Non-parametric identification of feedback system without excitation

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ABSTRACT

This paper describes the non-parametric identification of feedback system by two different controllers without exterior excitation. The proposed method doesn't necessarily require any prior information for processing and, furthermore, it can assume time delay and modeling degree with accuracy. Its efficiency is proved by simulation.

Keywords: system identification, non-parameter model, external excitation

1. Introduction

The identification of feedback system is done necessarily when the open loop is unstable [1,2]. This problem has attracted lots of attention in science [3].

Non-parametric identification of feedback system with exterior excitation was widely studied in the past[4-6]. In this case, it normally requires increasing amplitude of exterior excitation signal so as to get enough signals to noise ratio. The exterior excitation signal works on the system as a disturbance signal, though. Here, the larger the amplitude of signal is, the larger the dispersion of output signal is, and it results lower accuracy of identification.

So, it is considered to be effective in practice [7]. [8] decided possible conditions for identification of feedback system. [9, 10] proposed identification of back coupling system when the set signal changes or not. This method is used in case the limitation of time delay, controller order and plant order is given. PID controllers are still widely used in practice.

When PID controller is used, the excitation condition for feedback system was not fully considered. So, in practice, the problem of identification of feedback system appears to be important when PID is used.

My paper proposes the non-parametric identification of back coupling system which can be processed with output data from two different controllers. We can get parametric modeling from non-parametric one.

The content of my paper is as follows.

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Section 2 covers preliminaries and section 3 covers the proposed non-parametric identification method. Section 4 covers the analysis on the simulation. Section 5 covers Conclusion.

2. **Preliminaries**

The paper studies on the feedback system in Fig.1.

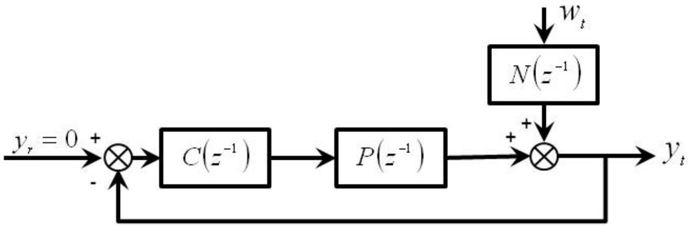

Figure1.Feedback system

Where w_t is output noise, $P(z^{-1}), C(z^{-1}), N(z^{-1})$ is plant, controller, disturbance respectively.

$$P(z^{-1}) = z^{-d} P_0(z^{-1}) = z^{-d} \sum_{i=0}^{\infty} p_i z^{-i} \quad , \quad C(z^{-1}) = \sum_{i=0}^{\infty} c_i z^{-i} \quad , \quad N(z^{-1}) = \sum_{i=0}^{\infty} n_i z^{-i} \qquad (2.1)$$

Where z^{-1} is the one-step backward shift operator, d is the time delay; p_i, c_i and n_i in are the impulse response coefficients; $P_0(z^{-1})$ represents the n-order transfer function without time delay. The reference input y_r s set to zero for convenience.

Assume that $\omega_t \sim N(0,1)$, polynomial of denominator and numerator is minor each other, two controllers $C_1(z^{-1}), C_2(z^{-1})$ make the closed system asymptotically stable.

The output signal y_t is expressed as follows;

$$y_t = \frac{N(z^{-1})}{1 + z^{-d} P_0(z^{-1}) C(z^{-1})} \omega_t = G(z^{-1}) \omega_t \qquad (2.2)$$

Hankel matrix for $P_0(z^{-1})$ is defined as follows in Eq.(2.1).

$$H(h,i) = \begin{bmatrix} p_i & p_{i+1} & \cdots & p_{i+h-1} \\ p_{i+1} & p_{i+2} & \cdots & p_{i+h} \\ \cdots & \cdots & \cdots & \cdots \\ p_{i+h-1} & p_{i+h} & \cdots & p_{i+2h-2} \end{bmatrix} \in R^{h \times h} \qquad (2.3)$$

Where, $i \ge d+1$ and $h \in N(h \ge n)$.

