Revisiting the Michelson-Morley Paradox

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Abstract

The Michelson-Morley experiment and its resolution by the special theory of relativity form a foundational truth in modern physics. In this paper I propose an equivalent relativistic experiment involving a single-source interferometer having infinite arms. Further, we debate the possible outcomes from such an experiment and in doing so uncover a conflict between special relativity and the symmetry of nature. I demonstrate this conflict by the method of *reductio ad absurdum.*

Keywords— Michelson-Morley, symmetry of nature, special relativity

1 Introduction

The aim of this paper is to conduct an in-depth theoretical revisitation of the paradigm shifting Michelson-Morley (MM) experiment and its famous null result [4]. An attempt is made to generalise the events within an MM interferometer (i.e. a single source interferometer having equal length orthogonal arms) so as to arrive at a theoretical construct we refer to as the *infinite arm interfeormeter* (IAI). The thought provoking questions that arise from this exercise are comprehensible at the under-graduate level.

1.1 Premise

This work is premised on the following:

1. The MM null result [1] [2] is interpreted as follows : Under ideal conditions, the interference fringe pattern generated by an MM interferometer having equal length orthogonal arms moving under inertial rules with respect to some rest frame will show no deviation *across* the line of relative motion. This remains true over all orientations of the interferometer with respect to the line of relative motion.
2. Symmetry in physics [3] is interpreted as follows: If two physical experiments involving identical components yield identical sequences of events within identical spatial geometries, then they are in theory, one experiment and the predictions associated with one may be applied to the other .

1.2 Methodology

Let us now investigate and generalise the geometry and sequence of events within a MM interferometer in the steps below:

1. We examine two flat triangles that are relevant to the discussions at hand.

2. We show that the triangles discussed above form a template for the geometry and sequence of events within a MM interferometer.
3. We generalise the MM experiment's null result to create a theoretical interferometer having infinite equal length arms i.e. an IAI.
4. We consider a thought experiment involving travelling waves that reflect and interfere with each other within the confines of a circular boundary. Further, we establish that our thought experiment generates an event sequence equivalent to the IAI we have earlier theorised.
5. We argue the predicted outcome from our thought experiment
6. We debate the physical implementation of our thought experiment in order to arrive at our conclusion.

2 Euclidean Geometry

On a flat surface, we draw any angle θ at origin Q bounded by two equal length line segments $QB = QB' = h$. We join points B and B' to points A and C such that the line segment AC is perpendicular to QB and centred at Q. We will restrict our arguments to the domain $x < h$. Fig. 1 illustrates.

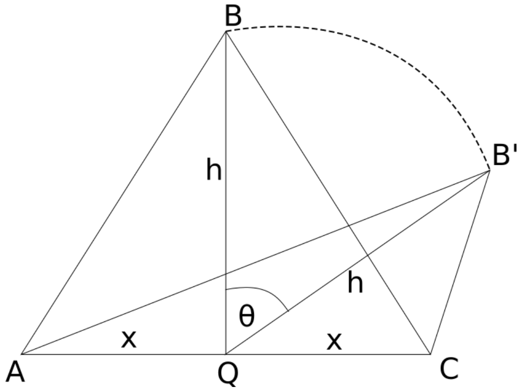


Figure 1: Triangles ABC and $AB'C$ rendered on a flat surface.

From fig. 1, we posit the following:

1. If $x > 0$, physical measurements will verify the theoretical statement $AB + BC \neq AB' + B'C$ remains true for all $\theta \neq 0, \pi, 2\pi...$
2. Since h is constant, curve BB' will take the form of a circle as $0 \leq \theta \leq 2\pi$ independent of x.
3. If $x > 0$, physical measurements will verify the theoretical statement $\angle AB'Q \neq \angle QB'C$ remains true over all $\theta \neq 0, \pi/2, \pi...$

3 A Template of the MM Experiment

Now we turn to theoretical aspects of relativistic optical interferometry to demonstrate that the geometry and sequence of events within an MM interferometer always templates to that of fig. 1.

