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A new law of gravitational force beyond the limits of applicability of Newton's law of gravity.

Abstract: *The law of cosmological gravitational force is proposed in addition to Newton's law of gravitation. The law operates beyond the limit of applicability of Newton's law of gravitation and is applicable to the gravitational interaction of the universe. The new law of gravitational force shows that any body of mass* **m** *is subject to the cosmological force proportional to the mass of the body and the cosmological constant* ***Λ****.*

The formula for the law of cosmological force is:

F = (mc^2)√Λ.

Instead of the gravitational constant ***G****, the cosmological constant* ***Λ*** *is included in the cosmological force law. The new law gives the value of the force very close to the value of the Pioneer anomaly.*

Keywords*: Pioneer effect, Pioneer anomaly, gravitation, Newton's law, large numbers, cosmological equations, cosmological force.*

1. Introduction

The dominant force in the Universe is the gravitational force. Newton's law of gravity was a real breakthrough in science.

$$F_N = G\frac{m_1 m_2}{r^2} \qquad (1)$$

Newton's law of gravitation impresses with its simplicity and mathematical perfection. Gravitational interaction has become the fourth fundamental interaction. Newton's law of gravitation allows us to explain and predict with great accuracy the movements of celestial bodies. The attractive thing about Newton's law of gravitation is the simple dependence of force on the parameters of interacting bodies.

At the same time, simple and perfect in mathematical representation, Newton's law of gravitation has limitations and limits of applicability. Newton's law of gravitation describes the interaction of two point masses. Newton's law of gravity determines with great accuracy the force of interaction between objects within the universe. But it does not take into account the gravitational interaction of bodies with the mass of the Universe. It doesn't answer:"*with what force interacts a body with the mass of the Universe distributed in space?", "on what parameters of the Universe depends the cosmological force?"*.

The limits of applicability of Newton's law of gravitation end where masses cannot be considered as point masses. Attempts to extend it to the Universe lead to Seeliger's Gravitational Paradox [1, 2].

Newton's law of gravitation contains the law of inverse squares. But in relation to the Universe even the very notion of "*distance from a body to the Universe*" becomes incorrect. The reason why Newton's law is not applicable to cosmology is the following restrictions:

- masses are considered point masses;

- the formula includes the law of inverse squares, which is not applicable to the Universe.

Repeated attempts were made to modify Newton's law and make it applicable to cosmology. In 1745 Alexis Clairaut [3] proposed a modification of Newton's law in which the law of inverse squares was changed. In 1894 *Hall A.*[4] proposed replacing the square of the distance by a slightly greater degree. Hugo von Seeliger proposed a modification of the law with a faster than Newton's law of gravitation decreasing with distance [5].

No refinements and cosmetic corrections of Newton's law of gravitation did not make it applicable in cosmology. The simple and mathematically perfect formula of Newton's law was not applicable in cosmology. The law of inverse squares and point masses are the main limiting factors for extending Newton's law to the Universe.

The law of gravitation as the law of inverse squares was formulated in 1645 by Ismail Bullialdus [5]. The law of inverse squares proved to be very productive for solving the two-body problem. This was shown by Newton's law of gravitation. The same law of inverse squares became an insurmountable obstacle when trying to extend the action of Newton's law of gravitation to the Universe. Obviously, another yet undiscovered law of gravitation, different from Newton's law of gravitation, applies to the Universe.

2. Law of Cosmological Force

Instead of trying to extend Newton's law of gravitation to the Universe, we are looking for a new law of gravitation applicable to the Universe. Newton's law as applied to the Universe cannot be considered as a solution to the two-body problem, since it is impossible to correctly introduce the concept of distance between a body that is part of the Universe and a second body (the Universe). On the other hand, the N-body problem has no solution.

Unsuccessful attempts to extend Newton's law to the Universe allow us to formulate additional requirements to the law applicable to the Universe. The law of gravitational force applicable to the Universe must satisfy additional requirements:

- the law must not have restrictions like Newton's law;

- it should not include the law of inverse squares;

- the requirement of point masses for interacting objects should be removed.

The main obstacle is the law of inverse squares. Is it possible to derive a law of force for the Universe as simple as Newton's law, but not burdened by the problem of inverse squares and the problem of point masses? Such a possibility is given by the cosmological constant $\mathbf{\Lambda}$ [6]. Below is a new cosmological force law (Fig. 1), which satisfies the additional requirements for the law of the universe.

The formula of the cosmological force law has the form:

$$F_{Cos} = mc^2\sqrt{\Lambda}$$

Fig. 1. Law of cosmological force. Where: $\mathbf{F_{cos}}$ - cosmological force, **m** - body mass, **c** - speed of light in vacuum, $\mathbf{\Lambda}$ - cosmological constant.

