Approaching Goldbach's conjecture using the asymmetric relationship between primes and composites within a limited even boundary

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Abstract

The wave of primes less than integer n is known to determine new primes by removing composites within a limited boundary between n and n^2. As the boundary initiated from n also includes $2n$, it is possible to apply the prime wave analysis to Goldbach's conjecture within a limited $2n$ boundary. Prior to analysis, the $2n$ boundary is divided by two: a first boundary between 0 and n and a second boundary between n and $2n$. From the point of n, the waves of primes in the first boundary directly but asymmetrically connect to composites in the second boundary. Due to asymmetrical connection between primes and composites, the composites are removed and the remaining numbers are all new primes. As a result, the passively remaining new primes in the second boundary, which is not affected by the continued waves of primes, are partially but symmetrically related to the primes in the first boundary. As long as primes and new primes are symmetrical through maintaining the same distance from n, the sum of a prime and a new prime is always equal to $2n$.

Introduction

Goldbach's conjecture has been studied with two major streams: strong conjecture and weak conjecture. The strong conjecture states that every even number greater than 2 is the sum of two primes while the weak conjecture is composed of odd numbers greater than 5 and the sum of three primes (Mattmüller and Lemmermeyer 2015). Goldbach's conjecture is an extension of the prime rules, so it has been studied for over 300 years to understand the primes, but there still remain unsolved problems in the field of number theory.

Goldbach's conjecture is ruled by primes, thus understanding the rule of primes is an essential key to approach the conjecture. Recently, Eom (2024) suggests that primes less than integer n determine the new primes within a limited boundary between n and n^2, which is proved by sine wave analysis. The limited boundary initiated from n also includes $2n$ which means that Goldbach's conjecture can also be explained using the sine wave analysis. In this paper, therefore, Goldbach's conjecture is initially approached by a defined equation, $p + q = 2n$, where p and q are primes, and it is discussed and visualized using the sine wave analysis in the discussion.

Results and Conclusions

Goldbach's conjecture is satisfied when every even integer can be expressed as the sum of two primes. So, let $p + q = 2n$, where p and q are primes ($p < q$) and n is a positive integer (> 1).

The equation can be organized around n.

$$n = \frac{p+q}{2}$$

Based on the boundary explanation (Eom 2024), the organized equation can be written as follows between 0 and $2n$.

$$\mathbf{0}, 1, 2, \ldots, (n-3), (n-2), (n-1), \boldsymbol{n}, (n+1), (n+2), (n+3), \ldots, (2n-2), (2n-1), \mathbf{2}\boldsymbol{n}$$

Two different boundaries are considered from n: the first boundary between 0 and n and the second boundary between n and $2n$. In the first boundary, the largest number is $(n - 1)$ while the smallest one is $(n + 1)$ in the second boundary.

Suppose $(n - 1)$ and $(n + 1)$ are primes, and assume that they are each summed with themselves.

$$(n-1) + (n-1) = 2n - 2 < 2n$$

$$(n+1) + (n+1) = 2n + 2 > 2n$$

If the primes are each summed with themselves in each first or second boundary, the sum of the two primes is not equal to $2n$. Therefore, two primes should be selected from each of the first and second boundary respectively.

Suppose that $(n - \alpha)$ and $(n + \beta)$ are primes, p and q, selected from the first and second boundary respectively, where α and β are any positive integers ($\alpha, \beta < n$).

$$p = (n - \alpha)$$

$$q = (n + \beta)$$

, where $0 < (n - \alpha) < n$ and $n < (n + \beta) < 2n$.

Using the definition of Goldbach's conjecture, two primes are summed.

$$p + q = (n - \alpha) + (n + \beta)$$

$$p + q = 2n - \alpha + \beta$$

At first, the sum of primes is defined by $2n$. Thus,

$$2n = 2n - \alpha + \beta$$

$$\alpha = \beta.$$

As a result, two primes, p and q, from their respective boundaries can be written as follows.

$$p = n - \alpha \text{ and } q = n + \alpha$$

or

$$p = n - \beta \text{ and } q = n + \beta$$

Overall, it is concluded that Goldbach's conjecture is satisfied when the two primes, p and q, are placed in the first and second boundaries respectively by maintaining the same distance from n. Within a limited $2n$ boundary, even numbers including 2, 4, and 6 are excluded because they require 1 as part of the primes in the first boundary. Therefore, the equation $p + q = 2n$ is satisfied when n is greater than or equal to 4 within a limited even boundary.