3.1 Frames of Reference

Consider two imaginary euclidean reference frames that are in relative motion with respect to each other. Let us arbitrarily assume one of these frames is at rest and the other moves with some velocity v with respect to the rest frame. Accordingly we refer to fig. 1 and declare,

1. A rest frame I_0 centered at point Q.
2. A moving frame I_1 that translates from point A to point C with some velocity v relative to rest frame I_0.

3.2 Geometry and Sequence of Events

Now let us consider the structure of an MM interferometer [4](see fig. 2). By fixing $\angle B_1'QB_2' = \pi/2$, line segments QB_1' and QB_2' form the arms of the interferometer. Mirrors B_1 and B_2 are aligned perpendicular to their respective arms. The apparatus may be rotated about its source and consequently each arm subtends its own angle θ measured from a perpendicular to line segment AC. Let us affix moving frame I_1 to the source of the interferometer. Now let us imagine this interferometer moving through space under inertial rules such that,

1. v remains constant ($AQ = QC$).
2. The interferometer orientation (θ) with respect to line segment AC remains constant.

Reference frame I_1 (affixed to the source) translates with constant velocity v from point A to point C. From the perspective of the rest frame I_0, a discrete event cycle begins with the source at point A marking the simultaneous emission of a pair of photons (wavelength=λ). As the entire apparatus moves with some constant ($AQ = QC$) velocity v relative to origin Q along line segment AC, the photons are emitted at point A, reflect from mirrors B_1 and B_2 to finally arrive simultaneously (in phase with each other) at point C. This geometry and sequence of events remains true over all possible orientations of an MM interferometer [2] and over all $0 \leq v < c$ where c represents the velocity of light in free space [5].

