From: Derivation of Ancient Approximations of the Circle Figure π
to: a Complete Solution of Leonardo da Vinci's Vitruvian Man

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1) Introduction:

In this report the author tries to handle with four themes.
Referring the first topic (Chapter 2) derivations of ancient approximations for the Circle Figure π are presented using the figures of the two- and three-dimensioninal case for the straight and the round.

Chapter 3 deals with possible connections between the Circle Figure π and the Golden Section Φ using modified terms as presented at the first topic.

At Chapter 4 a complete solution of the puzzle around the drawing Vitruvian Man of Leonardo da Vinci is presented, which is mostly based on the informations given by two german authors.

At Chapter 5 a system is described, which is based on the geometrical system of the drawing *Vitruvian Man* of Leonardo da Vinci and by which the attempt of a connection between the Circle Figure π and the Golden Section Φ is done.

2) Derivation of Ancient Approximations of the Circle Figure π :

Since the antiquity the approximation ***22:7*** (=3.14285714) for the Circle Figure π (=3.14159265) is known. This approximation is accurate regarding two descendants and delivers a relative percentage deviation of 0.04%.
The Circle Figure approximation ***355:113*** (=3.14159292) is described to the Chinese Tsu Ch'ung Chi, who lived from 340 to 510 after Christus. This approximation is accurate regarding 6 descendants and delivers a relative percentage deviation of 8.5E-06%.
The history of these approximations can be taken from the internet page [1].

In the following a starting stage is presented, which is based on two- and three-dimensional relations between circle and square and between sphere and cube and which leads to the just mentioned approximations for the Circle Figure π.
The diameters of circle and sphere and the side lengths of square and cube are treated as equivalent and set to the unit-less value 1.

In the two-dimensionial case one gets the following values for the surfaces and circumferences:

Square Area: $A_{Sq} = L_{Sq}^2 = 1$;	Circle Area $A_C = D_C^2 * \pi : 4 = \pi : 4$
Square Circumference: $C_{Sq} = 4 * L_{Sq} = 4$;	Circle Circumference $C_C = D_C * \pi = \pi$

In the above results there are the figure 4 and the Circle Figure π besides the figure 1.

In the three-dimensionial case one gets the following values for the volumes and surfaces:

Cube volume $V_{Cu} = L_{Cu}^3 = 1$;	Sphere volume $V_{Sp} = D_{Sp}^3 * \pi : 6 = \pi : 6$
Cube surface $S_{Cu} = 6 * L_{Cu}^2 = 6$;	Sphere Surface $S_{Sp} = D_{Sp}^2 * \pi = \pi$

These results contain the figure 6 and the Circle Figure π besides the figure 1.

Looking for further informations the figures 4, 6 und π , which are given by the two- and three-dimensional comparision, are connected in the following way:

1) $(6 + \pi) : (4 + \pi) = 1.28005 \approx 1.28$ (1.1)

One sets the approximation Pi^1 for the Circle Figure π and the result value 1.28 in the upper relation and dissolves the equation for Pi^1:

$(6 + Pi^1) : (4 + Pi^1) = 1.28$
$(6 + Pi^1) : (4 + Pi^1) = 128 : 100$
$100 * (6 + Pi^1) = 128 * (4 + Pi^1)$
$Pi^1 * (128 - 100) = 600 - 512 = 88$
$\mathbf{Pi^1 = 88 : 28 = 22 : 7}$

2) $(6 - \pi) : (4 - \pi) = 3.329896 \approx 10 : 3$ or ≈ 3.33 (1.2)

Now one sets the approximation Pi^{2a} for the Circle Figure π and the fraction "10:3" as result in the upper relation and dissolves the equation for Pi^{2a}:

$(6 - Pi^{2a}) : (4 - Pi^{2a}) = 10 : 3$ (1.3)
$3 * (6 - Pi^{2a}) = 10 * (4 - Pi^{2a})$
$Pi^{2a} * (10 - 3) = 40 - 18 = 22$
$\mathbf{Pi^{2a} = 22 : 7}$

At least one sets the approximation Pi^{2b} for the Circle Figure π and the result value 3.33 in the Equation (1.2) and dissolves the equation for Pi^{2b}:

$(6 - Pi^{2b}) : (4 - Pi^{2b}) = 3.33 = 333 : 100$ (1.4)
$333 * (4 - Pi^{2b}) = 100 * (6 - Pi^{2b})$
$Pi^{2b} * (333 - 100) = 4 * 333 - 6 * 100$
$\mathbf{Pi^{2b}} = (4 * 333 - 6 * 100) : (333 - 100) \mathbf{= 732 : 233 = 3.14163090}$

The approximation Pi^{2b} for the Circle Figure π ist accurate regarding 3 descendants and delivers a relative percentage deviation of 1.2E-03 %.

If one subtracts the nominator or denominator, respectively of the approximation Pi^{2a} from the nominator or denominator, respectively of the approximation Pi^{2b}, one gets (analog to [1]):

(732 - 22) : (233 - 7) = 710 : 226 = 355 : 113

The presented way, which connects the two-dimensional with the three-dimensional referring to the straight and the round, is very simple and leads to the Circle Figure approximations ***22:7*** and ***355:113*** (by use of the Circle Figure approximation ***732:233)*** which are well known since more than 1500 years.

3) Possible Connections of the Circle Figure *π* to the Golden Section Φ:

There is a well known approximation for the Circle Figure π, which is performed with the root of the Quotient Φ of the Golden Section:

$Pi^{R\Phi} = 4 : \sqrt{\Phi} = 3.1446046$

The Quotient Φ of the Goldenen Section is given by: $\Phi = 0.5*(1+\sqrt{5}) = 1.6180340$
and the root of Φ: $\sqrt{\Phi} = 1.27202$ Further is valid: $\Phi^2 = \Phi + 1$

The approximation $Pi^{R\Phi}$ for the Circle Figure is accurate regarding two descendants and delivers a relative percentage deviation of 0.096%. Therefore the approximation $Pi^{R\Phi}$ is less accurate than the approximation 22:7.

[There are better well-known approximations for the Circle Figure dependent on the Quotient Φ, for example: $Pi^{\Phi 2} = 1.2 * \Phi^2 = 3.1416408$]

What is special regarding the Circle Figure approximation $Pi^{R\Phi}$?

If the Circle Figure π is replaced by the Approximation $Pi^{R\Phi}$ in Equation (1.1) and Equation (1.2), it delivers the Equations (2.1) and (2.2):

$$(6 + Pi^{R\Phi}) : (4 + Pi^{R\Phi}) = 1.2799315 \qquad (2.1)$$

$$(6 - Pi^{R\Phi}) : (4 - Pi^{R\Phi}) = 3.3381025 \qquad (2.2)$$

If one adds the result of Equation (2.1) to the one of Equation (2.2), one gets the eye falling result:

$$(6 + Pi^{R\Phi}) : (4 + Pi^{R\Phi}) + (6 - Pi^{R\Phi}) : (4 - Pi^{R\Phi}) = 4.6180340 = \Phi + 3 = \Phi^2 + 2 \qquad (2.3)$$

Could it be, that starting with the Equations (1.1) and (1.2) in connection with the Equations (2.1) and (2.3) one can find unknown informations about the Circle Figure π dependent on the Quotient Φ of the Golden Section?

Up to the present the Golden Section Φ got only little attention in the science. And as presented in this chapter, particularly the root of the Quotient ($\sqrt{\Phi}=1.27202$) could get a great significance.
One looks at the length measure inch with a length of 2.54 cm, which is close to the term $2*\sqrt{\Phi}$ *cm*.
And the term $\Phi*\pi:2$ *cm* amounts to 2.5416 cm.

These two terms connected with each other lead to the already mentioned Circle Figure approximation: $Pi^{R\Phi} = 4 : \sqrt{\Phi} = 3.1446046$
Please see some well-known special features of the Golden Section Φ (Figure 1, page 19).

4) Determination of the Circle Diameter at the Drawing *Vitruvian Man* of Leonardo da Vinci, his Proportion Study and a graphical Solution for the Leg Positions

Because of possible missing rights referring the Reproduction Image and therefore possible legal complaints this Extract of an author's report, which is not yet publicly available and which deals only with the drawing of Leonardo da Vinci, is presented without an Image of the Drawing *Vitruvian Man*! At Figure 1 to Figure 4 the reader has to imagine the presence of an Image of the Drawing *Vitruvian Man*. This is regrettable, but nevertheless the reader can get an impression about the ideas behind the derivation of the Circle Diameter and the Solution of the - in the past - unknown leg positions!
One can also get a good impression about this topic by the Images at the homepage of the author Klaus Schröer [2].

