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New large numbers of scales 10^{140} , 10^{160} , 10^{180} , new cosmological equations and a mathematical method for obtaining the parameters of the universe.

***Abstract.** Measurements of the parameters of the observed Universe is a very difficult task and does not give the necessary accuracy. A mathematical method for obtaining the parameters of the Universe has been found. The method is based on the revealed relationship between the parameters of the Universe and the dependence of their values on the fundamental physical constants. New large numbers on the previously unknown scales 10^{140} , 10^{160} and 10^{180} were derived. The new large numbers allowed us to obtain new cosmological equations linking the parameters of the Universe with fundamental physical constants. The number of new cosmological equations and their constituent parameters was sufficient to unite the equations into a system of cosmological equations. This made it possible to form a system of algebraic equations containing all parameters of the Universe. As a result, it became possible to obtain the parameters of the Universe by mathematical method. The parameters of the Universe are the roots of the system of algebraic equations of the Universe. The theory based on the law of scaling of large numbers allows us to obtain the parameters of the observed Universe with an accuracy close to the accuracy of the Newtonian constant of gravitation G . The results obtained show that the Universe is tuned with high mathematical accuracy.*

***Keywords:** large numbers, cosmological equations, parameters of the observable Universe, fine-tuning of the Universe.*

1. Introduction

Large numbers in physics and cosmology and their multiple coincidences have attracted the attention of many scientists for more than 100 years. For more than 100 years the problem of coincidence of large numbers has not found a solution. The coincidence of large numbers was first noticed by H. Weyl. He also obtained large numbers as the ratio of the electric force to the gravitational force for the electron ($\approx 10^{42}$) and the ratio of the radius of the Universe to the radius of the electron ($\approx 10^{40}$) [1]. In attempts to explain the coincidences of large numbers, well-known scientists have proposed cosmological equations that include the parameters of the observable Universe. These include the Dirac equation [2, 3], Stewart equation [4, 5], Eddington-Weinberg equation [6, 7], Teller equation [8, 9], Rice equation [10], Nottale equation [11, 12], Milne equation [13, 14], Bleksley equation [15], Hoyle equation [16], Carvalho equation [17].

Some equations are approximate and the coincidences in them are evaluated only by order of magnitude. In [18], the approximate equations are refined to exact equations. The table in Fig. 1 summarizes the known cosmological equations obtained in different years.

Name	Parameters Universe	Formula	Note
Weyl's equations	G, R_U	$e^2/Gm_e^2 = 4.17 \times 10^{42}$ $R_U/re \approx 10^{40}$	1918, [1]
Approximate equation Rice	G, R_U	$\frac{4\pi}{\alpha} \approx \frac{r_e^2 c^2}{6R_U G m_e}$	1925, [10]
Exact equation		$\frac{R_U G m_e}{r_e^2 c^2} = \alpha$	
Stewart's equation	G, H	$Gm_e^2 / r_e = \alpha^2 \hbar H$	1931, [4]
Milne equation	G, M_U, T	$M_U = c^3 T_U / G$	1936, [13]
Approximate Dirac equation	G, H	$m_e c^3 / (H e^2) \approx e^2 / (G m_e^2)$	1937, [2,3]
Exact equation		$m_e c^3 / (H e^2) = \alpha e^2 / (G m_e^2)$	[18]
Bleksley equation	G, M_U, R_U	$M_U = c^2 R_U / G$	1951, [15]
Approximate Eddington- Weinberg equation	G, H	$\hbar^2 H \approx G m_p^3$	1972, [6]
Exact equation		$\hbar^2 H \alpha^3 = G m_e^3$	[18,19] [20]
Approximate Teller's equation	G, H	$2 \frac{G \hbar H}{c^4 l_{pl}} = 2 t_{pl} H = 2 \frac{G m_{pl} H}{c^3} \cong \exp(-1/\alpha)$	1948 [8]
Exact equation		$\frac{G \hbar H}{c^4 l_{pl}} = t_{pl} H = \frac{G m_{pl} H}{c^3} = (\sqrt{\alpha D_0})^{-3}$	[18]
Approximate Nottale equation	G, Λ	$c^3 / G \hbar \Lambda \approx 10^{120}$	1993 [11,12]
Exact equation		$c^3 / G \hbar \Lambda = \alpha^3 D_0^3$	[18]
Carvalho equation	G, M_U, H	$M_U = c^3 / GH$	1995, [22]