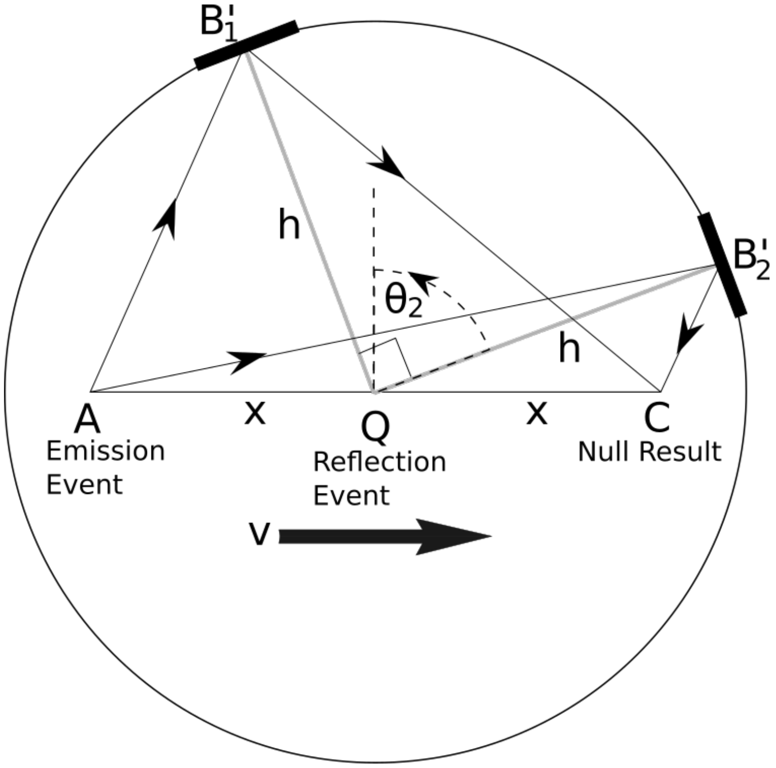


Figure 2: Geometry of the Michelson-Morley experiment depicting the general case $v \neq 0$ and $\theta_i \neq 0, \pi/2, \pi....$ Point Q is chosen as the origin. Only the events within the interferometer that are relevant to relativistic discussion are shown. Independent of the orientation of the interferometer, we find triangle $AB_i'C$ is a generalisation of triangle $AB'C$ in fig. 1. Identical to fig.1, physical measurements of the geometry of events will confirm that $AB_i'+B_i'C \neq AB_j'+B_j'C$ for all $\sin\theta_i \neq \sin\theta_j$ (inequality in path lengths) and $\angle AB'Q \neq \angle QB'C$ (inequality in angles of incidence and reflection). It is evident from the diagram that h and x in fig. 1 are equivalent in both magnitude and direction to c and v in the MM experiment and that curve BB' will take the form of a circle of radius h about point Q. By setting $v = 0$ ($x = 0$), the figure represents the observational perspective of moving frame I_1. By setting $v > 0$ ($x > 0$), the figure represents the observational perspective of rest frame I_0.

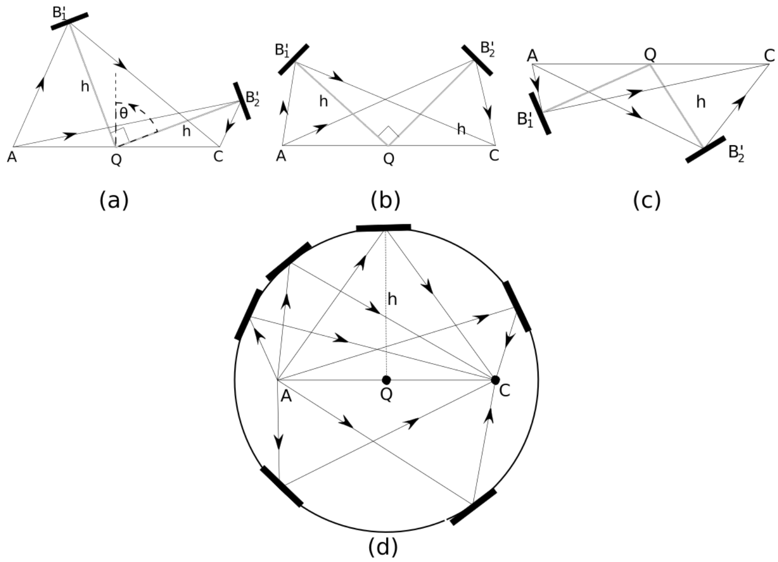
4 Constructing an Interferometer with Infinite Arms

Keeping in mind that in physics we are only interested in positions in space where physical events are occurring, consider the following:

1. In a traditional MM interferometer, the physical structure of the arms are real but irrelevant to the event sequence; the arms serve only to keep the mirrors in inertial motion and do not themselves merit theoretical consideration. Therefore, in theory, we may ignore the arms altogether.
2. We also recognise that in the MM experiment, the mirrors retain practical relevance only during reflection events, thus we may choose to ignore their existence during the emission and result events.
3. By similar arguments, in theory, the interferometric source needs our consideration only during the emission and result events.

4.1 Merging Geometries Over Origin Q

With the above considerations in mind, imagine n discrete cycles of an MM interferometer (arm length $= h$) moving with some velocity v with respect to rest frame I_0. It has been experimentally demonstrated [2] that each of these cycles would render a null result independent of v and the orientation of the interferometer (θ) with respect to the line of relative motion.



(a) (b) (c)

(d)

Figure 3: Merging Geometries. The figure shows 3 runs (a), (b) and (c) of a standard MM interferometer as observed from the perspective of rest frame I_0. The orientation of the interferometer θ is unique for each run. Were it possible to conduct these n runs simultaneously, it is reasonable to predict each discrete cycle would manifest its respective null result at its respective point C. Since all the runs were conducted at the same relative velocity v, we posit that all these null results will be manifested simultaneously. Therefore rest frame I_0 may superimpose these n runs (see figure (d)) over origin Q so as to generate a composite image and will predict a combined null result at point C. The triangles $AB_i'C$ depicted in the figure (d) are a generalisation of triangle $AB'C$ in fig. 1. In the figure (d), we have ignored the mirrors during the emission and result events and also the source during reflection events (Sec. 4 justifies).