In the case of the drawing *Vitruvian Man* the general question has been in the past: which dependence has the Circle Diameter on the Square Side Length?

After nearly 500 years the two german authors Klaus Schröer and Klaus Irle (see [3] - translated title into English: *But me, I square the circle)* got the idea - if they were the first one, is unknown to the author -, to sketch a second circle around the horizontal outstreched arms. And the very important information by that was, that this smaller circle is nearly equal in area to the drawn square of the drawing *Vitruvian Man* (The drawing is also named *Proportion Study*).
The circle, which is equal in area with the Square Sq_1 (see Sketch Extract1) is named as Circle C_1.

Consistently the authors drew a second bigger Square Sq_2, which is nearly equal – as far as possible because of the deviations of the drawn Circle C_{VM} to an ideal circle - in area to the Circle C_{VM} at the Proportion Study. Additionally a Circle C_2 was sketched, which is equal in area to the Square Sq_2.
The Diameter D_{VM} of the Circle C_{VM} has been a puzzle since centuries and was firstly determined by K. Schröer and K. Irle by an Iteration Method, which is presented in book [3].

Important: the calculated diameter D_2 of circle C_2 used in this report corresponds to the Diameter D_{VM} of Circle C_{VM} at the drawing *Vitruvian Man*.

Short desciption of the informations won by the book [3] and new insights:

What the authors of book [3] found out by this geometric constellation, is very important. A straight line (in the following named $g_2(x)$) runs **a)** through the left lower edge of Square Sq_1, **b)** through the navel point of the Vitruvian Man (or the mid point M_{C2} of Circle C_2, respectively) and furthermore **c)** through the upper intersection point S_{R2L2} of Circle C_2 and Square Sq_2 (see Sketch Extract1 or Figure 1).

Last one (c) is not self-evident and the key for the solution.

Figure M9 of book [3] at page 125 shows a line, which is equivalent to the just named straight line $g_2(x)$. At one of the author's first drafts the straight line $g_{1\#}(x)$ (see Sketch Extract1 at page 5) was sketched with the goal to get any informations. With the knowledge of book [3], that two pairs of square/circle exist behind the drawing, the author realized, that the additionally sketched straight line $g_1(x)$ is the line, by which one is able to get a simple solution for the diameter D_2 by the equalization of the straight lines $g_1(x)$ and $g_2(x)$. By that an Iteration method is not necessary. Please see the calculations at page 6, how the calculated straight lines $g_1(x)$, $g_2(x)$ and the geometry at the drawing *Vitruvian Man*, the Proportion Study correlates to each other.

The formula of the diameter D_2 of Circle C_2 in dependence of the side length L_{Sq2} of Square Sq_2 amounts in the case of the Proportion Study:

$$\mathbf{D_2} = L_{Sq2} * \sqrt{(4:\pi)} = \mathbf{L_{Sq1} * [1 - 0.5*\sqrt{(4:\pi)}] : [\sqrt{(4:\pi)} - 1]^{0.5}} = \mathbf{L_{Sq1} * 1.216327} \qquad \mathbf{(3.1)}$$

One can clearly see at Equation (3.1), that besides the square side length L_{Sq1} the diameter D_2 is only dependent on the Circle Figure $\boldsymbol{\pi}$. Equation (3.1) is equivalent to the term "$2 * L_{Sq1} * x_{i+1}$" with the quantity x_{i+1} given in book [3].

According to the author's knowledge the authors K. Schröer and K. Irle are the first ones, who published the formula of the radius $\mathbf{x_{i+1}}$ for the Circle C_{VM} at the drawing of Leonardo da Vinci (given at page 123 in book [3]).

The factor $F_{\sqrt{4/\pi}}$ ($=\sqrt{(4:\pi)}=D_1:L_{Sq1}=D_2:L_{Sq2}$) used at Equation (3.1) means area equality of square and circle. Besides the square side length L_{Sq1}, the factor $F_{\sqrt{4/\pi}}$ is a starting value.

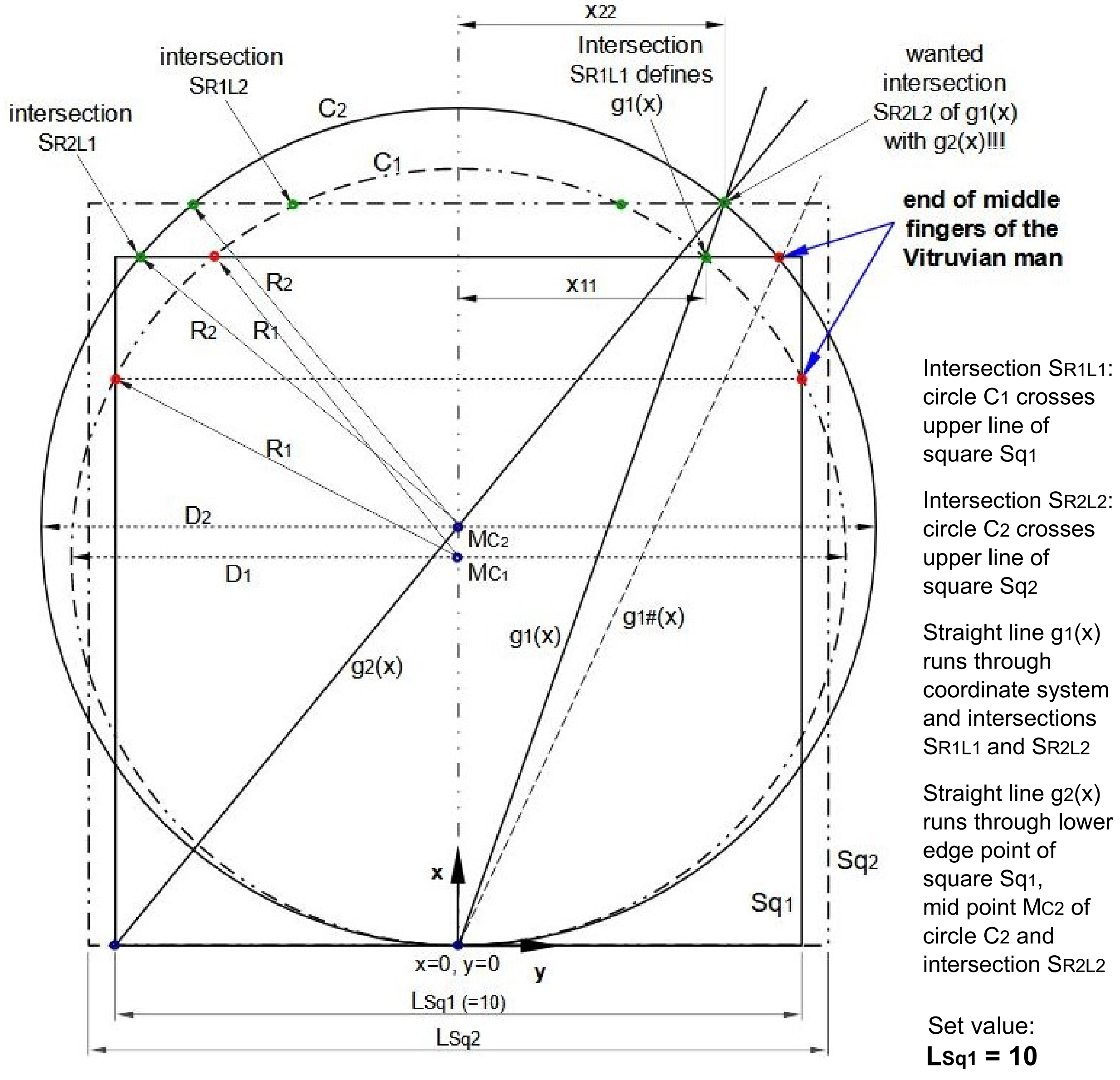
Derivation and proof of Equation (3.1), which is also presented grahicly at Sketch Extract2:

1) $L_{Sq1} = 10$ ---> $D_1 = L_{Sq1} * \sqrt{(4:\pi)} = 11.283792$ ---> $R_1 = 0.5 * D_1 = 5.641896$
2) $g_1(x) = L_{Sq1} * x : x_{11}$ with $x_{11} = [R_1^2 - (L_{Sq1}-R_1)^2]^{0.5} = L_{Sq1} * [\sqrt{(4:\pi)} - 1]^{0.5} = 3.5830039$
3) $g_2(x) = 0.5*D_2 + D_2 : L_{Sq1} * x$; D_2 can be firstly determined at step 6, because the not yet known quantity D_2 correlates with the variable x at Equation $g_1(x) = g_2(x)$
4) Quantity x_{22} is special because of the formulas (see also the solution vector at Sketch Extract2):
   $g_{2\_i}(D_i) = 0.5*D_i + D_i : L_{Sq1} * x_{22} = L_{Sqi}$ and $D_i : L_{Sqi} = \sqrt{(4:\pi)}$
   Diameter D_1 set in above Formulas is: $g_{2\_1}(D_1) = 0.5*D_1 + D_1 : L_{Sq1} * x_{22} = L_{Sq1}$ and $D_1 : L_{Sq1} = \sqrt{(4:\pi)}$ ---> $\mathbf{x_{22} = L_{Sq1} * (\sqrt{(\pi:4)} - 0.5) = 3.8622693}$
5) $g_1(x_{22}) = L_{Sq1} * x_{22} : x_{11} = 10 * 3.8622693 : 3.5830039 = 10.7794169 = \mathbf{L_{Sq2}}$ **!!!**
6) $g_1(x_{22})$ is equal to the square side length L_{Sq2}, by that diameter D_2 can be derived from it:
   $$\mathbf{D_2 = L_{Sq2} * \sqrt{(4:\pi)} = 10.7794169 * \sqrt{(4:\pi)} = L_{Sq1} * [1 - 0.5*\sqrt{(4:\pi)}] : [\sqrt{(4:\pi)} - 1]^{0.5}} =$$
   $$= \mathbf{12.1632695} \quad ---> \quad R_2 = 0.5 * D_2 = 6.0816347$$

7) Test: $g_2(x_{22}) = 0.5*D_2 + D_2 * x_{22} : L_{Sq1} = 6.0816347 + 12.1632695 * 3.8622693 : 10 =$
$= 10.7794169$ **[= $g_1(x_{22})$**, see step 5**]**

Value x_{22} with "$x_{22} = L_{Sq1}*(\sqrt{(\pi:4)}-0.5)$" is the only solution value of Equation "$g_1(x)=g_2(x)$"

Please keep in mind the mid-points of the circles and squares, which will have some importance for further examinations in this report, at the sketches and figures!



Sketch Extract1: Circles and Squares of this sketch correspond to those of Figure 1

According to the report [4] the measure value of the diameter D_C of the circle C at the drawing *Vitruvian Man* is D_C=220 mm and the square side length L_{Sq}=181.5 mm. The ratio D/L_{Sq} amounts to 1.21212, which means a relative deviation of -0.35% to the calculated values of this report:

$[D/L_{Sq} - (D_2/L_{Sq2})] : (D_2/L_{Sq2}) = [1.212121 - 1.216327] : 1.216327 = -0.0035$

Leonardo da Vinci's Vitruvian Man-System:

inspired by the book of K. Schröer / K. Irle.
Wanted is the intersection of straight lines $g_1(x)$ and $g_2(x)$ at equality in area of square and circle.
$g_1(x)$ and $g_2(x)$ cross each other at the intersection of upper line of Square Sq2 with Circle C2. The values of L_{Sq2} and D_2 are solvable graphically and analytically.

Square length $L_{Sq1} = 10$
Circle-Ø $D_1 = L_{Sq1} \cdot \sqrt{(4:\pi)} = 11.283792$
from "$L_{Sq1} \cdot L_{Sq1} = D_1 \cdot D_1 \cdot \pi : 4$" (area equality)

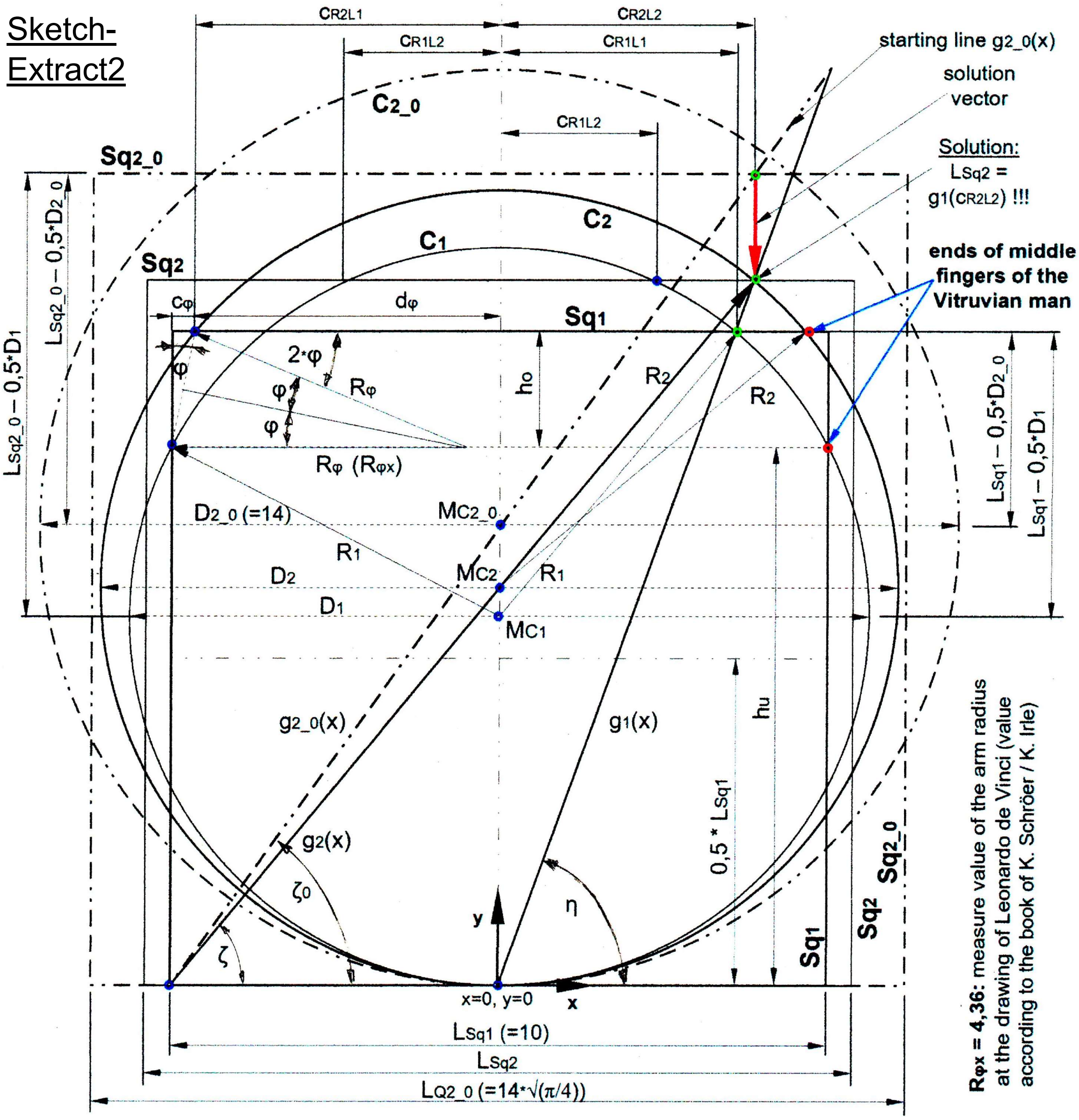
Circle-Ø $D_{2\_0} = 14$ (Start values are marked with _0)
Square length $L_{Sq2\_0} = D_{2\_0} \cdot \sqrt{(\pi:4)} = 12.407177$