Fig. 1. Cosmological equations containing the parameters of the Universe. **M_U** is the mass of the observable Universe, α is the fine structure constant, \hbar is Planck's constant, **G** is the Newtonian gravitational constant, Λ is the cosmological constant, **R_U** is the radius of the observable Universe, **T_U** is the time of the Universe, **H** is the Hubble constant, **r_e** is the classical radius of the electron; **c** - speed of light in vacuum; **m_e** - electron mass, **D₀** - large Weyl number, **t_{pl}** - Planck time, **l_{pl}** - Planck length, **m_{pl}** - Planck mass.

The known cosmological equations contain a limited number of parameters of the Universe. Equations link 2 and 3 parameters of the Universe. Equations linking more than 3 parameters of the Universe are unknown. The limited number of parameters of the Universe in equations did not allow us to obtain the values of the parameters of the Universe. The same reason did not give an opportunity to reveal the possible relationship between the parameters of the Universe. An even greater unsolved mystery remains the relationship between the parameters of the Universe and the fundamental physical constants.

New large numbers, obtained from the law of scaling of large numbers [21], lead to new cosmological equations linking more than 3 parameters of the Universe. These results open new possibilities for astrophysics and cosmology.

2. The law of scaling of large numbers as a function of the fine structure constant “alpha”

The values of large numbers that follow from the relations of the dimensional parameters of the Universe allowed us to derive the law of scaling of large numbers [21]. The law of scaling of large numbers is represented by the degree dependence of the combination of two dimensionless constants: fine structure constant alpha and the Weyl number.

The law of scaling of large numbers has the form (Fig. 2).

$$D_i = (\sqrt{\alpha D_0})^i$$

$$i = 0, \pm 1, \pm 2, \pm 3, \pm 4, \pm 5, \pm 6, \pm 7, \pm 8, \pm 9.$$

Fig. 2: Fine structure constant alpha in the scaling law of large numbers. D_0 is a large Weyl number ($D_0 = 4.16561... \times 10^{42}$), α is the fine structure constant.

The scaling law gives a new method of calculating the values of large numbers. Its peculiarity is that the large numbers are obtained from the dimensionless fine structure constants alpha and Weyl number. The scaling law generates large numbers up to scale 10^{180} with high accuracy. The large numbers obtained from the scaling law are close to the accuracy of the Newtonian constant of gravitation G. This opens the possibility to obtain by mathematical method the parameters of the Universe with accuracy close to the accuracy of the Newtonian constant of gravitation G. The values of the large numbers and the formulas for their calculation are given in Fig. 3.

$$(\sqrt{\alpha D_0})^0 = 1$$

$$D_{20} = (\sqrt{\alpha D_0})^1 = 1.74349... \cdot 10^{20}$$

$$D_{40} = (\sqrt{\alpha D_0})^2 = 3.03979... \cdot 10^{40}$$

$$D_{60} = (\sqrt{\alpha D_0})^3 = 5.29987... \cdot 10^{60}$$

$$D_{80} = (\sqrt{\alpha D_0})^4 = 9.24033... \cdot 10^{80}$$

$$D_{100} = (\sqrt{\alpha D_0})^5 = 16.1105... \cdot 10^{100}$$

$$D_{120} = (\sqrt{\alpha D_0})^6 = 28.088... \cdot 10^{120}$$

$$D_{140} = (\sqrt{\alpha D_0})^7 = 48.972... \cdot 10^{140}$$

$$D_{160} = (\sqrt{\alpha D_0})^8 = 85.383... \cdot 10^{160}$$

$$D_{180} = (\sqrt{\alpha D_0})^9 = 148.86... \cdot 10^{180}$$

Fig. 3. Family of large numbers and formulas for their computation from the law of scaling large numbers.