Now let us superimpose these n cycles of the MM interferometer as described in fig. 3 and argue further : were it physically possible to construct an apparatus with not 2, or 4 or 8 but rather an infinite number of arms, this too would render a combined null result at point C and by the arguments of sec. 3.2, rest frame I_0 is assured that the locus of all points in space where a reflection event can occur is a circle of radius h about point Q. This IAI theoretical construct generalises the notion of two arms, each oriented at some angle θ, into an infinite number of arms (i.e. over all $0 \leq \theta \leq 2\pi$), each terminated at a point mirror. Since the arms are infinite in number, this circle of point mirrors takes the form of a continuous circle of radius h about point Q. By selecting point Q as origin, rest frame I_0 is assured that this circle of reflection remains fixed in space over all $0 \leq v < c$. Further, at the instant of emission, the role of two photons in the MM experiment is generalised into a two dimensional equivalent of Einstein's spherical wave [6]. Let us refer to this thought process as generalising a standard MM interfometer over all $0 \leq \theta \leq 2\pi$.

4.2 A Thought Experiment

Imagine an ideal homogeneous flat surface S1 enclosed by an ideal rigid boundary of geometrically circular shape (radius $= h$) and capable of transporting a travelling wave of the form,

$$\frac{1}{c^2}\frac{\delta^2 y}{\delta t^2} = \frac{\delta^2 y}{\delta x^2} \tag{1}$$

where the terms are as follows:

1. x (m) represents the displacement of the measurement point from the origin of the wave measured along the direction of travel,
2. c (m/s), a constant, represents the velocity of the wave measured along the direction of travel,
3. y (m) represents the instant displacement of surface S1 measured perpendicular to its equillibrium state.
4. t (s) represents the time elapsed since the instant that the wave was created.

From directly above, we may project fig.1 onto S1 without distortion such that the boundary of S1 is defined by curve BB', a circle of radius h about point Q. It is reasonable to agree that surface S1 supports the geometry of fig. 1 over all $0 \leq \theta \leq 2\pi$ and $0 \leq x < h$.

We choose any point A on S1 and disturb the equilibrium causing an isotropic [3] [6] sinusoidal wave (wavelength = λ) to emanate from that point and travel outwards at constant velocity c. As this primary wave expands, its wavefront will interact with S1's boundary generating innumerable secondary waves as it does so. Each reflection event along curve BB' generates its own isotropic [3] wave and from physical measurements of fig. 1, we find that if $x \neq 0$ the statement $AB + BC \neq AB'_1 + B'_1C ... \neq AB'_i + B'_iC$ is true (See fig. 4 which is a generalisation of fig. 1 over all $0 \leq \theta \leq 2\pi$). Let us invoke the following assumptions to debate the nature of the interference pattern at point C:

1. The wave we generate originates from a single point and travels according to eq. 1.
2. λ remains constant in accordance with the law of conservation of energy [7].
3. Reflections are instantaneous and lossless.

