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Unexpected connection of the parameters of the observed Universe with the fine structure constant "alpha".

Abstract: *The paper demonstrates a new method of obtaining values of the Universe parameters. The method is based on the revealed relationship between the parameters of the Universe and fundamental physical constants. New ratios of the dimensional parameters of the observable Universe are derived, which give the fine structure constant alpha. This is an unexpected result, since the fine structure constant refers to the microcosm, but not to the Universe. There are many of these equations. They have no explanation. There is no answer as to why, on such enormous scales, the ratios of the dimensional parameters of the universe give the alpha constant. Despite the lack of explanation, the new equations open up new possibilities in cosmology. The constant "alpha" and the parameters of the Universe are present together in one equation. This makes it possible to use the high precision of the alpha constant to calculate the values of the parameters of the observable Universe. This provides a high accuracy of the parameters of the observable Universe close to the accuracy of the Newtonian constant of gravitation G. New cosmological equations are derived, from which the value of the cosmological acceleration is obtained. This result allows us to solve the long-standing Pioneer-anomaly problem.*

Keywords: *large numbers, fine structure constant, cosmological constant, Pioneer-anomaly, parameters of the observable Universe.*

1. Introduction

The Universe is based not on arbitrary, but on strictly defined values of fundamental constants included in physical laws [1]. The fundamental constants set the mathematically fine-tuned parameters of the Universe. In [2] the origin of the fundamental parameters of the Universe from the electron constants is shown.

All parameters of the observable Universe come from fundamental physical constants and are scaled electron constants. The scaling factors are large numbers. The large numbers, in turn, are related to each other by the law of scaling of large numbers. Such strict regularities provide a mathematically precise tuning of the parameters of the Universe. The fine structure constant "alpha" plays an essential role in this fine tuning of the parameters of the Universe.

2. The law of scaling of large numbers

The law of scaling of large numbers has the form (Fig. 1):

$$D_i = (D_{20})^i = (\sqrt{\alpha D_0})^i$$

$$i = 0, \pm 1, \pm 2, \pm 3, \pm 4, \pm 5, \pm 6, \pm 7, \pm 8, \pm 9$$

Fig. 1. The scaling law of large numbers. D_0 is a large Weyl number (D_0 = 4.16561...x 10^{42}), α - fine structure constant.

The law of scaling gives a second method of calculating the values of large numbers.This is a new method for obtaining large numbers.Its advantage is that large numbers are obtained from dimensionless constants.The scaling law of large numbers complements the well-known method of obtaining large numbers from relations of dimensional constants. The scaling law generates large numbers up to scale 10^{180} with high accuracy. The large numbers obtained from the scaling law are close to the accuracy of the Newtonian constant of gravitation G.

The values of large numbers and formulas for their calculation are given in Fig. 2.

$$(\sqrt{\alpha D_0})^0 = 1$$

$$D_{20} = (\sqrt{\alpha D_0})^1 = 1.74349... \bullet 10^{20}$$

$$D_{40} = (\sqrt{\alpha D_0})^2 = 3.03979... \bullet 10^{40}$$

$$D_{60} = (\sqrt{\alpha D_0})^3 = 5.29987... \bullet 10^{60}$$

$$D_{80} = (\sqrt{\alpha D_0})^4 = 9.24033... \bullet 10^{80}$$

$$D_{100} = (\sqrt{\alpha D_0})^5 = 16.1105... \bullet 10^{100}$$

$$D_{120} = (\sqrt{\alpha D_0})^6 = 28.088... \bullet 10^{120}$$

$$D_{140} = (\sqrt{\alpha D_0})^7 = 48.972... \bullet 10^{140}$$

$$D_{160} = (\sqrt{\alpha D_0})^8 = 85.383... \bullet 10^{160}$$

$$D_{180} = (\sqrt{\alpha D_0})^9 = 148.86... \bullet 10^{180}$$

FIG. 2. Large numbers and formulas for their calculation from the law of scaling of large numbers.