3. Extension of the family of large numbers to scales 10^{140} , 10^{160} and 10^{180} .

Studies of large numbers usually end with the scale 10^{120} . At the same time, it is well known that the ratio of the volume of the Universe to the Planck volume gives a large number of scale 10^{180} . The law of scaling of large numbers showed that the large number of scale 10^{120} is not the maximum number in the family of large numbers. The law of scaling has led to new large numbers on scales 10^{140} , 10^{160} and 10^{180} . The values of the large numbers ($D_{140} = 48.972 \dots \times 10^{140}$, $D_{160} = 85.383 \dots \times 10^{160}$, $D_{180} = 148.86 \dots \times 10^{180}$) were confirmed by dimensional relationships. The relations of dimensional quantities coincided with the large numbers obtained from the mesoscaling law with high accuracy. As a result, new cosmological equations are obtained on scales 10^{140} , 10^{160} and 10^{180} , which demonstrate a fundamental relationship between the parameters of the Universe [21, 22, 23].

4. Ten scales and ten large numbers from the law of scaling of large numbers

The table in Fig. 4 summarizes the dimensional relations that lead to large numbers at 10 scales. The many coincidences of large numbers make it possible to derive cosmological equations for various combinations of cosmological parameters and fundamental physical constants.

Ratios of dimensional constants		Scale
$\frac{Gm_e^2}{r_e \alpha^2 \hbar H} = \frac{G\hbar}{r_e^3 H c^2} = \frac{Gm_e}{r_e^2 \alpha H c} = \frac{Gm_e^3 c}{\alpha^3 \hbar^2 H} = \frac{c^2}{M_U R_U G \Lambda} = \frac{c^3}{M_U G H} = \frac{c^2 R_U}{M_U G} = \frac{l_{Pl}^4}{\Lambda r_e^6} =$ $= \frac{c^3 T_U}{M_U G} = \frac{\Lambda c^2}{H^2} = \frac{c^3 T_U^3}{R_U^3} = \frac{H^2}{M_U R_U G \Lambda^2} = \frac{c r_e^3 A_0}{G \hbar} = \frac{c^4}{M_U R_U H^2 G} = \frac{M_U R_U H A_0 G}{c^5} = 1$		10^0
$D_{20} = \frac{r_e}{l_{Pl}} = \frac{t_0}{t_{Pl}} = \frac{\alpha m_{Pl}}{m_e} = \frac{c l_{Pl}}{r_e^2 H} = \frac{l_{Pl} R_U}{r_e^2} = \frac{c^2 l_{Pl}}{r_e^2 A_0} = \sqrt{\alpha D_0}$		10^{20}
$D_{40} = \frac{T_U}{t_0} = \frac{R_U}{r_e} = \frac{m_e c^2}{\alpha \hbar H} = \frac{1}{t_0 H} = \frac{r_e^2}{l_{Pl}^2} = \frac{t_0^2}{t_{Pl}^2} = \frac{\alpha^2 m_{Pl}^2}{m_e^2} = \frac{c^2}{r_e A_0} = (\sqrt{\alpha D_0})^2$		10^{40}
$D_{60} = \frac{T_U}{t_{Pl}} = \frac{R_U}{l_{Pl}} = \frac{M_U}{m_{Pl}} = \frac{c}{l_{Pl} H} = \frac{r_e^3}{l_{Pl}^3} = \frac{t_0^3}{t_{Pl}^3} = \frac{c^3}{G m_{Pl} H} = \frac{c^2}{l_{Pl} A_0} = (\sqrt{\alpha D_0})^3$		10^{60}
$D_{80} = \frac{R_U^2}{r_e^2} = \frac{H M_U^2 \alpha G}{c^3 m_e} = \frac{c^2}{r_e^2 H^2} = \frac{c r_e}{H l_{Pl}^2} = \frac{1}{r_e^2 \Lambda} = (\sqrt{\alpha D_0})^4$		10^{80}
$D_{100} = \frac{m_e c^3}{l_{Pl} \alpha \hbar H^2} = \frac{r_e \alpha M_U}{l_{Pl} m_e} = \frac{H M_U^2 \alpha G r_e}{c^3 m_e l_{Pl}} = \frac{R_U^2}{r_e l_{Pl}} = \frac{1}{r_e l_{Pl} \Lambda} = (\sqrt{\alpha D_0})^5$		10^{100}
$D_{120} = \frac{T_U^2}{l_{Pl}^2} = \frac{R_U^2}{l_{Pl}^2} = \frac{M_U^2}{m_{Pl}^2} = \frac{c^2}{l_{Pl}^2 H^2} = \frac{R_U^3}{r_e^3} = \frac{M_U c^2}{\hbar H} = \frac{G M_U^2}{\hbar c} = \frac{c^5}{G \hbar H^2} = \frac{c^3}{G \hbar \Lambda} = \frac{1}{l_{Pl}^2 \Lambda} = (\sqrt{\alpha D_0})^5$		10^{120}
$D_{140} = \frac{r_e^2 m_e c^3}{l_{Pl}^3 \alpha \hbar H^2} = \frac{r_e^3 \alpha M_U}{l_{Pl}^3 m_e} = \frac{R_U^3}{l_{Pl} r_e^2} = \frac{1}{t_{Pl} t_0^2 H^3} = \frac{c}{l_{Pl} r_e^2 H \Lambda} = \frac{c^2}{l_{Pl} r_e^2 A_0 \Lambda} = (\sqrt{\alpha D_0})^7$		10^{140}
$D_{160} = \frac{M_U R_U c^2 \alpha^2}{G m_e^2} = \frac{M_U^2 R_U G \alpha}{c^2 r_e^2 m_e} = \frac{1}{r_e^4 \Lambda^2} = (\sqrt{\alpha D_0})^8$		10^{160}
$D_{180} = \frac{r_e^4 m_e c^3}{l_{Pl}^5 \alpha \hbar H^2} = \frac{r_e^5 \alpha M_U}{l_{Pl}^5 m_e} = \frac{R_U^3}{l_{Pl}^3} = \frac{c^3}{l_{Pl}^3 H^3} = \frac{c}{l_{Pl}^3 H \Lambda} = \frac{c^2}{l_{Pl}^3 A_0 \Lambda} = \frac{G M_U T_U^2 l_{Pl}}{\Lambda r_e^6} = (\sqrt{\alpha D_0})^9$		10^{180}