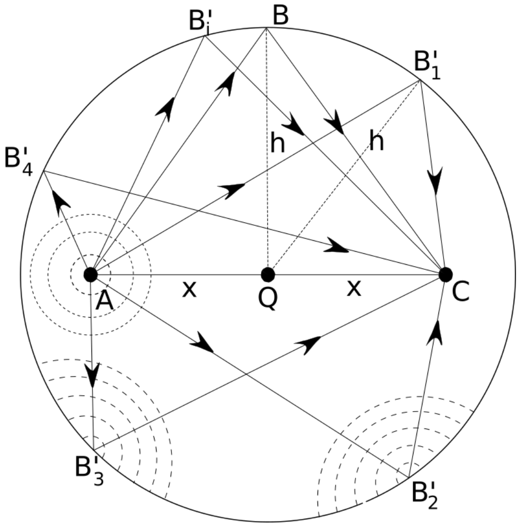


Figure 4: A single isotropic sinusoidal wave is emitted from point A and reflects from the circular boundary generating innumerable secondary wavefronts. We predict (a) if $x = 0$, (i) every wave that is created will expand equally in all directions and the wave velocity c in the medium will remain constant and (ii) reflection events from any two points B_i' and B_j' occur at the same instant in time and (b) by setting $x > 0$, (i) every wave will expand isotropically, c will remain constant and (ii) the reflection events from B_i' and B_j' will be separated in time except if the line joining B_i' and B_j' is perpendicular to AC. In the figure, we see $x \approx 2h/3$ and readily agree that the reflection event from B_2' must occur *after* the reflection event from B_3'. We consider these predictions further in sec. 5.

4.2.1 Predicted Outcome

To predict the outcome of our thought experiment, let us consider the geometry of the sequence of events from the perspective of an observer anchored to point Q. These are:

1. The emission of an isotropic wave [6] at point A.
2. Reflection events from an infinite number of point mirrors, all of which are located on a continuous circle of radius h about point Q.
3. Some outcome at point C, which we seek to determine by argumentative means.

Since the components involved and the sequence and geometry of events from our thought experiment are identical to those manifested within a theoretical IAI, we may now invoke premise 2 (refer sec. 1.1) to predict a null result at point C over all $0 \leq x < h$. We note here that since $AB_i' + B_i'C \neq AB_j' + B_j'C$ for all $\sin\theta_i \neq \sin\theta_j$, this prediction (a null result) disagrees with the application of eq. 1 to the geometry under consideration in cases where $x > 0$.

4.3 Practical Implications

The thought experiment presented may be brought a step closer to realisation by imagining a pair of isotropic radio antennae placed diametrically opposite each other within a reflective boundary of circular shape. This apparatus would form a physical implementation of the theoretical IAI described in sec. 4.

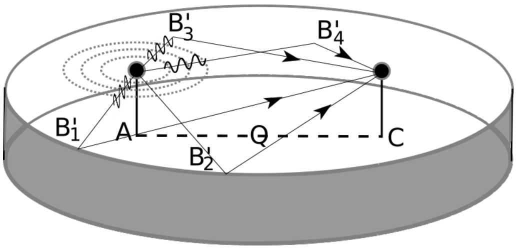


Figure 5: Two isotropic radio antennae placed diametrically opposite each other within a circular reflective boundary. When viewed from directly above, this physical setup is identical to fig. 4. Invoking premise 2 (refer sec. 1.1) we posit that when energised, this setup should generate events and manifest relativistic effects identical to a Michelson-Morley interferometer moving from point A to point C under inertial rules but generalised over all $0 \leq \theta \leq 2\pi$.

An isotropic source of electromagnetic waves is placed at some random point A within a circular shaped reflective boundary of arbitrary radius h. An isotropic receiver is placed diametrically opposite (point C). By energising the system, we posit that this IAI setup will generate events and manifest relativistic effects equivalent to the MM experiment, but generalised over all $0 \leq \theta \leq 2\pi$. When viewed from directly above, we see that within the IAI, if $x > 0$ then $AB + BC \neq AB'_1 + B'_1C ... \neq AB'_i + B'_iC$ and should this apparatus present a null result as predicted, we would encounter an identical inequality in path lengths as presented by the MM experiment in cases where $v > 0$.