When the dimensionality of $P_0(z^{-1})$ is n [11], we get follow as;

$$rank\{H(h,i)\} = n \ , \ h \ge n \ , \ \mathrm{i} \ge \mathrm{d}+1$$

If $\eta < n$, $rank\{H(h,i)\} = h$. Then the system $P_0(z^{-1})$ is asymptotically stable, $\lim_{i \to \infty} p_i = 0$.

So $rank\{H(h,1)\}$ decides the object's dimensionality. Let's see the method to decide transfer function described as fractional function from impulse response coefficient. First, the transfer function $P_0(q^{-1})$ of plant is truncated by $h_0 + 1$ term as

$$P_0(z^{-1}) = p_d + p_{d+1}z^{-1} + \cdots + p_{d+n}z^{-n} + \cdots + p_{d+h_0}z^{-h_0}, (h_0 \in N, h_0 > 2n) \quad (2.4)$$

Then, $P_0(h^{-1})$ is approximated as a n-dimensional fractional polynomial as follows;

$$P_0(z^{-1}) = \frac{b_0 + b_1 z^{-1} + \cdots + b_n z^{-n}}{1 + a_1 z^{-1} + \cdots + a_n z^{-n}} \quad (2.5)$$

Eqs.(2.4) and(2.5) enables the following relation;

$$\begin{cases} \mathbf{b} = \Psi(\mathbf{a})\mathbf{p}_b \\ -\mathbf{p}_a = \Psi_p(\mathbf{p}, n)\mathbf{a} \end{cases} \quad (2.6)$$

Where

$$\mathbf{b} = \begin{bmatrix} b_0 \\ b_1 \\ b_2 \\ \ldots \\ b_n \end{bmatrix}, \Psi(\mathbf{a}) = \begin{bmatrix} 1 & 0 & 0 & \ldots & 0 \\ a_1 & 1 & 0 & \ldots & 0 \\ a_2 & a_1 & 1 & \ldots & 0 \\ \ldots & \ldots & \ldots & \ldots & \ldots \\ a_n & a_{n-1} & a_{n-2} & \ldots & 1 \end{bmatrix}, \mathbf{p}_b = \begin{bmatrix} p_d \\ p_{d+1} \\ p_{d+2} \\ \ldots \\ p_{d+n} \end{bmatrix} \quad (2.7)$$

$$\mathbf{p}_a = \begin{bmatrix} p_{d+n+1} \\ p_{d+n+2} \\ p_{d+n+3} \\ \ldots \\ p_{d+h_0} \end{bmatrix}, \Psi_p(\mathbf{p}, n) = \begin{bmatrix} p_{d+1} & p_{d+2} & \cdots & p_{d+n-1} & p_{d+n} \\ p_{d+2} & p_{d+3} & \cdots & p_{d+n} & p_{d+n+1} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ p_{d+h_0-n} & p_{d+h_0-n+1} & \cdots & p_{d+h_0-2} & p_{d+h_0-1} \end{bmatrix}, \mathbf{a} = \begin{bmatrix} a_n \\ a_{n-1} \\ a_{n-2} \\ \ldots \\ a_1 \end{bmatrix} \quad (2.8)$$

If we decide dimensionality n and impulse response coefficient $\{p_i\}$ correctly, then $\Psi_p(\mathbf{p}, n)\big|_{h_0=2n} = H(n, d+1) \in R^{n\times n}$. So $rank\{H(n, d+1)\} = n$.

Then, $\mathbf{a}$ is calculated as follows;

$$\mathbf{a} = -\left[\Psi_p^T(\mathbf{p}, n)\Psi_p(\mathbf{p}, n)\right]^{-1}\Psi_p^T(\mathbf{p}, n)\mathbf{p}_a \quad (2.9)$$

Thus, if we know $\Psi(\mathbf{a})$, we can decide $\mathbf{b}$ using Eq.(2.7).

3. The proposed identification method.

3.1 The case with known output noise

Let's see non-parametric modeling identification in the case with known output noise.