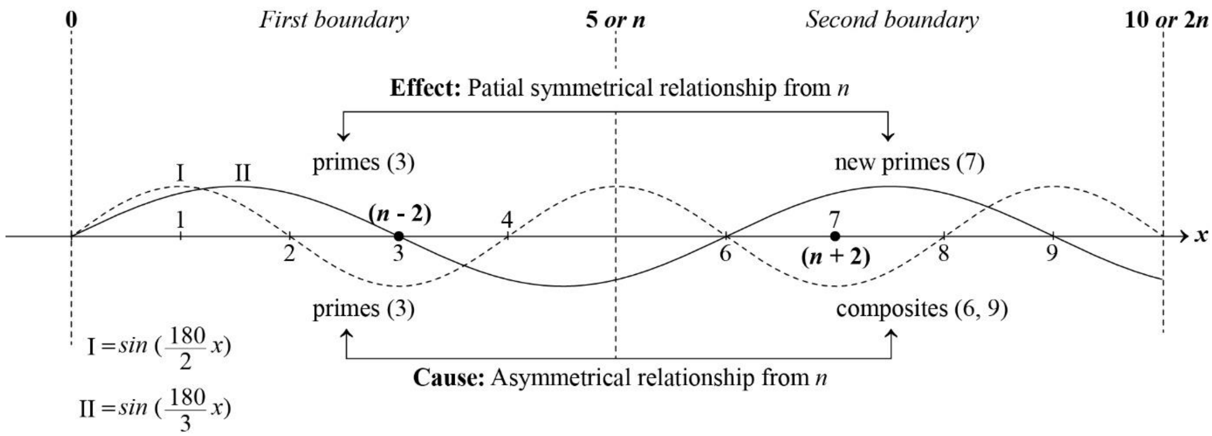
Discussions

Eom (2024) reports that primes less than n determine the new primes by removing composites within a limited boundary, n^2, and it is explained by the prime wave analysis. As the boundary defined between n and n^2 also includes $2n$, Goldbach's conjecture may also be interpreted using the prime wave analysis as discussed below.

In the main text, it is concluded that the two primes should maintain the same distance from n, and their sum is equal to $2n$ by satisfying Goldbach's conjecture. In the wave analysis, it can be explained that the series of prime waves less than n determine new primes by removing the composites between n and $2n$. In other words, the primes in the first boundary between 0 and n and the composites and new primes in the second boundary between n and $2n$ have a cause-and-effect relationship. The wave of primes initiated in the first boundary is continued to the second boundary by selecting the composites, and the wave values on the *x-axis*, where $y = 0$, are asymmetrical from the point of n. In results, the remaining numbers or new primes in the second boundary, which is not affected by the continued waves of primes from the first boundary, can maintain a partial but symmetrical relationship with the primes in the first boundary. If the primes in the first boundary are used as factors for $2n$, these waves of primes including the wave of 2 are excluded because they form the symmetry to the composites in the second boundary.

For example, the wave of prime, 3 or $(n - 2)$, in the first boundary removes the composites, 6 or $2 \cdot (n - 2)$ and 9 or $3 \cdot (n - 2)$, in the second boundary, and they form an asymmetry from the point of 5 or n (Figure 1). As a result, the new prime, 7 or $(n + 2)$, is symmetrical to the prime, 3 or $(n - 2)$. As 3 or $n - 2$ and 7 or $n + 2$ maintain the same distance from 5 or n, the sum of two primes, 3 or $(n - 2)$ and 7 or $(n + 2)$, is equal to 10 $(2n)$. The explanation of wave analysis in Figure 1 is identical to the results and conclusions in the main text. Theoretically, the number of primes is estimated $\frac{n}{ln(n)}$ and $\frac{2n}{ln(2n)} - \frac{n}{ln(n)}$ in the first and second boundaries respectively (Stein 2000), and the estimated number of primes in the second boundary over the first boundary, $\frac{ln(n) - ln2}{ln(n) + ln2}$, converges to 1. It means that the primes in the first boundary increase correspondingly as n increases, which can lead to an almost equal increase in the new primes in the second boundary. Consequently, the number of prime pairs that satisfy Goldbach's conjecture between the first and second boundaries also increases.

Figure 1. The wave of primes less than n in the first boundary determines the new primes by removing the composites in the second boundary. Due to the asymmetrical relationship between primes and composites from n, primes and new primes can be partially but symmetrically related between the first and second boundaries, and their sum is always equal to $2n$. The waves of primes used as factors for $2n$ and the wave of 2 are excluded. Even numbers 2, 4, and 6 are also excluded for $2n$ as these numbers should be satisfied by adding 1 and a prime. Thus, the equation $p + q = 2n$ is satisfied when n is greater than or equal to 4 within a limited even boundary.



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