Remark: $c_{R1L1} = x_{11}$ **and** $c_{R2L2} = x_{22}$

$R_1 = 0.5 \cdot D_1$ and $R_2 = 0.5 \cdot D_2$
$g_1(x) = L_{Sq1} : c_{R1L1} \cdot x$ with
$c_{R1L1} = [R_1^2 - (L_{Sq1} - R_1)^2]^{0.5} = 3.5830$
$g_2(x) = 0.5 \cdot D_2 + D_2 : L_{Sq1} \cdot x$
$\tan(\eta) = L_{Sq1} : c_{R1L1}$; $\eta = 70.2874°$
$\tan(\zeta_0) = D_{2\_0} : L_{Sq1}$
$\zeta_0 = 54.4623°$

$h_u = R_1 + (R_1^2 - 0.25 \cdot L_{Sq1}^2)^{0.5} =$
$= 8.2555$;
$h_o = L_{Sq1} - h_u = 1.7445$

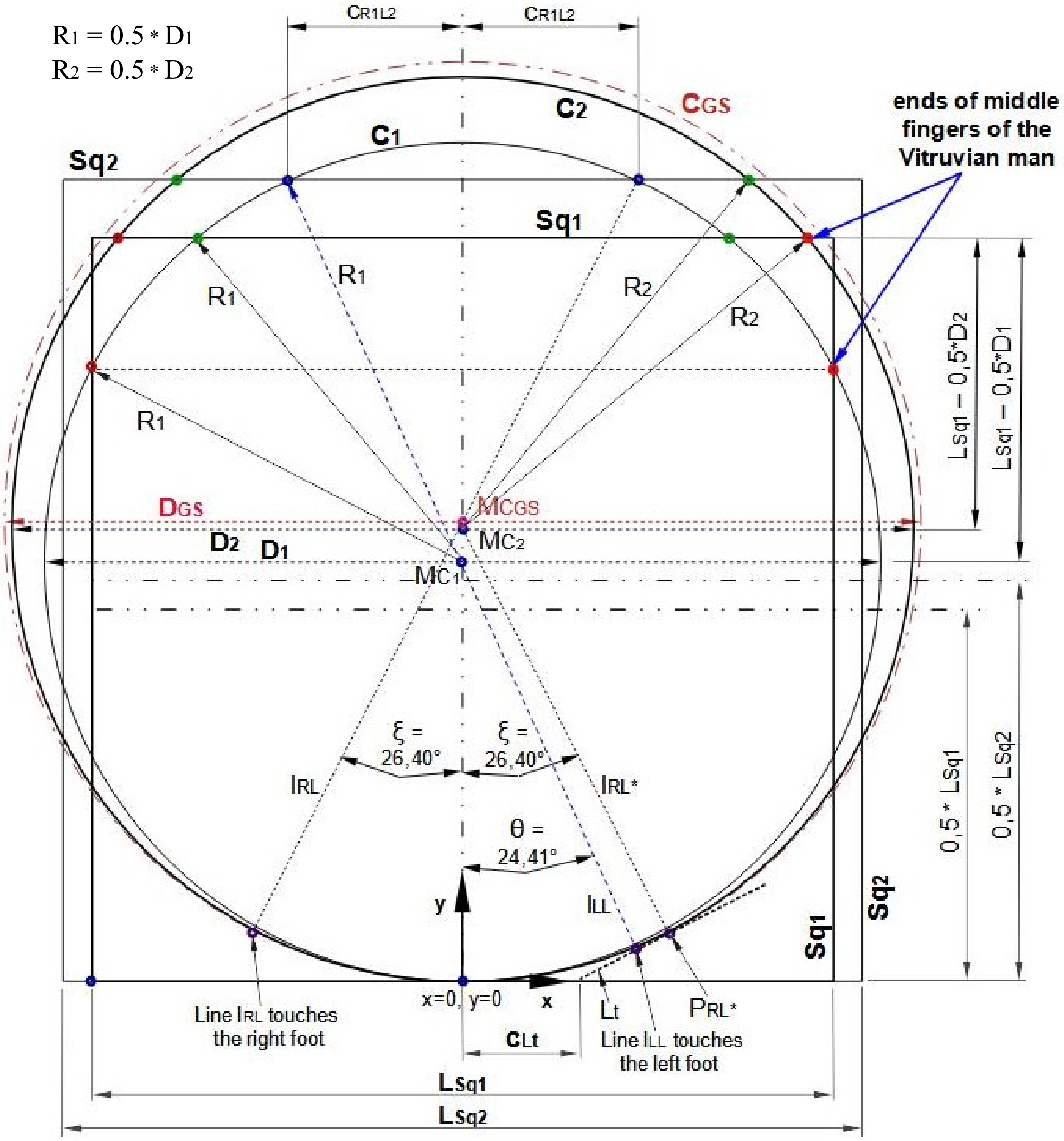
Sketch-Extract2



Solution: D_2 and L_{Sq2} are wanted quantities
$c_{R2L2} = L_{Sq2\_0} : \tan(\zeta_0) - 0.5 \cdot L_{Sq1} = 3.86227$
$L_{Sq2} = c_{R2L2} \cdot \tan(\eta) = 10.77942$
$D_2 = L_{Sq2} \cdot \sqrt{(4:\pi)} = 12.16327$
$R_2 = D_2 : 2 = 6.081635$
$\zeta = \arctan[L_{Sq2} : (0.5 \cdot L_{Sq1} + c_{R2L2})] = 50.57476°$
$g_1(x = c_{R2L2}) = L_{Sq1} : c_{R1L1} \cdot x = 10.77942$
$g_2(x = c_{R2L2}) = 0.5 \cdot D_2 + D_2 : L_{Sq1} \cdot x = 10.77942$

$d_\varphi = c_{R2L1} = [(R_2^2 - (L_{Sq1} - R_2)^2]^{0.5} = 4.6511$
$c_\varphi = L_{Sq1} : 2 - d_\varphi = 0.3489$
$\varphi = \arctan(c_\varphi : h_o) = 11.3101°$
$R_\varphi = h_o : \sin(2 \cdot \varphi) = 4.5356$ (R_φ: Arm Radius)

Measure value: $R_{\varphi x} = 4.36$; $\varphi_x = \arcsin(h_o : R_{\varphi x}) : 2$
$\varphi_x = 11.7926°$; $c_{\varphi x} = h_o \cdot \tan(\varphi_x) = 0.3642$
$d_{\varphi x} = L_{Sq1} : 2 - c_{\varphi x} = 4.6358$
$D_{2\varphi x} = (L_{Sq1}^2 + d_{\varphi x}^2) : (L_{Sq1}) = 12.1491$



Sketch Extract3: Geometries of this sketch correspond to those of Figure 2

Straight line Lt is orthogonal to straight line L_{RL*} and tangential to Circle C_2.
Dimension c_{Lt} is nearly 1:7 of square side length L_{Sq1}. This is proved by the following calculation!

$\xi = \arctan[c_{R1L2} : (L_{Sq2} - R_2)] = 26.3972°$ with $c_{R1L2} = [R_1^2 - (L_{Sq2}-R_1)^2]^{0.5} = 2.3317$

$x_{PRL\#} = R_2 * \sin(\xi) = 2.7038$ and $y_{PRL\#} = R_2 * (1 - \cos(\xi)) = 0.6341$

$c_{Lt} = x_{PRL\#} - y_{PRL\#} : \tan(\xi) = 1.4263 \approx L_{Sq1} : 7 = 10 : 7 = 1.4286$ (length of the foot[a])

Relative deviation of the dimension c_{Lt} to the length of the foot ($=L_{Sq1} : 7$) takes the value -0.16%.

Angle θ: $\theta = \arctan[c_{R1L2} : (L_{Sq2} - R_1)] = 24.4113°$

Diameter D_{GS} of the Circle C_{GS} is the diameter, which corresponds to the Golden Section Φ.

$D_{GS} = 2 * (1 : \Phi) = 2 * 0.618034 = 1.236068$

a) At page 74 of the book [3] the quotation of Leonardo da Vinci is given: *“.... The foot may be the seventh part of the man.”* [Translation by the author may not be correct]

Calculation of the Arm Radius R_φ:

The Arm Radius R_φ, which is dependent on the Circle Diamter D_2, is calculated to:

$R_\varphi = ho : \sin(2*\varphi) = 4.5356$

The calculation of the angle φ is listed at the end of page 6 (right side) and the quantity ho is derived at the top of page 6 (also right side).