All large numbers contain the fine structure constant "alpha".FIG. 3 shows some formulas leading to large numbers. These are not all possible formulas. There are many more such formulas. All formulas are equivalent. The matches of values obtained from equivalent formulas are not approximate, but exact. Why are there so many equivalent formulas for large numbers? There is no answer to this question yet.

Ratios of dimensional constants	Scale
$\frac{Gm_e^2}{r_e\alpha^2\hbar H}=\frac{G\hbar}{r_e^3Hc^2}=\frac{Gm_e}{r_e^2\alpha Hc}=\frac{Gm_e^3c}{\alpha^3\hbar^2H}=\frac{c^2}{M_UR_UG\Lambda}=\frac{c^3}{M_UGH}=\frac{c^2R_U}{M_UG}=$ $=\frac{c^3T_U}{M_UG}=\frac{\Lambda c^2}{H^2}=\frac{H^2}{M_UR_UG\Lambda^2}=.\frac{cr_e^3A_0}{G\hbar}=\frac{c^4}{M_UR_UH^2G}=\frac{M_UR_UHA_0G}{c^5}=1$	10^0
$D_{20}=\frac{r_e}{l_{Pl}}=\frac{t_0}{t_{Pl}}=\frac{\alpha\, m_{Pl}}{m_e}=\frac{cl_{Pl}}{r_e^2H}=\frac{l_{Pl}R_U}{r_e^2}=\frac{c^2l_{Pl}}{r_e^2A_0}=\sqrt{\alpha D_0}$	10^{20}
$D_{40}=\frac{T_U}{t_0}=\frac{R_U}{r_e}=\frac{m_ec^2}{\alpha\hbar H}=\frac{1}{t_0H}=\frac{r_e^2}{l_{Pl}^2}=\frac{t_0^2}{t_{Pl}^2}=\frac{\alpha^2m_{Pl}^2}{m_e^2}=\frac{c^2}{r_eA_0}=(\sqrt{\alpha D_0})^2$	10^{40}
$D_{60}=\frac{T_U}{t_{Pl}}=\frac{R_U}{l_{Pl}}=\frac{M_U}{m_{Pl}}=\frac{c}{l_{Pl}H}=\frac{r_e^3}{l_{Pl}^3}=\frac{t_0^3}{t_{Pl}^3}=\frac{c^3}{Gm_{Pl}H}=(\sqrt{\alpha D_0})^3$	10^{60}
$D_{80}=\frac{R_U^2}{r_e^2}=\frac{HM_U^2\alpha G}{c^3m_e}=\frac{c^2}{r_e^2H^2}=\frac{cr_e}{Hl_{Pl}^2}=\frac{1}{r_e^2\Lambda}=(\sqrt{\alpha D_0})^4$	10^{80}
$D_{100}=\frac{m_ec^3}{l_{Pl}\alpha\hbar H^2}=\frac{r_e\alpha M_U}{l_{Pl}m_e}=\frac{HM_U^2\alpha Gr_e}{c^3m_el_{Pl}}=\frac{R_U^2}{r_el_{Pl}}=\frac{1}{r_el_{Pl}\Lambda}=(\sqrt{\alpha D_0})^5$	10^{100}
$D_{120}=\frac{T_U^2}{t_{Pl}^2}=\frac{R_U^2}{l_{Pl}^2}=\frac{M_U^2}{m_{Pl}^2}=\frac{c^2}{l_{Pl}^2H^2}=\frac{R_U^3}{r_e^3}=\frac{M_Uc^2}{\hbar H}=\frac{GM_U^2}{\hbar c}=\frac{c^5}{G\hbar H^2}=\frac{c^3}{G\hbar\Lambda}=\frac{1}{l_{Pl}^2\Lambda}=(\sqrt{\alpha D_0})^6$	10^{120}
$D_{140}=\frac{r_e^2m_ec^3}{l_{Pl}^3\alpha\hbar H^2}=\frac{r_e^3\alpha M_U}{l_{Pl}^3m_e}=\frac{R_U^3}{l_{Pl}r_e^2}=\frac{1}{t_{Pl}t_0^2H^3}=\frac{c}{l_{Pl}r_e^2H\Lambda}=(\sqrt{\alpha D_0})^7$	10^{140}
$D_{160}=\frac{M_UR_Uc^2\alpha^2}{Gm_e^2}=\frac{M_U^2R_UG\alpha}{c^2r_e^2m_e}=\frac{1}{r_e^4\Lambda^2}=(\sqrt{\alpha D_0})^8$	10^{160}
$D_{180}=\frac{r_e^4m_ec^3}{l_{Pl}^5\alpha\hbar H^2}=\frac{r_e^5\alpha M_U}{l_{Pl}^5m_e}=\frac{R_U^3}{l_{Pl}^3}=\frac{c^3}{l_{Pl}^3H^3}=\frac{c}{l_{Pl}^3H\Lambda}=\frac{c^2}{l_{Pl}^3A_0\Lambda}=(\sqrt{\alpha D_0})^9$	10^{180}