When controller $C_i(i=1,2)$ is used, output signal ($\omega_t^{(i)}, G_i(z^{-1})$ $(i=1,2)$ is output noise and system transfer function respectively) is decided as follows;

$$y_t^{(i)} = G_i(z^{-1})\omega_t^{(i)}, (i=1,2) \quad (3.1)$$

Where

$$G_i = N/(1 + PC_i), (i=1,2) \quad (3.2)$$

$$G_i(z^{-1}) = \sum_{j=0}^{\infty} g_j^{(i)} z^{-j} \ , \ C_i(z^{-1}) = \sum_{j=0}^{\infty} c_j^{(i)} z^{-j} \ , (i=1,2) \quad (3.3)$$

Because of $G_1 - G_2 = P(C_2G_2 - C_1G_1)$ by Eq.(3.2),

$$\Phi_i(z^{-1}) = C_iG_i = \sum_{j=0}^{\infty} \xi_j^{(i)} z^{-j} \ , \ (i=1,2) \quad (3.4)$$

Then

$$\bar{g}_0 + \bar{g}_1 z^{-1} + \cdots = \left(p_0 + p_1 z^{-1} + \cdots\right)\left(\bar{\xi}_0 + \bar{\xi}_1 z^{-1} + \cdots\right) \tag{3.5}$$

Where $\bar{g}_i = g_i^{(1)} - g_i^{(2)}$, $\bar{\xi}_i = \xi_i^{(2)} - \xi_i^{(1)}$.

Also, if

$$\begin{aligned} e_t &= \omega_t^{(1)} + \omega_t^{(2)} \\ \tilde{y}_t &= C_2 y_t^{(2)} - C_1 y_t^{(1)} = \Phi_2 \omega_t^{(2)} - \Phi_1 \omega_t^{(1)} \\ \bar{y}_t &= y_t^{(1)} - y_t^{(2)} = G_1 \omega_t^{(1)} - G_2 \omega_t^{(2)} \end{aligned} \tag{3.6}$$

It is a stationary random process of $\tilde{y}_t, \bar{y}_t$ by assumption. Where, the coefficient of correlation between $\bar{y}_t$ and e, $\tilde{y}_t$ and e is decided as follows;

$$\begin{aligned} \bar{R}_{\bar{y}e}(j) &= E\left\{\bar{y}_t e_{t-j}\right\} = E\left\{\left(G_1 \omega_t^{(1)} - G_2 \omega_t^{(2)}\right)\left(\omega_{t-j}^{(1)} + \omega_{t-j}^{(2)}\right)\right\} \\ &= g_j^{(1)} E\left\{\omega_t^{(1)} \omega_t^{(1)}\right\} - g_j^{(2)} E\left\{\omega_t^{(2)} \omega_t^{(2)}\right\} = \bar{g}_j \sigma^2 \\ \tilde{R}_{\tilde{y}e}(j) &= E\left\{\tilde{y}_t e_{t-j}\right\} = E\left\{\left(\Phi_2 \omega_t^{(2)} - \Phi_1 \omega_t^{(1)}\right)\left(\omega_{t-j}^{(1)} + \omega_{t-j}^{(2)}\right)\right\} \\ &= \xi_j^{(2)} E\left\{\omega_t^{(2)} \omega_t^{(2)}\right\} - \xi_j^{(1)} E\left\{\omega_t^{(1)} \omega_t^{(1)}\right\} = \bar{\xi}_j \sigma^2 \end{aligned} \tag{3.7}$$

From the Eq.(3.5) we obtain that

$$\sum_{i=0}^{\infty} \bar{R}_{\bar{y}e}(i) z^{-1} = \sum_{i=0}^{\infty} p_i z^{-i} \sum_{i=0}^{\infty} \tilde{R}_{\tilde{y}e}(i) z^{-i} \tag{3.8}$$

Then, the following equation is defined;

$$\bar{R}_{\bar{y}e}(j) = p_0 \tilde{R}_{\tilde{y}e}(j) + p_1 \tilde{R}_{\tilde{y}e}(j-1) + \cdots + p_j \tilde{R}_{\tilde{y}e}(0), (j = 0,1,\cdots) \tag{3.9}$$