4.4 Estimating Relative Velocity

Given the geometry of the sequence of events within the IAI, let us estimate the relative velocity between frames I_0 and I_1. Recall from fig. 2 that v and c in the MM experiment are equivalent to x and h in our thought experiment. Further, since c is known[5], determining relative velocity between I_0 and I_1 does not require a clock and v may be obtained by measuring rod alone,

$$v \approx \frac{x}{h} 299792458(m/s) \tag{2}$$

Thus by setting $x \approx h$ in a practical IAI experiment, we have arranged experimental conditions that according to our predictions must generate events and manifest relativistic effects equivalent to an MM interferometer having $v \approx c$ and generalised over all $0 \leq \theta \leq 2\pi$.

4.5 Observational Perspectives in the IAI experiment

From a practical viewpoint, let us discuss the method of realising the observational perspectives of both the rest frame (I_0) and the moving frame (I_1). Curiously, the IAI apparatus i.e. two isotropic antennae placed diametrically opposite each other within a circular reflector does not itself move with respect to origin Q. A human experimenter may follow the procedure below to generate events equivalent to those experienced by either the rest or moving frames of a single cycle in an MM interferometer:

1. To reveal the observational perspective of moving frame I_1, we select points A and C to coincide with point Q. This creates the geometry of fig. 4 such that $x = 0$. In this special case frames I_0 and I_1 are coincident and at rest with respect to a human experimenter over the full emission-reflection-result cycle.
2. To reveal the observational perspective of rest frame I_0, we choose points A and C such that $0 < x < h$. See also fig. 4 such that $x > 0$. In this general case of the IAI experiment, the human experimenter continues to remain affixed to origin Q

and recognises moving frame I_1 translate with velocity v from point A to point C within the time interval T of the emission-reflection-result cycle. By choosing points A and C near the periphery of the reflective boundary, we create the condition $x \approx h$ thereby arranging the a geometry and sequence of events identical to an MM interferometer (aboard frame I_1) moving relative to I_0 having $v \approx c$ (equation 2 refers). In a traditional MM interferometer, it is difficult to realise the perspective of rest frame I_0 [8] in cases where $v \approx c$. However we see that within the IAI, this may be achieved simply by setting $x \approx h$.

5 Is the IAI Truly a Relativistic Experiment?

Let us consider the various temporal and spatial effects manifested in a relativistic experiment and debate whether a physical conduct of the IAI experiment would manifest these effects or not.

5.1 Postulates of Special Relativity

1. *The laws of physics are equal in all frames of reference* [6]. This postulate flows directly from symmetry in nature [3]. Since the IAI experiment is a natural one, it follows that this postulate is satisfied.
2. *Light is always propagated in empty space with a definite velocity c which is independent of the state of motion of the emitting body* [6]. This property of light waves is equivalently satisfied in the IAI experiment for we readily predict (refer fig. 4) that independent of x, i.e. from the perspective of either frame I_0 or I_1, all the wavefronts will be isotropic [6] and all the waves will travel according to eq. 1 with equal and constant velocity c.

5.2 Reflection Geometry

The equivalence of MM and IAI experiments is also manifest in the peculiarities of the reflection event if $v > 0$ (the perspective of the rest frame) and $\theta \neq 0, \pi/2, \pi..$ (Refer fig. 1). It can be seen from the figure that in the case of the MM interferometer, the angles of incidence ($\angle AB'Q$) and reflection ($\angle QB'C$) are *unequal.* Since the IAI event sequence and geometry is a generalisation of the MM interferometer over all $0 \leq \theta \leq 2\pi$, this relativistic spatial effect may also be evidenced by physical measurements of triangle AB'_iC in fig. 4. In an astronomical scale experiment, this relativistic effect (as observed at point A or point C) is known as stellar aberration [9] and is proportional to v/c (x/h in the IAI experiment) and $\sin 2\theta$.