Fig. 3. Set of coincidences of large numbers. M_U is the mass of the observable Universe, α is the fine structure constant, ħ is Planck's constant, G is the Newtonian gravitational constant, Λ is the cosmological constant, R_U is the radius of the observable Universe, T_U is the time of the Universe, H is the Hubble constant, A_0 is the cosmological acceleration, r_e is the classical radius of the electron; c - speed of light in vacuum; $t_0= r_e/c$, m_e - electron mass, D_0 - large Weyl number, t_{pl} - Planck time, l_{pl} - Planck length, m_{pl} - Planck mass.

3. Cosmological equations

There are two types of equations in cosmology. The Friedman cosmological equation describes the dynamics of the expansion of the Universe [9]. This equation does not reveal the possible relationship between the parameters of the Universe and does not reveal the relationship of the parameters to the fundamental physical constants. The relationship between the parameters of the Universe and the fundamental physical constants is revealed in other cosmological equations. These include: Dirac equation, Stewart equation, Eddington-Weinberg equation, Teller equation and others.

The Dirac equation [10, 11] is of the form:

$$\mathbf{m_e c^3/(He^2) \approx e^2/(Gm_e^2)} \qquad (1)$$

Stewart's equation [4, 5] has the form:

$$, \; Gm_e^2 / r_e = \alpha^2 \hbar H \qquad (2)$$

The Eddington-Weinberg equation [12] has the form:

$$\hbar^2 H \approx Gcm_p^3 \qquad (3)$$

The Teller equation [6, 7] is of the form:

$$2\frac{G\hbar H}{c^4 l_{Pl}} = 2t_{Pl}H = 2\frac{Gm_{Pl}H}{c^3} \cong \exp(-1/\alpha) \quad (4)$$

Equations (1) - (4) include constants G and H. A large number of coincidences of large numbers (Fig. 3) allow us to obtain new cosmological equations using other parameters of the Universe M_U, R_U, Λ, A_0, H, T_U in various combinations. The new cosmological equations complement the known equations.Combining equations becomes a powerful tool in cosmology.Both the known cosmological equations and the new equations combine very precise fundamental physical constants and very imprecise parmeters of the Universe. This unification of heterogeneous constants opens a wide range of possibilities in cosmology. The exact fundamental physical constants in the cosmological equations provide the key to calculate the parameters of the Universe with high accuracy. We will single out for study the cosmological equations containing the fine structure constant "alpha".

4. Attempts to search for the connection of the Universe parameters with the fine structure constant "alpha"

The analysis of published works on this problem has shown that attempts to relate the fine structure constant to the parameters of the Universe have been made earlier.