Then, p_j is calculated as follows;

$$p_j = \frac{1}{\tilde{R}_{\tilde{y}e}(0)} \left[\bar{R}_{\bar{y}e}(j) - \sum_{i=0}^{j-1} p_i \tilde{R}_{\tilde{y}e}(j-i) \right] \tag{3.10}$$

Meanwhile, from Eqs.(3.1) and (3.3), the coefficient of relation between $y_t^{(1)}$ and $\omega_t^{(1)}$, $y_t^{(2)}$ and $\omega_t^{(2)}$ is decided as follows;

$$R_{y\omega}^{(i)}(j) = E\left\{y_t^{(i)} \omega_{t-j}^{(i)}\right\} = g_j^{(i)} E\left\{\omega_t^{(i)} \omega_t^{(i)}\right\} = g_j^{(i)} \sigma^2 \ , \ (i = 1,2) \tag{3.11}$$

Then, we get

$$g_j^{(i)} = \frac{R_{y\omega}^{(i)}(j)}{\sigma^2} \ , \ (\mathrm{i} = 1,2) \tag{3.12}$$

and disturbance model can be obtained the as follows;:

$$N\left(z^{-1}\right) = \left(1 + \sum_{j=0}^{\infty} p_j z^{-j} \sum_{j=0}^{\infty} c_j^{(i)} z^{-j}\right) \sum_{j=0}^{\infty} g_j^{(i)} z^{-j} \ , \ (i = 1,2) \tag{3.13}$$

3.2 The case with unknown output noise

Now, let's see the identification of non-parametric model in case output noise is unknown.

In practice, $\omega_t^{(1)}, \omega_t^{(2)}$ is unknown quantity and the sample size M of $y_t^{(1)}, y_t^{(2)}$ is always finite.

The estimate of $\omega_t^{(1)}, \omega_t^{(2)}$ is $\hat{\omega}_t^{(1)}, \hat{\omega}_t^{(2)}$, then it can be represented as

$$\begin{aligned} \hat{\omega}_t^{(1)} &= \omega_t^{(1)} + \varepsilon_t^{(1)} \\ \hat{\omega}_t^{(2)} &= \omega_t^{(2)} + \varepsilon_t^{(2)} \end{aligned} \tag{3.14}$$

We use the following notations

$$\hat{e}_t = \hat{\omega}_t^{(1)} + \hat{\omega}_t^{(2)} = e_t + \varepsilon_t \tag{3.15}$$

Where $e_t = \omega_t^{(1)} + \omega_t^{(2)}$, $\varepsilon_{\mathrm{t}} = \varepsilon_t^{(1)} + \varepsilon_t^{(2)}$.

By whitening process for dataset $\{y_t^{(1)}\}, \{y_t^{(2)}\}$ on M data point and using the estimate $\hat{\sigma}_1 = 1, \hat{\sigma}_2 = 1$, we can get $\hat{\omega}_t^{(1)}, \hat{\omega}_t^{(2)}$. Then, $\overline{R}_{\overline{y}e}, \widetilde{R}_{\widetilde{y}e}$ can be assumed as follows;

$$\hat{\overline{R}}_{\overline{y}e} = \frac{1}{m}\sum_{t=0}^{m-\tau} \overline{y}_t \hat{e}_{t-\tau} \quad , \quad \hat{\widetilde{R}}_{\widetilde{y}e} = \frac{1}{m}\sum_{t=0}^{m-\tau} \widetilde{y}_t \hat{e}_{t-\tau} \tag{3.16}$$

Based on above expression, p_j is assumed as follows;

$$\hat{p}_j = \frac{1}{\hat{\widetilde{R}}_{\widetilde{y}e}(0)} \left[\hat{\widetilde{R}}_{\widetilde{y}\hat{e}}(j) - \sum_{i=0}^{j-1} \hat{p}_i \hat{\widetilde{R}}_{\widetilde{y}\hat{e}}(j-i) \right] \tag{3.17}$$