The measure value of the Arm Radius $R_{\varphi x}$ at the drawing of Leonardo da Vinci takes the value **$R_{\varphi x}$=0.436** according to book [3] (at page 97 this measure value is named "x" and is related to a square side length L_{Sq1}=1. Arm Radius $R_{\varphi x}$ means 43.6% of the Vitruvian Man width). The Arm Radius $R_{\varphi x}$=4.36 corresponds to the square side length L_{Sq1}=10, which is applied for the calculations of this report. This measure value is about **4%** smaller than the calculated Arm Radius R_φ (= 4.5356). Percentage value is related to the Radius $R_{\varphi x}$.

The Circle Diameter $D_{2\varphi x}$, which corresponds to the measure Arm Radius $R_{\varphi x}$ (=4.36), amounts according to the formulas at the end of page 6 (right side) to:

$D_{2\varphi x} = (L_{Sq1}^2 + d_{\varphi x}^2) : L_{Sq1} = 12.14906$

The value of $D_{2\varphi x}$ is about **0.12%** [=(12.16327-12.14906) : 12.14906] smaller than the Diameter D_2.

Notice: in the radius range of the Circle Diameter D_2 (=12.16327) small changes of diameter D_2 cause relative big changes of the Arm Radius R_φ.
See the calculated deviations before: a deviation of 0.12% at D_2 causes a deviation of 4% at R_φ!

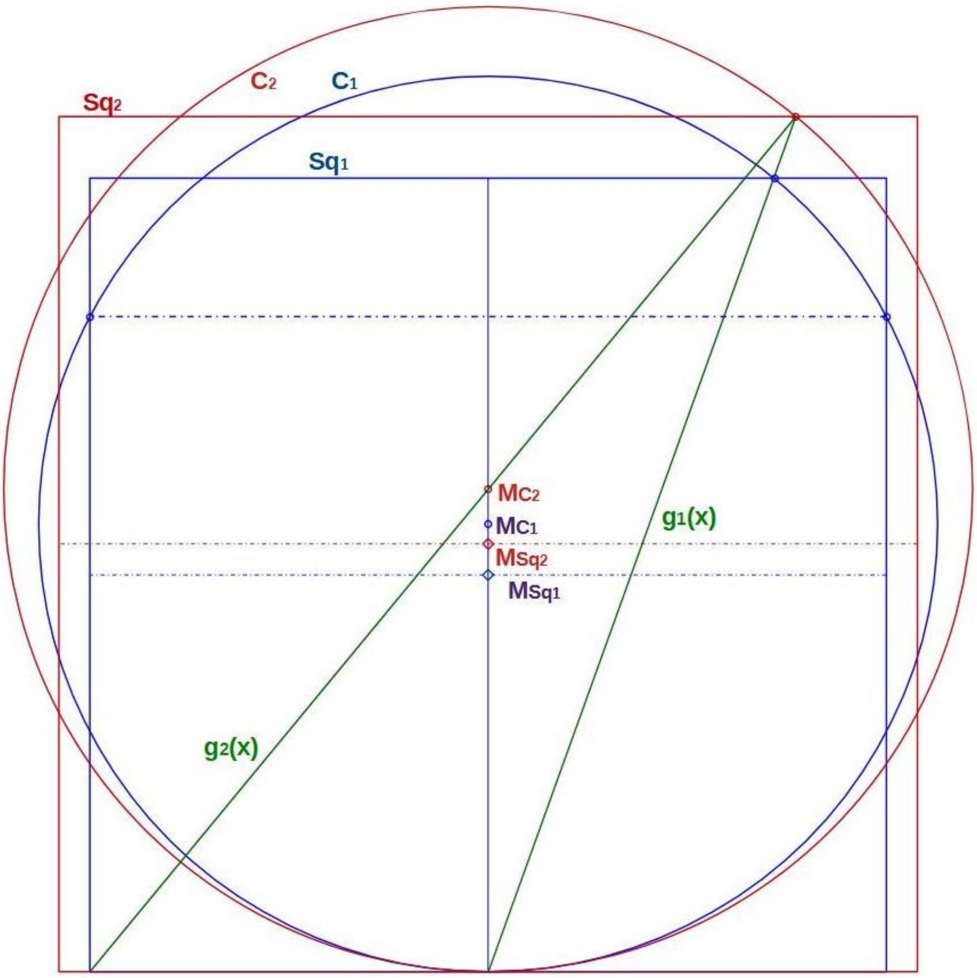


Figure 1: Equalization of the two straight lines $g_1(x)$ and $g_2(x)$

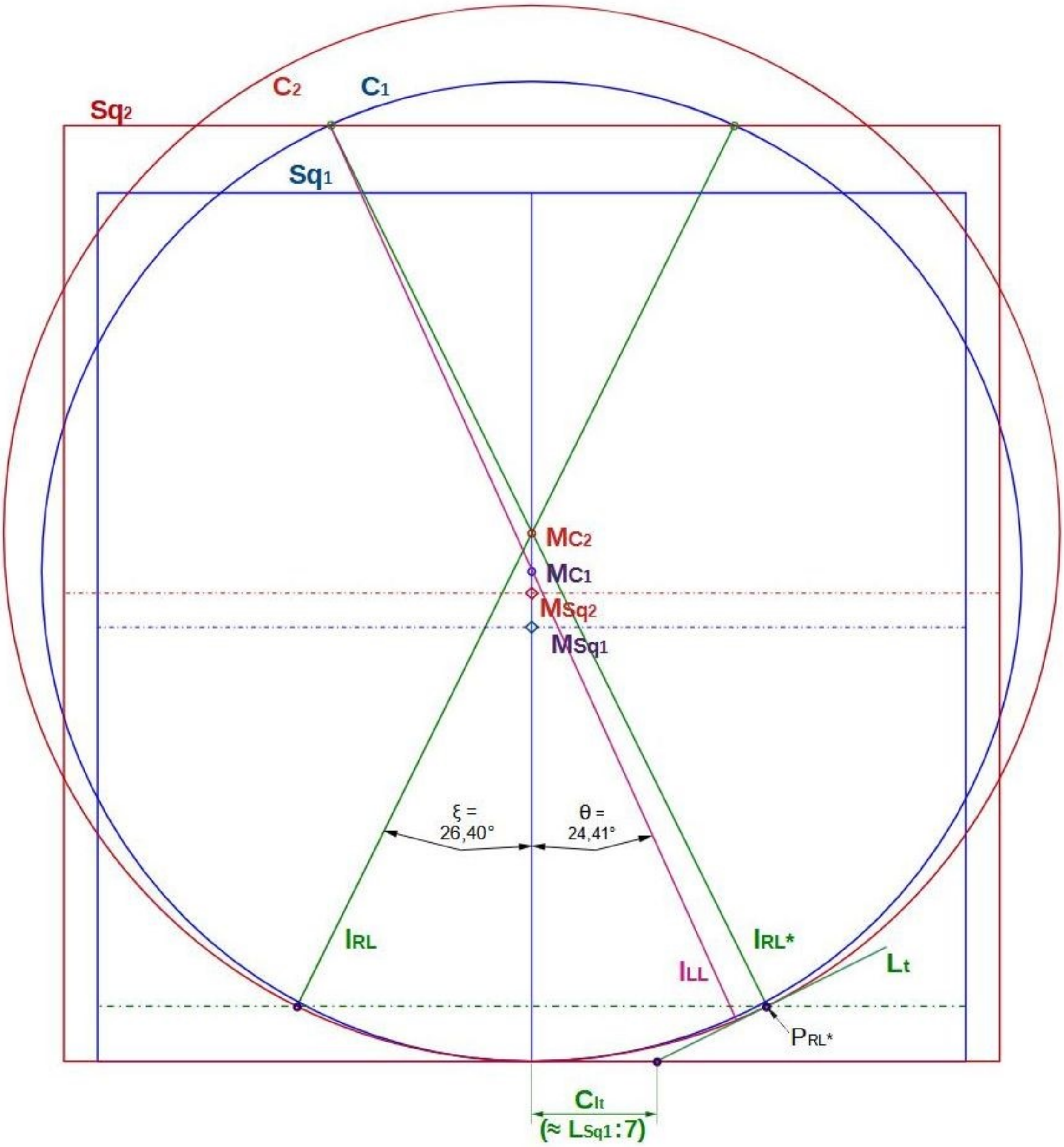


Figure 2: Leg Positions dependent on Mid Points M_{C1} and M_{C2}

Leg Positions:

The straddled leg position at the left side of the Proportion Study could comprehend with a line named line l_{RL} (see Figure 2). [RL are the initials for Right Leg. LL are the initials for Left Leg] Line l_{RL} runs through the mid point M_{C2} of the Circle C_2 and through the upper intersection point of Circle C_1 with Square Sq_2. The angle ξ between the vertical and line l_{RL} is **26.397°** (see also Sketch Extract3 at page 7).
Line l_{RL} nearly tangentially touches the foot of the straddled leg at the left side of the drawing (see Figure 2).