In 1925 James Rice [3] attempted to relate the constant alpha to the radius of the Universe. He proposed an approximate formula:

$$\frac{4\pi}{\alpha} = \frac{r_e^2 c^2}{6R_U G m_e} \qquad (5)$$

Stewart J. O. in 1931 [4, 5] proposed a cosmological equation that relates the constant alpha to the Newtonian constant of gravitation G and to the Hubble constant:

$$\frac{Gm_e^2}{\hbar H r_e} = \alpha^2 \qquad (6)$$

E.Teller in 1948 [6, 7, 8] proposed a cosmological equation (4) that contains the fine structure constant alpha:

$$2\frac{G\hbar H}{c^4 l_{Pl}} = 2t_{Pl}H = 2\frac{Gm_{Pl}H}{c^3} \cong \exp(-1/\alpha)$$

James Rice equation and E. Teller equation are approximate. Stewart J. O. accurate equation. Although the James Rice equation is approximate, his idea to relate the parameters of the observable Universe to the fine structure constant alpha was a remarkable foresight.

5. New cosmological equations containing the constant "alpha"

The high accuracy of the large numbers obtained from the scaling law and the many coincidences of large numbers make it possible to derive a number of new cosmological equations.

These new ratios of dimensional parameters of the Universe turned out to be equal to the fine structure constant alpha.

This is an unexpected result, since alpha is a constant of the microcosm and does not relate to the parameters of the Universe. Its appearance in the cosmological equations is the evidence of not yet revealed deep interrelation of parameters of the observable Universe and fundamental physical constants.

These equations are given below:

$$\frac{M_U G^2 m_e}{c^4 r_e^2} = \alpha \quad (7), \qquad \frac{m_e}{M_U \Lambda r_e^2} = \alpha \quad (8),$$

$$\frac{Gm_e}{R_U \Lambda c^2 r_e^2} = \alpha \quad (9), \qquad \frac{Gm_e}{r_e^2 A_0} = \alpha \quad (10),$$

$$\sqrt{\frac{1}{r_e^2 \Lambda D_0^2}} = \alpha \quad (11), \qquad \sqrt[3]{\frac{c^3}{G\hbar \Lambda D_0^3}} = \alpha \quad (12),$$

$$\frac{Gm_e A_0}{\Lambda c^4 r_e^2} = \alpha \quad (13), \qquad \frac{R_U G m_e}{r_e^2 c^2} = \alpha \quad (14),$$

$$\frac{Gm_e}{r_e^2 c^2 \sqrt{\Lambda}} = \alpha \quad (15), \qquad \frac{m_e c^4}{M_U A_0^2 r_e^2} = \alpha \quad (16),$$

where : α - fine-structure constant, ħ - Planck constant, M_U - mass of the observable Universe, G - Newtonian constant of gravitation, Λ - cosmological constant, R_U - radius of the observable Universe, A_0 - cosmological acceleration [13 - 16], r_e - classical electron radius; c - speed of light in vacuum; m_e - electron mass, D_0 - large number, r_e - classical electron radius; c - speed of light in vacuum; m_e - electron mass.

Thus, there are many coincidences ratios of constants that lead to constant alpha.

$$\frac{M_U G^2 m_e}{c^4 r_e^2} = \frac{m_e}{M_U \Lambda r_e^2} = \sqrt{\frac{1}{r_e^2 \Lambda D_0^2}} = \sqrt[3]{\frac{c^3}{G\hbar\Lambda D_0^3}} = \frac{Gm_e}{R_U \Lambda c^2 r_e^2} = \frac{Gm_e}{r_e^2 A_0} = \frac{Gm_e A_0}{\Lambda c^4 r_e^2} = \frac{R_U G m_e}{r_e^2 c^2} = \ldots = \alpha \quad (17)$$

In each equation there is a very precise constant "alpha" and very imprecise parameters of the Universe. All equations are linked by a single law of scaling of large numbers. The given equations (7) - (16) are equivalent. The equations are exact. The high accuracy of the constant alpha and the acceptable accuracy of G make it possible to obtain values of the parameters of the Universe with an accuracy close to the accuracy of the Newtonian constant of gravitation G. This is an unprecedented accuracy for the parameters of the Universe! As the accuracy of the constant G increases, it will be possible to calculate the parameters of the Universe with greater accuracy.

