Buffon’s needle without π.

Mieczysław Szyszkowicz

mszyszkowicz@yahoo.ca

Buffon's needle problem, posed by Georges-Louis Leclerc, Comte de Buffon in the 18th century, stands as a cornerstone in the realm of geometric probability. The problem encapsulates a scenario where a needle of a given length is dropped randomly onto a floor composed of parallel strips of equal width. The inquiry revolves around determining the likelihood that the needle will intersect a line between two strips. Here a new solution of this classical problem is proposed.

1. Introduction

Buffon's needle technique for estimating π is an example of applying a simulation process to estimate a probability. This experiment is very well known and is widely presented in many books related to statistical simulation and geometrical probability. The description of this experiment is as follows: A needle of length L is dropped on a floor marked by a set of parallel equidistant lines, which are D units apart. This setup can be imagined as a hardwood floor covered by boards with a spacing of D. A set of N needles is dropped on this floor, and intersections with the lines are counted as successes. Of course, a simulation of such a process can be easily realized on a computer, making it an example of a Monte Carlo method.

2. A classical solution

A traditional approach to simulate the Buffon-Laplace needle experiment can be described as follows: The midpoint of the needle is randomly located on the considered grid, and the rotation angle of the needle is also randomly chosen on the interval [0, 2π]. The positions of the ends of the needle are then determined, and their coordinates allow judgment of intersections with the parallel lines on the floor.

Assuming L ≤ D, and using trigonometry and calculus, it can be shown that the probability P that the needle intersects one of the lines is described by the following formula:

$$P = \frac{2}{\pi}\frac{L}{D}.$$

This correct formula was obtained by Comte de Buffon [Buffon 1777]. In the simulation process, the probability is estimated by the proportion H/N, where H is the number of times the needle intersects a line in N tries. Thus, P is approximated as P=H/N. This relation allows the estimation of the value of the number π, as first observed and concluded by Laplace [Lazzarini 1901]. The relation between P and π can be easily inverted, and the estimation of this transcendental number is given accordingly as:

$$\pi = \frac{2}{P}\frac{L}{D} \approx \frac{2N}{H}\frac{L}{D}.$$

3. An approach without the angle (π)

In this presentation, the Buffon-Laplace needle experiment is realized by simulating the position of one of the ends of the needle uniformly on the grid. Two coordinates of the ends, x and y, are randomly chosen on the intervals $0<x\leqslant L$ and $0<y\leqslant D$, respectively. The point with the coordinate (0, y) is chosen to represent one of the ends of the needle. The coordinates (x, y) are used to estimate the distance of the end at the point (0, y) to the point (x, D). The distance represents a length of the segment under the line. If this distance is shorter than L, then the needle intersects the line. According to the specifications, the following relation holds (see also Figures 1 and 2).

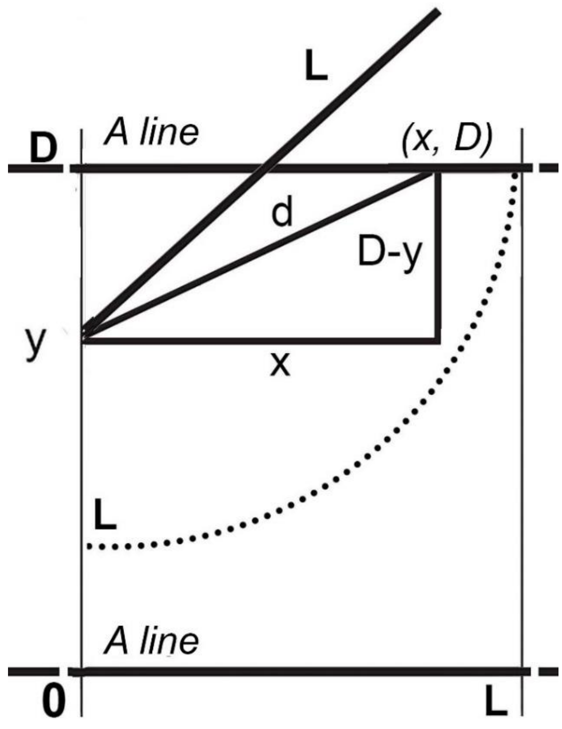

Figure 1. The area where the needle intersects the line (a circle, d<L) and where does not (outside of the circle, d>L).

The criterion for the intersection of the needle with the line is described by the following relation:

$$x^2 + (D-y)^2 \leq L^2$$

This represents one quarter of the circle with the center at the point (0, D) and radius L. The proportion of its area to the area of the rectangle with sides L and D determines the probability P. Thus, this probability is represented in the following form:

$$P = \frac{\pi L^2}{4 * L * D} = \frac{\pi}{4}\frac{L}{D}$$

Finally, inverting this relation results in the formula that can be used to approximate the number π:

$$\pi = 4P\frac{D}{L} \approx 4\frac{H}{N}\frac{D}{L}$$

In this process, dropping the needle at random means that x and y are distributed uniformly within their intervals. The presented process does not have a paradoxical issue. In some implementations of the Buffon needle simulation, the value of the number π is used to realize the rotation of the needle.

4. A trigonometric approach

Ii is the equivalent solution as it presented in the above. In this case the angles are used. Figure 2 illustrates the situation.

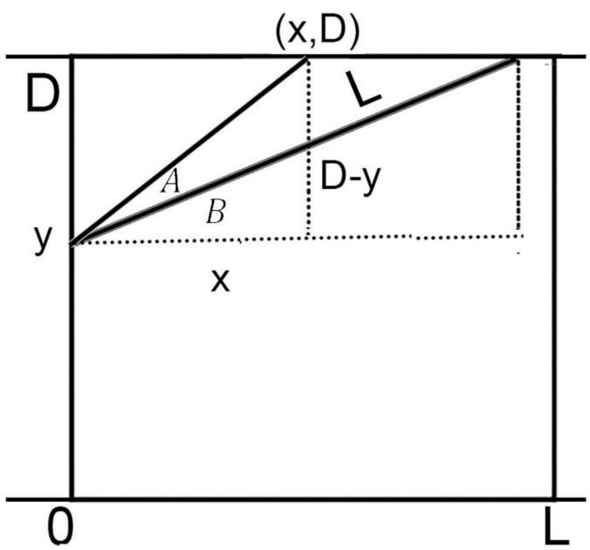

Figure 2. The criterion of the crossings: the angle A is greater than B.

The associations among the segments allow to obtain the following relations

$\frac{D-y}{x} = tgA$ and $\frac{D-y}{L} = \sin B$.

If the angle A is greater than the angle B, then the needle intersects the line. The angle B corresponds to the segments of the length L, i.e. to the needle. Any shorter segment results in the intersection.. It is easy to observed that this criterion is equivalent to the presented above (without using the trigonometry). By symmetry all four cases are covered (left and right, lower and upper corners on the grid).

It should be emphasized that the presented approach is a new point of view on the aold mathematical problem.

Disclosure statement

No potential conflict of interest was reported by the author(s).

Biblograpy

Buffon, Comte de (Georges-Louis LeClerc), . *Essai d’Arithmétique morale, Supplément à l’Histoire Naturelle*, volume 4. (1777), Paris: H. Champion.
Lazzarini, Mario, Un’applicazione del calcolo della probabilita alla ricerca esperimentale di un valore approssimato di π�. *Periodico di Matematica per l'Insegnamento Secondario*, 4 (1901):140–143.