The law of cosmological force gives the same direct dependence of the interaction force on the parameters of interacting bodies as Newton's law. The parameters of the universe in the law are represented by the cosmological constant $\mathbf{\Lambda}$ and **c** - speed of light in vacuum. The parameter of a body is mass ***m.***

The law of gravitational force of the Universe is represented by a simple formula, which is not inferior in simplicity and perfection to the formula of Newton's law of gravitation. The law of cosmological force does not contain constant **G**. Instead of constant **G**, it includes the cosmological constant $\mathbf{\Lambda}$. The cosmological constant $\mathbf{\Lambda}$ acts as a coupling constant in the cosmological force law. This determines the name of the law. The law shows with what force a body interacts with the mass of the Universe distributed in space. The law of cosmological force shows that any body of mass **m** is affected by the cosmological force of the Universe proportional to the mass of the body and the cosmological constant $\mathbf{\Lambda}$.

3. The additional cosmological force.

Thus, the complete law of gravitation must contain two laws:

- Newton's law of gravitation;

- the law of cosmological force.

The law of cosmological force complements Newton's classical theory of gravitation. The complete gravitational interaction includes the pairwise interaction of bodies with an interaction force that obeys Newton's law of gravitation. The gravitational interaction of bodies with the mass of the Universe is added to the pairwise interaction. The force of the additional interaction is subject to the law of cosmological force. The law of gravitation becomes complete with the inclusion of the law of cosmological force.

The additional cosmological force remained unnoticed for a long time. The reason is the small value of the cosmological constant Λ ($\Lambda = 1.36285...\text{x } 10^{-52}\ m^{-2}$) [6]. For the first time, an unknown force was noticed in the Pioneers experiment.

4. Coincidence of the cosmological force value with the Pioneer anomaly.

The range of the cosmological force manifests itself at distances of cosmological scales. The unknown force was first experimentally detected in the Pioneer effect [7, 8, 9]. The Pioneers effect still has no convincing explanation. The new force, which follows from the cosmological force law, was surprisingly found to be close to the Pioneer anomaly.

The value of the cosmological acceleration that follows from the cosmological force law:

$$c^2\sqrt{\Lambda} = 10.4922... \bullet 10^{-10}\, m/s^2 \tag{2}$$

The significance of the cosmological force:

$$F_{Cos} = m \bullet 10.4922 \bullet 10^{-10}\ N \tag{3}$$

Value of the unknown force found in the pioneer effect:

$$F_{Pioneer} = m \bullet (8.74 \pm 1.33) \bullet 10^{-10}\, N \tag{4}$$

In addition to the Pioneer-10 and Pioneer-11 experiment, there is anomalous acceleration data from Galileo and Ulysses [10 - 13]. The value of the unknown force for Galileo:

$$F_{Galileo} = m \bullet (8 \pm 3) \bullet 10^{-10}\, N \tag{5}$$

Value of unknown force for Ulysses:

$$F_{Ulysses} = m \bullet (12 \pm 3) \bullet 10^{-10}\, N \tag{6}$$

The cosmological force law gives a force value very close to both the value of the Pioneer anomaly and the values for Galileo and for Ulysses. The force value F = m(10.4922... × 10^{-10}) N is very close to the experimental values F = m(8.74 ± 1.33) x 10^{-10} N, F = m(8±3) x 10^{-10} N, F = m(12±3) x 10^{-10} N. The coincidence of the force values casts doubt on attempts to explain the pioneer anomaly by a temperature effect.

The value of the cosmological acceleration (A0 = 10.4922... × 10^{-10} m/s^2) is obtained from the system of cosmological equations of the Universe (Fig. 2). This acceleration value is very close to the Pioneer anomaly.

$$
\begin{cases}
G\hbar/r_e^3A_0 = c^1 \\
1/T_U^2\Lambda = c^2 \\
M_UA_0G = c^4 \\
M_UR_UA_0G/T_U = c^5 \\
M_UR_UA_0^2G = c^6
\end{cases}
$$

Fig. 2. System of cosmological equations for calculating the parameters of the observed Universe.

As we can see, the value of the cosmological acceleration ($A_0 = 10.4922... \times 10^{-10}$ m/s^2) is very close to the experimental values $(8.74 \pm 1.33) \times 10^{-10}$ m/s^2, $(8 \pm 3) \times 10^{-10}$ m/s^2, $(12 \pm 3) \times 10^{-10}$ m/s^2.

5. Conclusion

The classical theory of gravitation must include the law of cosmological force in addition to Newton's law of gravitation. The cosmological force is always present as an additional force to Newton's gravitational force. The fundamental gravitational interaction without the cosmological force is incomplete.

There is no need for the cosmological force law to consider masses as point masses. The new law of gravitation does not lead to the Seeliger's Gravitational Paradox. The formula of the cosmological force law does not include the law of inverse squares. The classical theory of gravitation with the introduction of the cosmological force law becomes suitable for solving cosmological problems.

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