An important formula for cosmology follows from equations (8) and (9): $\mathbf{G R_U M_U \Lambda^2 = H^2}$. This same "Universe equation" was obtained in [2] from the coincidence of large numbers on a scale of 10^{160}.

From equations (9) and (10) follows the formula for the relationship between the Hubble constant and the cosmological constant Λ: $\mathbf{H^2 = \Lambda c^2}$. The same formula was obtained in [2] from the coincidence of large numbers on a scale of 10^{120}.

6. System of cosmological equations containing the constant "alpha"

From the new equations, a system of algebraic equations is composed, where the unknown parameters are the mass M_U, the lambda Λ, and the radius R_U, and the acceleration A_0 (Fig. 4):

$$\begin{cases} \dfrac{M_U G^2 m_e}{c^4 r_e^2} = \alpha \\ \dfrac{m_e}{M_U \Lambda r_e^2} = \alpha \\ \dfrac{Gm_e}{R_U \Lambda c^2 r_e^3} = \alpha \\ \dfrac{Gm_e A_0}{\Lambda c^4 r_e^2} = \alpha \end{cases}$$

Fig. 4. System of cosmological equations containing the constant "alpha".

Since all equations are equal to "alpha", the system of equations allows us to express the unknown parameters M_U, Λ, R_U, A_0 using the Newtonian constant of gravitation G.

The acceptable accuracy of the constant G allows us to obtain the parameters of the Universe from the system of cosmological equations (Fig. 4) with an accuracy close to the accuracy of the Newtonian constant of gravitation G. For the parameters of the Universe this is an unprecedented accuracy.

The values of the Universe parameters (mass M_U, lambda Λ, radius R_U, acceleration A_0) obtained from the system of cosmological equations (Fig. 4) are given in Fig. 5.

$$M_U = 1.15348... \bullet 10^{53}\, kg$$

$$R_U = 0.856594... \bullet 10^{26}\, m$$

$$\Lambda = 1.36285... \bullet 10^{-52}\, m^{-2}$$

$$G = 6{,}6743... x 10^{-11}\, kg^{-1} m^3 s^{-2}$$

$$A_0 = 10.4922...\, x\, 10^{-10}\, m/s^2$$

Fig. 5. Values of the fundamental parameters of the Universe from equations (7) through (16) and from the system of equations (Fig. 4).

7. The value of cosmological acceleration and Pioneer-anomaly

The solution of the system of cosmological equations (Fig. 4) gives the value of cosmological acceleration $A_0 = 10.4922... x\ 10^{-10}\ m/s^2$. The value of cosmological acceleration was first obtained in the Pioneer-10, Pioneer-11, Galileo and Ulysses experiments, which was called Pioneer-anomaly [13 - 16].

The experimental values of the acceleration (Pioneer-anomaly) are as follows:

For Pioneer:

$$\mathbf{A_0 = (7{,}41 - 10{,}07) \times 10^{-10}\ m/s^2} \qquad (18)$$

For Galileo:

$$\mathbf{A_0 = (8 \pm 3)\ x\ 10^{-10}\ m/s^2} \qquad (19)$$

For Ulysses:

$$\mathbf{A_0 = (12 \pm 3)\ x\ 10^{-10}\ m/s^2} \qquad (20)$$

As we can see, the value of the acceleration $A_0 = 10.4922...x\ 10^{-10}\ m/s^2$, obtained by solving the system of equations (Fig. 4), is very close to the experimental value (Pioneer-anomaly).

