{G} = \hbar\,, \qquad \frac{M_U r_e^3\sqrt{\Lambda}}{T_U} = \hbar\,.$$

Fig. 11. Cosmological constant Λ and Planck constant in cosmological equations. Where : ħ - Planck constant, M_U - mass of the observable Universe, G - Newtonian constant of gravitation, Λ - cosmological constant, R_U - radius of the observable Universe, A_0 - cosmological acceleration, r_e - classical electron radius; c - speed of light in vacuum; m_e - electron mass, Do - large number.

All equations give the following values of Λ:

$$\Lambda = 1.36285...\bullet 10^{-52}\, m^{-2}$$

Among the new cosmological equations were those that contain the cosmological constant Λ and the speed of light constant.

$$
\begin{gathered}
\mathbf{M_U R_U \Lambda^2 G T_U^2 = c^0} \\
\mathbf{M_U R_U \Lambda^2 G A_0 T_U^3 = c} \\
\mathbf{M_U G \sqrt{\Lambda} = c^2} \\
\mathbf{R_U / T_U^3 \Lambda = c^3} \\
\mathbf{G^2 m_e^2 / \alpha^2 r^4 \Lambda = c^4} \\
\mathbf{A_0 / \Lambda^2 T_U^3 = c^5} \\
\mathbf{M_U A_0^2 G / \sqrt{\Lambda} = c^6}
\end{gathered}
$$

Fig. 12. The cosmological constant Λ and the speed of light in cosmological equations.

All equations give the following values of Λ:

$$\Lambda = 1.36285... \bullet 10^{-52} m^{-2}$$

Fig. 13 shows 10 equivalent formulas for calculating the cosmological constant Λ.

$$\mathbf{\Lambda} = \left\{ \begin{matrix} \dfrac{1}{r_e^2 \alpha^2 D_0^2}, & \dfrac{m_{Pl}^2}{l_{Pl}^2 M_U^2}, & \dfrac{\hbar c}{l_{Pl}^2 M_U^2 G}, & \\ \dfrac{A_0}{G M_U}, & \dfrac{1}{R_U^2}, & \dfrac{r_e^3}{l_{Pl}^2 R_U^3}, & \dfrac{c^4}{G^2 M_U^2}, \\ \dfrac{c^2}{M_U R_U G}, & \dfrac{R_U}{T_U^3 c^3}, & \dfrac{\hbar}{M_U c r_e^3}. & \end{matrix} \right\} = \mathbf{1.36285 ... x\ 10^{-52}\ m^{-2}}$$

Fig. 13. Equivalent formulas for calculating the cosmological constant Λ.

All equivalent formulas give the same value Λ = 1.36285...x 10^{-52} m^{-2}. A very close value of Λ (Λ = 1.36281(41) x 10^{-52} m^{-2}.) was obtained by L. Nottale [13, 20].

Thus the theory based on the law of scaling of large numbers predicts the following value of the constant Λ:

$$\boxed{\Lambda = 1.36285... \bullet 10^{-52} m^{-2}}$$

Fig. 14. Calculated value of the cosmological constant Λ.

Despite the "abundance" of equations for the cosmological constant Λ, this is only a part of possible equivalent formulas for calculating Λ. There are many more equivalent formulas. The constant Λ is included in many other relations of dimensional constants that yield large numbers.

Many formulas for calculating the cosmological constant Λ lead to many relations that give coincidences of large numbers.

10. Conclusion

The theory based on the law of scaling of large numbers predicts the value of the constant Λ close to the experimental one. The accuracy of the obtained value of Λ is close to the accuracy of the Newtonian constant of gravitation G. The new large numbers on scales 10^{140}, 10^{160} and 10^{180} allowed us to obtain new cosmological equations linking the constant Λ with fundamental physical constants. The number of new cosmological equations turned out to be very large. On the one hand, it became possible to combine the equations into systems of cosmological equations. This allowed us to obtain by mathematical method the value of the constant Λ and other parameters of the Universe. On the other hand, the "abundance" of equivalent equations indicates the existence of some mystery of the constant Λ.