From the Eqs.(3.11) and (3.12), the impulse response coefficient of closed system can be assumed as follows;

$$\hat{g}_i^{(1)} = \frac{1}{\hat{\sigma}_1^2}\frac{1}{m}\sum_{t=0}^{m-i} y_t^{(1)} \hat{\omega}_{t-i}^{(1)} \quad , \quad \hat{g}_i^{(2)} = \frac{1}{\hat{\sigma}_2^2}\frac{1}{m}\sum_{t=0}^{m-i} y_t^{(2)} \hat{\omega}_{t-i}^{(2)} \tag{3.18}$$

Where, $i = 0,1,\cdots,\lambda$ and the non-parametric model of disturbance is obtained as follows;

$$\hat{N}(z^{-1}) = \sum_{i=0}^{\lambda} \hat{n}_i z^{-i} = \left(1 + \sum_{i=0}^{\lambda} \hat{p}_i z^{-i} \sum_{i=0}^{\lambda} c_i^{(1)} z^{-i}\right) \sum_{i=0}^{\lambda} \hat{g}_i^{(1)} z^{-i} \tag{3.19}$$

Once the non-parametric model of the object is decided, we can decide parametric model by Eqs.(2.4)-(2.9).

4. Simulation examples

The continuous-time model in Equation 12 is as follows;

$$P(s) = \frac{e^{-5s}}{1.5s+1} \quad , \quad N(s) = \frac{4.9}{13.2s+1}$$

The length of the sampling interval is $T_s = 1$, the discrete model is as follows;

$$P(z^{-1}) = z^{-d}\frac{b_0}{1+a_1 z^{-1}} \quad , \quad N(z^{-1}) = \frac{d_0}{1+c_1 z^{-1}}$$

The following PID controllers are used as the first and second controller.

$$C_1^{PID}(z^{-1}) = \frac{0.8692 - 9933z^{-1} + 0.2656z^{-2}}{1 - z^{-1}} ,$$

$$C_2^{PID}(z^{-1}) = \frac{0.08692 - 993.3z^{-1} + 0.02656z^{-2}}{1 - z^{-1}}$$

The number of samples is $M = 2000$. Fig 2 shows the estimated impulse response of object and disturbance.

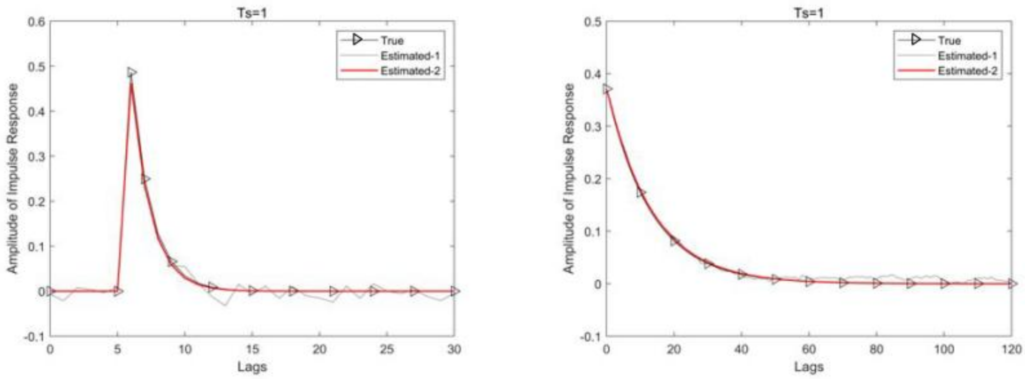

Figure2. Estimated impulse response (object on left, disturbance on the right)

We can see the estimators in Fig 3.

table1. Estimated parameters

	d	a_1	b_0	b_1
True	6	-0.5134	0.4866	0
$T_s=1$	6	-0.4979	0.4618	0.005

The simulation result proves the efficiency of the proposed algorithm for the non-parametric identification.

5. Conclusion

The proposed non-parametric identification doesn't require any prior knowledge of object and disturbance, and performs identification correctly by two different controllers.

The parametric model is obtained from non-parametric. The proposed methods have advantages of estimating time delay and the degree of a sample of a certain object.

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