The straddled leg at the right side of the Proportion Study is distorted compared to the one at the left side. What might be the reason for this distortion drawn by Leonarso da Vinci? It has to have a reason for it.
The answer might be: the adjustment of the left foot heel (at the right drawing side) to the line L_{LL}.

Line l_{LL} runs through the mid point M_{C1} of the Circle C_1 and through the upper intersection point of Circle C_1 with Square Sq_2 and nearly tangentially touches the foot of the straddled leg at the right side of the drawing The angle **θ** between the vertical and line l_{LL} amounts to **24.411°** (see also Sketch Extract3 at page 7).

Please imagine now that line l_{RL} is mirrowed at the vertical axis at x=0 (see Figure 2).
This mirrowed line l_{RL^*} intersects the Circle C_2 down at the right side of the drawing at a certain point, which is named P_{RL^*}. And now one takes the tangential straight line Lt to the Circle C_2 at this point P_{RL^*}. As shown at Figure 2, the straight line Lt crosses the lower side lines of the squares (at y=0) at the x-value c_{Lt}. And the dimension c_{Lt} comprises nearly the foot length, which is 1/7 of the width of the Vitruvian Man. The absolute relative deviation of the dimension c_{Lt} related to the foot length (1/7 L_{Sq1}) is only 0.16%. Please see the calculation and also the quotation of Leonardo da Vinci at Sketch Extract3 at page 7. Geometry of Sketch Extract3 corresponds to the geometry of Figure 2!

By that one can assume, that the straight line Lt represents the sole plane, on which the foot of the left straddled leg stands. And this plane (or line Lt) indicates to the end of the left foot of the upright standing leg. **Attention:** the left legs of the Vitruvian Man are located at the right side of the drawing.

These statements could be the reason, why the left straddled leg (and by that the foot) at the right side of the drawing is distorted. Lines l_{LL} and l_{RL^*} define the foot position and by that its leg distortion!

Observations referring the mentioned Equilateral Triangle:

At page 72 of the book [3] the quotation of Leonardo da Vinci is given: "*If you spread the legs so far, that the height measured from the head is diminished by 1/14, that the centre of the outermost points of the outstretched extremities is the navel and the area between the legs is an equilateral triangle.*" [Translation by the author may not be correct]

From this quotation one can take the following four informations:

1) the vertical height 1/14 of the heigth of the Vitruvian Man. It means a vertical height "1/14 * L_{Sq1}" from the bottom.
2) the navel point, which corresponds to the Mid Point M_{C2} of the Circle C_2
3) an equilateral triangle: assumption: the peak of this triangle is located at the Mid Point M_{C2}
4) the area between the legs: assumption: a) this area is also one of a triangle and it is nearly equal to the area of the equilateral triangle b) its peak is located at the male limb (Mid Point M_{Sq1} of the Square Sq_1), because of the note "area between the legs" and c) dependent on some of the just mentioned quantities at the points 1 to 3.

The Equilateral Triangle $Tr_{60°}$ (every angle between the side lines possesses 60°; see red lined triangle at Figure 3), which peak is located at the mid point M_{C2} of the Circle C_2 and the lower edges are located at the height 1/14 of L_{Sq1}, is nearly equal in area to the triangle Tr_{bL} (blue lined triangle), which peak is located at the mid point M_{Sq1} (onset of the male limb) of the Square Sq_1. The lower edges of the triangle Tr_{bL} are located at the bottom lines. The right side line runs through the point P_{IS}, which is the intersection between the horizontal line "1/14 L_{Sq1}" and Circle C_2 (see details of point 1 and point 2 given above). Both side lines possess the angle 56.286°.
The abbreviation bL stands for "between the Legs".

The equilateral triangle $Tr_{60°}$ posseses the area value 16.633 in comparison to the triangle Tr_{bL} with the value 16.682. The relative deviation of the two values amounts only to about 0.3% (see calculated values at next page). For an observer this deviation is not visible at a sketch like Figure 3.

Calculation of the areas of the triangles:

Area of the equilateral triangle $Tr_{60°}$ with its peak at the mid point M_{C2} (y = 6.0816):

Half angle: 30°

$h_{1/14} = L_{Sq1} : 14 = 10 : 14 = 0.71429$

Area 1: $Ar_{60°} = [(0.5*D_{C2} - h_{1/14})^2 * \tan(30°)] = (6.08163 - 0.71429)^2 * \tan(30°) = 16.633$

Area of the triangle Tr_{bL} between the legs with its peak at the mid point M_{Sq1} (y = 5): side lines run through the Intersection Point P_{IS} at Circle C_2 and at the height "1/14 of the body":

half width of the triangle Tr_{bL} at $y = h_{1/14}$: $b_{1/14} = [R_2^2 - (R_2 - h_{1/14})^2]^{0.5} = 2.8597$

half angle: $\arctan[b_{1/14} / (0.5*L_{Sq1} - h_{1/14})] = 33.7137°$

Area 2: $Ar_{bL} = M_{Sq1}^2 * \tan(33.7137°) = 5^2 * \tan(33.7137°) = 16.682$

Relative deviation: $(Ar_{bL} - Ar_{60°}) : Ar_{60°} = (16.682 - 16.633) : 16.633 = 0.3\%$