Fig. 4. Relationships of dimensional constants at 10 scales giving 10 large numbers. M_U - mass of the observable Universe, α - fine-structure constant, \hbar - Planck constant, G - Newtonian constant of gravitation, Λ - cosmological constant, R_U - radius of the observable Universe, T_U - time of Universe, H - Hubble constant, A_0 - cosmological acceleration, r_e - classical electron radius; c - speed

of light in vacuum; $t_0 = r_e/c$, m_e - electron mass, D_0 is a large Weyl number, t_{pl} - Planck time, l_{pl} - Planck length, m_{pl} - Planck mass.

5. The scale is 10^0 , which gives the most important cosmological equations.

From the scaling law of large numbers, the number $(\sqrt{\alpha D_0})^0$. This is the lower limit of the family of large numbers:

$$(\sqrt{\alpha D_0})^0 = 1$$

The scale 10^0 (Fig. 4) is represented by the largest number of coincidences of dimensional magnitude relations. From the relations of dimensional quantities on the scale 10^0 , the most important cosmological equations follow. Among them are Stewart's equation, Carvalho equation, Eddington-Weinberg equation, Milne equation, Bleksley equation.

6. New cosmological equations

Below are new cosmological equations, which contain the parameters of the Universe in different combinations - 2 parameters, 3 parameters, 4 parameters, 5 parameters, 6 parameters.

The table in Fig. 5 shows cosmological equations, which include 2 parameters of the Universe:

Name	Parameters Universe	Cosmological equations	Note
GR-equations	G, R_U	$\frac{c^3 r_e^3}{GR_U} = \hbar \quad \frac{R_U G m_e}{r_e^2 c^2} = \alpha \quad \mathbf{GR_U \hbar / r_e^3 = c^3}$	[22], [23]
GT-equations	G, T_U	$\mathbf{G \hbar \alpha^3 D_0^3 / T_U^2 = c^5} \quad \frac{T_U G m_e}{r_e^2 c} = \alpha$	[23]
GA-equations	G, Λ	$\mathbf{G^2 m_e^2 = \Lambda c^4 r_e^4 a^2} \quad \frac{G m_e}{r_e^2 c^2 \sqrt{\Lambda}} = \alpha$ $\mathbf{G \hbar \Lambda \alpha^3 D_0^3 = c^3}$	[23]
GA ₀ -equations	G, A₀	$\frac{c r_e^3 A}{G} = \hbar \quad \frac{G m_e}{r_e^2 A_0} = \alpha$	[22], [23]
GM-equations	G, M_U	$\mathbf{M_U G^2 m_e = \alpha c^4 r_e^2} \quad \frac{c^5 r_e^3}{M_U G^2} = \hbar$ $\mathbf{M_U G^2 m_e / \alpha r_e^2 = c^4}$	[22]
MA-equations	M_U, Λ	$\mathbf{M_U \Lambda c r_e^3 = \hbar} \quad \frac{m_e}{M_U \Lambda r_e^2} = \alpha$	[23]
M R -equations	M_U, R_U	$\mathbf{M_U c r_e^3 = \hbar R_U^2} \quad \frac{m_e R_U^2}{M_U r_e^2} = \alpha$	[22], [23]
M T-equations	M_U, T_U	$\mathbf{M \alpha r_e^2 = T_U^2 m_e c^2} \quad \mathbf{M r_e^3 = T_U^2 \hbar c}$ $\mathbf{M r_e^3 / \hbar T_U^2 = c}$	
MA ₀ -equations	M_U, A₀	$\frac{m_e c^4}{M_U A_0^2 r_e^2} = \alpha \quad \mathbf{M A_0^2 r_e^3 = \hbar c^3}$	[23]
T Λ -equation	T_U, Λ	$\mathbf{\Lambda T_U^2 c^2 = 1}$	
A ₀ Λ -equation	A₀, Λ	$\mathbf{A_0^2 = \Lambda c^4}$	

FIG. 5. Cosmological equations that contain 2 parameters of the Universe each.

The table in Fig. 6 shows cosmological equations that contain 3 parameters of the Universe each:

Name	Parameters Universe	Cosmological equations	Note
MA ₀ G-equation	M _U , A ₀ , G	M _U A ₀ G = c ⁴	
MGT-equation	M _U , G, T _U	M _U G = c ³ T _U	Milne equation
MGR	M _U , G, R _U	M _U G = c ² R _U	Bleksley equation
G, A ₀ , Λ-equations	G, A ₀ , Λ	$\frac{c^5 r_\epsilon^3 \Lambda}{GA_0} = \hbar$ $\frac{Gm_\epsilon A_0}{\Lambda c^4 r_\epsilon^2} = \alpha$	[22], [23]
G, R _U , Λ-equations	G, R _U , Λ	$\frac{R_U \Lambda c^3 r_\epsilon^3}{G} = \hbar$ $\frac{Gm_\epsilon}{R_U \Lambda c^2 r_\epsilon^2} = \alpha$	[22], [23]
RTΛ-equation	R _U , T _U , Λ	R _U /T _U ³ Λ = c ³	

FIG. 6. Cosmological equations that contain 3 parameters of the Universe each.

The table in Fig. 7 shows cosmological equations that contain 4 parameters of the Universe each:

Name	Parameters Universe	Cosmological equations	Note
MGRT-equation	M _U , G, R _U , T _U	M _U G = R _U ³ /T _U ²	Kepler's law [28]
MAGA ₀ -equation	M _U , Λ, G, A ₀	M _U ΛG = A ₀	
MTAG-equation	M _U , T _U , Λ, G	M _U T _U ΛG = c	
MRAG-equation	M _U , R _U , Λ, G	M _U R _U ΛG = c ²	[23]
MRA ₀ G-equation	M _U , R _U , A ₀ , G	M _U R _U A ₀ ² G = c ⁶	

FIG. 7. Cosmological equations that contain 4 parameters of the Universe each.

The table in Fig. 8 shows cosmological equations that contain 5 parameters of the Universe each:

Name	Parameters Universe	Cosmological equations	Note
MRAGA ₀ -equation	M _U , R _U , Λ, G, A ₀	M _U R _U Λ ² Gc ² = A ₀ ²	
MRAGT-equation	M _U , R _U , Λ, G, T	M _U R _U Λ ² GT ² = 1	

FIG. 8. Cosmological equations that contain 5 parameters of the Universe each.

The table in Fig. 9 shows cosmological equations that contain 6 parameters of the Universe each:

Name	Parameters Universe	Cosmological equations	Note
MRAGA ₀ T-equations	M _U , R _U , Λ, G, A ₀ , T _U	M _U R _U Λ ² GA ₀ T _U ³ = c M _U R _U ΛGA ₀ T _U = c ³	

FIG. 9. Cosmological equations that contain 6 parameters of the universe each.