The large number of cosmological equations that contain the constant Λ remains a mystery. This problem is related to the problem of coincidence of large numbers. The large number of equivariant cosmological equations that contain the constant Λ and the problem of coincidence of large numbers are closely related. The origins of these problems should be sought in their connection with fundamental physical constants. Something very important, but not yet known to science, is stored in the fundamental physical constants.

References

1. Lee Smolin. The trouble with physics: the rise of string theory, the fall of a science, and what comes next. — Boston: Houghton Mifflin, 2006.
2. H.Weyl, Raum-Zeit-Materie (Berlin,1918).
3. Rice J. On Eddington's natural unit of the field // Philosophical Magazine Series 6. — 1925. — Vol. 49. — P. 1056—1057.
4. Stewart J. O. “Phys. Rev.”, 1931, v. 38, p. 2071.
5. Muradyan, R. M. Universal constants. Physical and astrophysical constants and their dimensional and dimensionless combinations. Fiz. ehlementar. chastits i atom. yadra, Vol. 8, No. 1, p. 175 - 192. 1977.
6. A.S. Eddington, Proc. Roy. Soc. A 133, 605 (1931)
7. A.S. Eddington, Proc. Phys. Soc. 44, 1 (1932)
8. A.S. Eddington, Mon. Not. R. Astron. Soc. 104, 200 (1944)
9. Mykola Kosinov. THE LAW OF SCALING FOR LARGE NUMBERS: origin of large numbers from the primary large number D20 = 1.74349...x 10^20. January 2024. DOI: 10.13140/RG.2.2.33664.20480
10. Mykola Kosinov. Unexpected connection of the parameters of the observed Universe with the fine structure constant alpha. DOI: 10.13140/RG.2.2.24039.42408
11. Christian Beck, Axiomatic approach to the cosmological constant, Physica A: Statistical Mechanics and its Applications, Volume 388, Issue 17, 2009, Pages 3384-3390, ISSN 0378-4371, https://doi.org/10.1016/j.physa.2009.05.023.

12. Pierre-Henri Chavanis. A mass scale law connecting cosmophysics to microphysics. Physics of the Dark Universe, 2024, 44, pp.101420. 10.1016/j.dark.2024.101420. hal-04018537
13. L. Nottale, Fractal Space-Time and Microphysics (World Scientific, London, (1993).
14. Laurent Nottale. Solution to the cosmological constant problem from dark energy of the quark vacuum. 2019. ⟨hal-02133255⟩.
15. Turyshev, S. G.; Toth, V. T.; Ellis, J.; Markwardt, C. B. (2011). "Support for temporally varying behavior of the Pioneer anomaly from the extended Pioneer 10 and 11 Doppler data sets". Physical Review Letters. 107 (8): 81103. arXiv:1107.2886
16. John D. Anderson, Philip A. Laing, Eunice L. Lau, Anthony S. Liu, Michael Martin Nieto, Slava G. Turyshev. Study of the anomalous acceleration of Pioneer 10 and 11 // Physical Review D. — 2002. — Vol. 65, no. 8. — P. 082004.
17. Mizony, M.; Lachièze-Rey, M. (2005). "Cosmological effects in the local static frame". Astronomy and Astrophysics. 434 (1): 45–52. doi:10.1051/0004-6361:20042195.
18. Lachièze-Rey, M. (2007). "Cosmology in the solar system: the Pioneer effect is not cosmological". Classical and Quantum Gravity. 24 (10): 2735–2742. doi:10.1088/0264-9381/24/10/016
19. Mykola Kosinov. Extending the limits of applicability of Kepler's third law to electromagnetism and cosmology. February 2024. DOI: 10.13140/RG.2.2.33381.58087
20. Nottale, L. (1993) Mach's Principle, Dirac's Large Numbers and the Cosmological Constant Problem. http://www.luth.obspm.fr/%7Eluthier/nottale/arlambda.pdf