Area $Ar_{60°}$ is chosen as reference area in the denominator

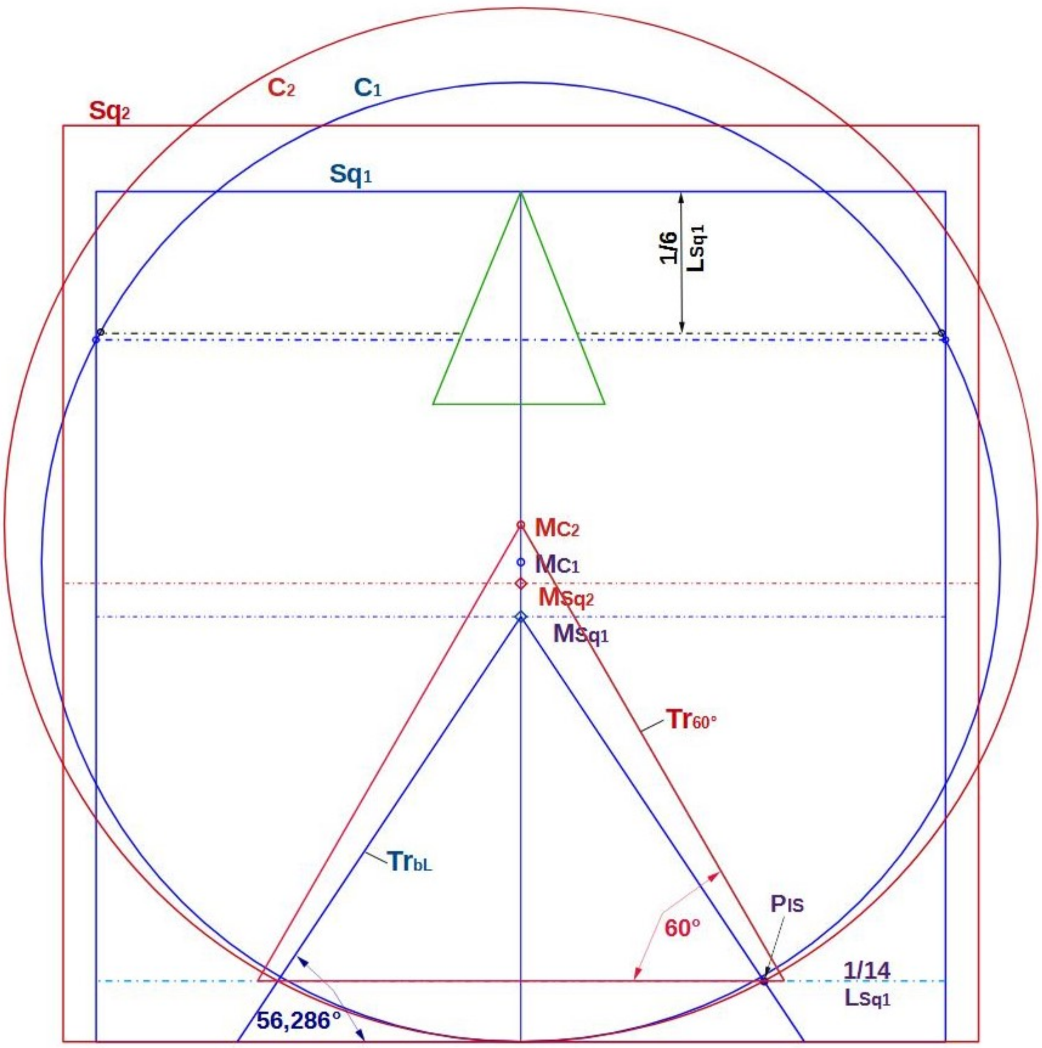


Figure 3: Peaks of the two Triangles at Mid Points M_{Sq1} and M_{C2}

There is another equilateral triangle named Tr_{eq2} (not drawn at Figure 3), which area is close to the area of the triangle Tr_{bL}. The peak of the triangle Tr_{eq2} is located at the mid point M_{Sq2} of the Square

Sq_2 and the lower edges are located at the bottom lines (y=0). The area Ar_{eq2} of the triangle Tr_{eq2} amounts to 16.771, which means a deviation of 0.54% to the area Ar_{bL} of the triangle Tr_{bL}.
The author is convinced, that the triangle Tr_{eq2} is not the one, which is concerned at the quotation of Leonardo da Vinci. The reason: the four listed arguments at page 10, which speak for the equilateral triangle $Tr_{60°}$. Its peak is located at the mid point M_{C2} as given in the qoutation.

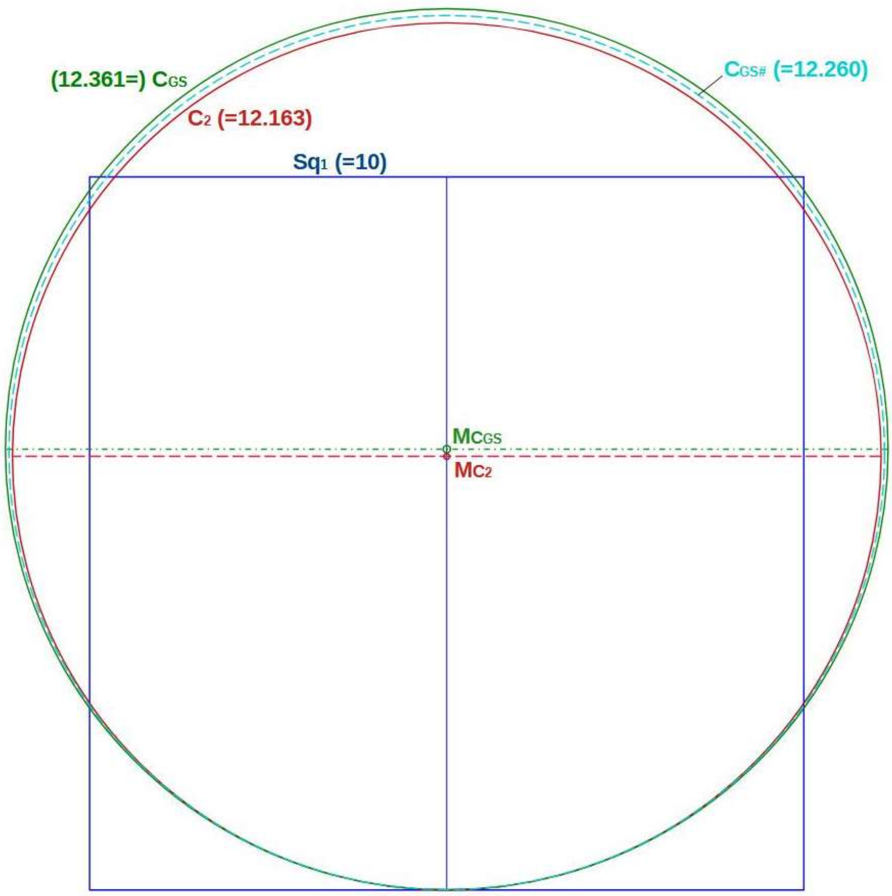
Remarkable: the figure 7 at the foot length (=1/7*L_{Sq1}) and the length $h_{1/14}$ (2*7=14) at the triangles.

<u>Description of the green coloured triangle (see Figure 3):</u>

If one connects the end points of the horizontal line at breast height with the mid of the head top at the drawing *Vitruvian Man* by a straight line on the left side as well on the right side, one gets an isoscele triangle, which peak is located at the mid of the upper line of the Square Sq_1.
The lower side line of this triangle is the horizontal line at breast height.
The isosceles side lines may run closely (how close may depend on the exactness of the used Reproduction Image) to the left and right end point of the drawn horizontal line at neck onset (upper breast end). Further the isosceles side lines nearly touch the eyes of the Vitruvian Man.

<u>Statement referring the Golden Section (GS):</u>

With a diameter D_{GS} (D_{GS} = 2 * 0.618034 = 1.236068 related to a square side length L_{Sq1} = 1 or D_{GS} = 12.36068 related to L_{Sq1} = 10), which depends on the Golden Section, the perfectness and harmony of the Proportion Study is not possible (see Figure 4 or Sketch Extract3 at page 7).



<u>Figure 4:</u> Golden Section Circle C_{GS}

The diameter D_{GS} does not allow an arm movement of the human body as drafted at the Proportion Study of Leonardo da Vinci.

With the constellation of the geometries as given as at the Proportion Study, a Circle C with a diameter D=12.5 (related to a square side length L_{Sq} = 10) overlaps the upper edge points of this square. And the diameter D_{GS} = 12.36068 is pretty narrow to the value D=12.5.

A diameter D_{GS} = 12.36068 delivers an Arm Radius R_{φ_GS} = 10.838. This value is bigger than the side length of the square L_{Sq}=10 (= width of the body) and therefore doesn't make any sense! Please look again at Figure 4 in this report or at the Figure 9 of the online report [4], at which the circle of the Golden Section C_{GS} is additionally drawn to the one of the Proportion Study. The Golden Section Circle C_{GS} gives less feeling of harmony as the Circle C_2 with its diameter D_2.

The deviation of the diameter D_{GS} to the diameter D_2 amounts to 0.1974 (=12.3607 - 12.1633) using a side length of the square L_{Sq}=10. The relative deviation is about 1.62% related to the diameter D_2. If one assumes an error, which might be done by Leonardo da Vinci at his sketching work for the drawing *Vitruvian Man*, of about the half of this deviation, this error results nearly to the diameter $D_{GS\#}$ = 12.26 (see Figure 4 at page 12).
With a diameter $D_{GS\#}$ the accompanying Arm Radius $R_{\varphi\_GS\#}$ amounts to: $R_{\varphi\_GS\#}$ = 6.31

Insight: even with the assumption of this drawing error, which leads to a circle diameter $D_{GS\#}$ (=12.26), the resulting Arm Radius $R_{\varphi\_GS\#}$ (=6.31) is bigger than the half of the square side length L_{Sq} (=5) and therefore the use of this smaller circle diameter $D_{GS\#}$ doesn't make sense, too.

For the calculation of any arm radius please use the formulas, which are given at the end of page 6 at the right side.
In the case of the diameter dependent on the Golden Section one has to consider the following for the determination of the quantities c_{R2L1_GS}, c_{φ_GS}, φ_{GS} and finally R_{φ_GS}: one can use the corresponding radius R_{GS}=0.5*D_{GS} (= 6.18034) for the calculation of the quantity c_{R2L1_GS} in the following way

$$d_{\varphi\_GS} = c_{R2L1\_GS} = [(R_{GS}^2 - (L_{Sq1}-R_{GS})^2]^{0.5} \qquad R_{GS} = 0.5 * D_{GS}$$

.