7. Mathematical method for obtaining the parameters of the observed Universe.

The new cosmological equations (Fig. 5 - Fig. 9), combined in a system of algebraic equations, provide a new method for obtaining the values of the parameters of the Universe. For the first time it was possible to obtain the parameters of the Universe as a result of solving a system of algebraic equations. The solution of systems of algebraic equations gives the parameters of the Universe with an accuracy close to the accuracy of the Newtonian constant of gravitation G

From cosmological equations it is possible to compose various blocks of algebraic equations in which combinations of the parameters of the Universe are equal to the speed of light.

$$\left\{ \begin{array}{l} \mathbf{M_U R_U \Lambda^2 G T_U^2 = c^0} \\ \mathbf{G \hbar / r_e^3 A_0 = c^1} \\ \mathbf{1/T_U^2 \Lambda = c^2} \\ \mathbf{M_U R_U \Lambda G A_0 T_U = c^3} \\ \mathbf{M_U A_0 G = c^4} \\ \mathbf{M_U R_U A_0 G / T_U = c^5} \\ \mathbf{M_U R_U A_0^2 G = c^6} \end{array} \right.$$

Fig. 10. Block of algebraic equations of the Universe

This attractive system of equations (Fig. 10) is redundant, so we can limit ourselves to this system of algebraic equations:

$$\left\{ \begin{array}{l} \mathbf{G \hbar / r_e^3 A_0 = c} \\ \mathbf{1/T_U^2 \Lambda = c^2} \\ \mathbf{G R_U \hbar / r_e^3 = c^3} \\ \mathbf{M_U A_0 G = c^4} \\ \mathbf{M_U R_U A_0 G / T_U = c^5} \end{array} \right.$$

Fig. 11. System of cosmological equations of the Universe

In this system of cosmological equations (Fig. 11), the known quantities are the fundamental physical constants G , \hbar , r_e , c . The unknown cosmological parameters M_U , R_U , Λ , A_0 , T_U are easily obtained by solving the system of algebraic equations.

All systems of cosmological equations of the Universe are t-invariant. They are valid for any epoch. The solution of the system of algebraic equations (Fig. 11) gives the following values of the parameters of the Universe (Fig. 12).

$$\begin{aligned}
 M_U &= 1.15348 \dots \bullet 10^{53} \text{ kg} \\
 R_U &= 0.856594 \dots \bullet 10^{26} \text{ m} \\
 T_U &= 2.85729 \dots \bullet 10^{17} \text{ s} \\
 \Lambda &= 1.36285 \dots \bullet 10^{-52} \text{ m}^{-2} \\
 G &= 6.67430 \dots \bullet 10^{-11} \text{ kg}^{-1} \text{ m}^3 \text{ s}^{-2} \\
 A_0 &= 10.4922 \dots \bullet 10^{-10} \text{ m / s}
 \end{aligned}$$

Fig. 12. Parameters of the Universe and their values.

8. Conclusion

In addition to the known cosmological equations of Stewart, Dirac, Eddington-Weinberg, Teller, Rice, Nottale, Milne, Bleksley, Hoyle, Carvalho, new cosmological equations are derived which reveal the fundamental relationship between the parameters of the Universe and physical constants. New cosmological equations are derived using previously unknown large numbers on scales 10^{140} , 10^{160} , and 10^{180} . The key to unlocking the mystery of large numbers is provided by new cosmological equations demonstrating the relationship between the parameters of the Universe and the fine structure constant alpha. Based on the revealed connection between the parameters of the Universe and fundamental physical constants, a mathematical method for obtaining the parameters of the Universe is developed. The mathematical method gives values of the parameters of the Universe with an accuracy close to the accuracy of the Newtonian constant of gravitation G. The system of algebraic equations of the Universe is composed of cosmological equations containing the parameters of the Universe. The parameters of the Universe are the roots of the system of algebraic equations. It strikes the imagination how mathematically fine-tuned the Universe is. The main discovery was the following: the values of the Universe parameters are scale-transformed electron constants; the scale factors are large numbers; the large numbers come from the fine structure constant "alpha".

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