5.3 Simultaneity of Relativity

Recall from fig. 1 that if $\theta \neq 0$ then in both IAI and MM experiments, points B and B'_i are separated in space from the perspective of both resting and moving observers. Therefore we may refer to fig. 4 and predict for both experiments:

1. If $x = 0$ (the perspective of moving frame I_1) then all the reflection events from the circular boundary occur simultaneously.
2. If $x > 0$ (the perspective of rest frame I_0) then reflection events from any two points B'_i and B'_j occur simultaneously only if $\sin\theta_i = \sin\theta_j$ i.e. only if the line segment $B'_iB'_j$ is perpendicular to AC (see fig. 4). In cases where $x > 0$ and $\sin\theta_i \neq \sin\theta_j$, these reflection events are separated in time as a function of x and the physical distance between B_i and B_j measured *along* the line of relative motion.

This difference in temporal observational perspectives between two frames in relative motion with each other is recognised as that of distant simultaneity [10].

6 Null Result

Finally let us debate whether or not the IAI will render a null result. The following are the only reasonable outcomes expected from a physical conduct of the IAI experiment:

1. Upon energising the system, rest frame I_0 observes a null result only if $x = 0$. But this experimental truth would disagree with our prediction from sec. 4.2.1 which predicts instead a null result over all $0 \leq x < h$. Should this outcome emerge from experiment, we are faced with a contradiction [11] between an experimental truth and a theoretical truth. The resulting paradox may be stated as follows: Given identical waves, identical mirrors and identical sequences of events within identical spatial geometries, has nature abandoned [12] her impartiality [3] and *preferred* to implement a null result within a two arm interferometer [4] but **not** in an identical experiment generalised over all $0 \leq \theta \leq 2\pi$?
2. Next we consider the case that when energised, the IAI returns a null result over all $0 \leq x < h$ in agreement with sec. 4.2.1 *and* the space-time distortions (lorentz contraction and time dilation) predicted by special relativity are also manifested proportional to the lorentz factor [6],

   $$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \tag{3}$$

   Eq. 3 predicts that if $v \approx c$, then lorentz contraction and time dilation grow to infinite magnitudes. In compliance with the method of argument known as *reductio ad absurdum* let us assume these predictions are true [12]. Thus we invoke eq. 3 to posit that if $v \approx c$, rest frame I_0 must predict the IAI null result event to be physically approached asymptotically in space and in time. Recall from sec. 4.5 that this observational perspective may be arranged within the IAI simply by setting $x \approx h$. Given that the apparatus is physically nothing more than a pair of radio antennae placed within a circular reflector, the expectation that setting $x \approx h$ (equivalently $v \approx c$) and energising the system will manifest infinitely large distortions in space and time strains the very bounds of reason. Rather, common sense [13] urges rest frame I_0 to predict:

   (a) Points A and C will remain fixed in space and the reflector will remain in circular shape.

   (b) The emission-reflect-result cycle will complete within some finite period of time.
3. Finally we consider the case that upon energising the system, the IAI returns a null result over all $0 \leq x < h$ in agreement with sec. 4.2.1, but manifests no evidence of lorentz contraction and time dilation. In this case rest frame I_0 is faced with the following questions:

   (a) This null result would be in conflict with the predictions of wave theory applied to the geometry at hand (refer sec. 4.2.1).

   (b) Given identical waves, identical mirrors, identical sequences of events within identical spatial geometries and identical relativistic effects presented in section 5, has nature abandoned her impartiality [3] and *preferred* to implement lorentz contraction and time dilation in the operation of a two arm MM interferometer [14] but **not** in an identical experiment generalised over all $0 \leq \theta \leq 2\pi$?

   (c) How do we reconcile the paradox of unequal path lengths presented by the IAI null result?

7 Conclusion

Traditionally, the MM problem is reconciled by selecting point A as the origin followed by the application of special relativity [14]. But we have seen that by selecting instead point Q as the origin, rest frame I_0 is assured that over all $0 \leq v < c$, the locus of all points in space where a reflection event can occur is a stationary circle of radius h about point Q. This intermediate truth and its theoretical consequences are presented for further scrutiny.

8 Statements and Declarations

The author has no competing interests to declare that are relevant to the content of this article. There are no data associated with this article.

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