Additional verification of the obtained value of the acceleration $A_0 = 10.4922...x\ 10^{-10}\ m/s^2$ on Newton's law gives the following value of the cosmological force:

$$\mathbf{F = M_U A_0 = 1.21025...\ x\ 10^{44}\ N} \qquad (21)$$

The value of the force from equation (33) coincides completely with the value of the Planck force obtained by the well-known formula:

$$\mathbf{F = c^4/G = 1.21025... \times 10^{44}\ N} \qquad (22)$$

Formulas (21) and (22) confirm the values of acceleration $A_0 = 10.4922... \times 10^{-10}$ m/s^2

8. Interrelation of the parameters of the Universe

The above shows that the parameters of the Universe are interrelated. They originate from fundamental physical constants. Such unity of microcosm constants and parameters of the Universe is not accidental. Some unknown fundamental reasons lead to such unity of constants.

From equations (21) and (22) follows the cosmological equation that links the three parameters of the Universe (M_U, G and A_0) :

$$\mathbf{M_U A_0 G = c^4} \qquad (23)$$

From the large numbers on scales 10^{120}, 10^{80} and 10^{40}, an equation that relates the 4 parameters of the Universe (M_U, Λ, G, A_0) is obtained:

$$\mathbf{M_U \Lambda G = A_0} \qquad (24)$$

From the large numbers at scales 10^{160}, 10^{140} and 10^{20}, an equation is obtained that relates the 5 parameters of the Universe (M_U, R_U, H, A_0, G):

$$\mathbf{M_U R_U H A_0 G = c^5} \qquad (25)$$

From the large numbers at scales 10^{180}, 10^{160} and 10^{20}, an equation that relates 6 parameters of the universe (M_U, R_U, Λ, H, A_0, G) is obtained:

$$\mathbf{M_U R_U \Lambda G A_0 = H c^3} \qquad (26)$$

9. The gravitational parameter of the universe.

Equations (23) through (26) include the product of two GM_U constants. This product of GM_U can be represented by equivalent formulas without using the Newtonian constant of gravitation G:

$$\mathbf{\mu_U = GM_U = c^2 R_U^3 \Lambda = R_U^3 H^2 = R_U^3 A_0^2/c^2 = R_U A_0^2/\Lambda c^2 = 7{,}69868...x\ 10^{42}\ m^3 s^{-2}} \qquad (27)$$

In addition to the formulas (27), the product GM_U can be represented by means of the fundamental physical constants re, c, α and a large scale number 1040. The formula for calculating GM_U is as follows:

$$\mathbf{\mu_U = GM_U = c^2 r_e(\alpha D_0) = 7.69868...x\ 10^{42}\ m^3 s^{-2}} \qquad (28)$$

Such a simple formula (28) for μ_U is obtained due to the fact that both the Newtonian constant of gravitation G and the mass of the Universe M_U are composite constants and come from the electron constants [2].

The value $\mu_U = GM_U$ Espen Gaarder Haug [17] called the "*gravitational parameter of the universe.*" He obtained the equation for GM_U using only three physical constants [17].

10. Other systems of cosmological equations

The law of scaling of large numbers allows us to derive many equations that contain the parameters of the Universe M_U, R_U, Λ, H, A_0, G in different combinations.Cosmological equations form systems of equations, the solution of which gives the value of the parameters of the Universe with high accuracy. In Fig. 6, there are beautiful equations that demonstrate the relationship between the parameters of the universe and the light speed constant. The equations form a system of cosmological equations.

$$\begin{cases} G\hbar/r_e^3 A_0 = c \\ M_U R_U \Lambda G = c^2 \\ M_U H G = c^3 \\ M_U A_0 G = c^4 \\ M_U R_U H A_0 G = c^5 \end{cases}$$

Фиг. 6. System of cosmological equations containing the constant "c".

The solution of the system of cosmological equations (Fig. 6), gives the same values of the parameters of the observable universe (Fig. 5), as the system of equations (Fig. 4), containing the fine structure constant "alpha".

11. Conclusion

The new equations containing the fundamental physical constant "alpha" allow us to take advantage of the high accuracy of the constant alpha to calculate values of cosmological parameters. This is an important result for practice, since experimental methods for determining the values of the parameters of the observed Universe are very complicated and do not give sufficient accuracy. In the system of cosmological equations (Fig. 4), only the Newtonian constant of gravitation G imposes a limitation on the accuracy. Nevertheless, its accuracy is sufficient to provide unprecedented precision for the parameters of the observable Universe.

12. Conclusions

1. New ratios of the dimensional parameters of the observable Universe are derived, which give the fine structure constant alpha.

2. Using the new cosmological equations, systems of cosmological equations are proposed, the solution of which gives the values of the parameters of the observable Universe with an accuracy close to the accuracy of the Newtonian constant of gravitation G.

3. The accuracy of the parameters of the observable Universe can potentially be approximated to the accuracy of the fine structure constant "alpha". For this purpose it is necessary to find a new method for determining the value of the Weyl number D_0 without using dimensional quantities.

4. All parameters of the observable Universe are composite quantities. They come from fundamental physical constants and are scaled electron constants. The scaling factors are large numbers.

References

1. Clark, Kelly James. Readings in the Philosophy of Religion. — Broadview Press, 2000. — P. 54.
2. Mykola Kosinov. THE LAW OF SCALING FOR LARGE NUMBERS: origin of large numbers from the primary large number D20 = 1.74349...x 10^20. January 2024. DOI: 10.13140/RG.2.2.33664.20480
3. Rice J. On Eddington's natural unit of the field // Philosophical Magazine Series 6. — 1925. — Vol. 49. — P. 1056—1057.)
4. Stewart J. O. "Phys. Rev.", 1931, v. 38, p. 2071.
5. Muradyan, R. M. Universal constants. Physical and astrophysical constants and their dimensional and dimensionless combinations. Fiz. ehlementar. chastits i atom. yadra, Vol. 8, No. 1, p. 175 - 192. 1977.
6. E. Teller, Phys. Rev. 73, 801(1948) E. Teller (1948). On the change of physical constants. Physical Review, vol.73 pp. 801—802. DOI:10.1103/PhysRev.73.801
7. RAINER W. KÜHNE. TIME-VARYING FINE-STRUCTURE CONSTANT REQUIRES COSMOLOGICAL CONSTANT. Modern Physics Letters AVol. 14, No. 27, pp. 1917-1922 (1999). https://doi.org/10.1142/S021773239900198X
8. Casado (2004). Connecting Quantum and Cosmic Scales by a Decreasing-Light-Speed Model. arxiv: astro-ph/0404130 [astro-ph].
9. Edward W. Kolb, Michael S. Turner. The Early Universe. 1990. 596 pages Published in: Front. Phys. 69 (1990) 1-547. DOI: 10.1201/9780429492860
10. Dirac, P. A. M., *Nature*, 139, 323 (**1937**).
11. Dirac, P. A. M., *Proc. R. Soc.*, **A 165**, 199 (1938)
12. S. Weinberg, "Gravitation and Cosmology," Wiley, New York, 1972.
13. Turyshev, S. G.; Toth, V. T.; Ellis, J.; Markwardt, C. B. (2011). "Support for temporally varying behavior of the Pioneer anomaly from the extended Pioneer 10 and 11 Doppler data sets". Physical Review Letters. 107 (8): 81103. arXiv:1107.2886
14. John D. Anderson, Philip A. Laing, Eunice L. Lau, Anthony S. Liu, Michael Martin Nieto, Slava G. Turyshev. Study of the anomalous acceleration of Pioneer 10 and 11 // Physical Review D. — 2002. — Vol. 65, no. 8. — P. 082004.
15. Mizony, M.; Lachièze-Rey, M. (2005). "Cosmological effects in the local static frame". Astronomy and Astrophysics. 434 (1): 45–52. doi:10.1051/0004-6361:20042195.
16. Lachièze-Rey, M. (2007). "Cosmology in the solar system: the Pioneer effect is not cosmological". Classical and Quantum Gravity. 24 (10): 2735–2742. doi:10.1088/0264-9381/24/10/016
17. Espen Gaarder Haug. Progress in the Composite view of the Newton Gravitational Constant and its link to the Planck scale. 2022. hal-03591779