A comprehensible way to understand, that the circle at the drawing *Vitruvian Man* isn't dependent on the Golden Section, is by sketching as follows:
Please just draw a square with a selected side length L and then draw a circle with the diameter D=1.23608*L (or the reduced diameter D=1.226*L) and compare this constellation with theone of the drawing *Vitruvian Man* (See again the closeness of the upper square edges to the line of circle C_{GS}, which is dependent on the the Golden Ratio, in report [4]). Using a CAD Software, which make just non-visible printing errors, it can be shown, too.

The upper four Intersection Points contribute to the Solution of the Proportion Study?

Assumption: Leonardo da Vinci has integrated all of the four upper intersection points of the two circles with the two squares – given by the dimensions c_{R1L1}, c_{R2L1}, c_{R1L2}, c_{R2L2} (see Sketch Extract2 at page 6) – at his drawing.

- dimension c_{R1L1} (intersection S_{R1L1}) delivers the straight line $g_1(x)$
- dimension c_{R2L2} (intersection S_{R2L2}) delivers the straight line $g_2(x)$ and therefore the radius R_2
- dimension c_{R2L1} (intersection S_{R2L1}) delivers the arm radius R_φ
- dimension c_{R1L2} (intersection S_{R1L2}) delivers the straight lines l_{RL} and l_{LL} and by that the solution for the straddled leg positions on the left side and the right side at the Proportion Study?

Important note of Leonardo da Vinci:

For a better understanding about the ideas of Leonardo da Vinci referring the Vitruvian Man, a quotation of him is given about the connection between arts and mathematics:
"It is the prince of mathematics, his knowledge is irrefutable – it has created the achitecture and the perspective and the divine painting."
[Translation may not be correct. The quotation is taken from the book [3] of Klaus Schröer and Klaus Irle (page 58)].

Used sketching software:

All of the sketches were performed with the LibreOpen-Impress and -Draw Software! The geometries were drawn as accurate as possible.

If someone is eager to reproduce the Figures 1 to 4 with an Reproduction Image, it is preferable to do the following when importing the Image:
Rotate the image by an certain angle clockwise or counterclockwise, until the lower square side line is visibly horizontal.

Then it is preferable to draw a (thin lined) equally four-part grid, whereby the lower line of the grid coincide with the lower square side line of the drawing and the upper line of the grid coincide with the mid point of the upper square side line of the drawing. By using the grid one can exactly see the mid point and height 3/4, by which one is able to compare these points and heights with the elements of the drawing.

In the following one should use the outer lines of this four-part grid for the square Sq_1, which is shown at Sketches Extract1 to Extract3. Now one can draw the geometric constellation described at Sketches Extract1 to Extract3 or Figures 1 to 4, respectively.

If one assumes a crooked lower line of the square of an Reproduction Image caused by an angle 0.5°, the height difference of the two line edges are about 1 mm related to a side length 120 mm. This height difference is clearly visible with the help of the just mentioned thin-lined four-part grid.
If the Reproduction Image is rotated by a certain angle to the state, where the lower line is visibly horizontal, it can be that the geometric constellation of square and circle is more symmetrically adapted or nearly symmetrical to the vertical line, which runs through the mid of the lower square line.

The deviations to the calculated values of the Vitruvian Man-System may vary for the different Reproduction Images dependent on their exactness to the original.
Therefore the author does not take any guarantee, that readers reach the sketching results, which are described in this report, by imitating the sketches or figures of this report.
Furthermore the figures and sketches presented in this report are performed with the LibreOpen-Impress and -Draw Software, by which one is not able to get the drawing exactness as with a CAD-Software.

5) Possible connections between π and Φ using the Vitruvian Man-System:

The Vitruvian Man-System (or Proportion Study-System):

At Sketch 2 at page 20 the solution way of the geometric constellation according to the Proportion Study-System of Leonardo da Vinci is drafted with the goal finding unknown informations.
The two Input Parameter are:

a) the side length L_{Sq2} of the square Sq_2 is set to the unit-less value $\mathbf{L_{Sq2}=\pi}$.
b) the gradient of the straight line $g_2(x)$ is set to the value $\sqrt{\Phi}$: $\mathbf{g_2'(x)=\sqrt{\Phi}}$

The general formula for the diameter of the Vitruvian Man-System is:

$$\mathbf{D_{LdV} = L_{LdV} * [1 - 0.5*F_{LdV}] : [F_{LdV} - 1]^{0.5}} \qquad \mathbf{(4.1)}$$

As already applied for the calculation at chapter 4, the Diameter D_{LdV} corresponds to the Diameter D_2 and the square side length L_{LdV} corresponds to the length L_{Sq1} (see D_2 and L_{Sq1} at Sketches 2 and 3).

The circle C_{2_0}, the square Sq_{2_0} are auxiliary quantities and the straight line function $g_{2\_0}(x)$ is a auxiliary function, which are used for enabling the calculation of the diameter D_{LdV} (or D_2) in dependence of the square side length L_{LdV} by a simple solution way (see Sketch Extract2).

Because of the chosen starting values ($L_{Sq2}=\pi$ and $g_2'(x)=\sqrt{\Phi}$) of the constellation according to Sketch 2 (page 20), the diameter D_2 or D_{LdV}, respectively and the side square length L_{Sq1} and diameter D_1 can be exactly determined by the equalization of the straight line functions $g_1(x)$ and $g_2(x)$ with their intersection at the x-value x_{22} and by the intersection of the smaller circle/square pair at the x-value x_{11}.

The following is valid for all of the straight line functions $g_2(x)$, $g_{2\_0}(x)$ and $g_{2\_1}(x)$: the upper side line of its square intersects with its appropriate circle at the the x-value x_{22} (see Sketch 2).

$g_2(x) = 0.5 * D_2 + D_2 : L_{Sq1} * x$	$g_2(x_{22}) = L_{Sq2}$
$g_{2\_0}(x) = 0.5 * D_{2\_0} + D_{2\_0} : L_{Sq1} * x$	$g_{2\_0}(x_{22}) = L_{Sq2\_0}$
$g_{2\_1}(x) = 0.5 * D_1 + D_1 : L_{Sq1} * x$	$g_{2\_1}(x_{22}) = L_{SQ1}$

The straight line $f_1(x)$ at Sketch 2, which runs through the right lower edge of the square Sq_2 and through the mid point of circle C_1, intersects the y-value $y=\pi$ at the x-value x_f, which is calculated to:

$x_f = \Phi * \pi * \sqrt{\Phi} : 4 = \Phi * \pi : Pi^{R\Phi} = 1.61648$ gained by equation $f_1(x) = 0.5 * D_1 + (D_1 : L_{Sq2}) * x$

In this context it may be mentioned again, that the result $\Phi+3$ is yielded using the Circle Figure approximation $Pi^{R\Phi}$. See Equation (2.3) at page 3.
Question: if there are some unknown informations for the Circle Figure π dependent on the Golden Section Φ, is it possible to find them by Sketch 2?

If one use the straight line function $g_3(x)$, one can derive a connection to the already mentioned Circle Figure approximation "$1.2 * \Phi^2$". Straight line $g_3(x)$ is orthogonal to $g_2(x)$, runs through the point (x_{22};0) and intersects the straight line function $g_2(x)$ at the y-value "$\pi : \Phi^2$" (see Sketch 2).

Further possible investigations:

One can overlap the constellation of Sketch 2, at which three squares and three circles are connected with each other, with a similar constellation, at which for example the factor F_{LdV} is seized with the value $F_{LdV}=\sqrt{(4:\pi)}$ and the side length L_{Sq2} of the square Sq_2 is seized with the value $L_{Sq2}=\pi$.

Maybe one can find unknown common grounds between the two constellations by this comparison?

Second constellation (marked with the double cross #):

In the following a second constellation is presented and is overlapped to the just presented constellation of Sketch 2 ($L_{Sq2} = \pi$ and the gradient of the straight line $g_2(x)$ is: $g_2'(x)=\sqrt{\Phi}$) with the intention to find common grounds.
At the second constellation the square side length $L\#_{Sq2}$ is set to the value "$4:\sqrt{\Phi}$" and the x-value $x\#_{22}$ of the upper intersection of square $Sq\#_2$ and circle $C\#_2$ is set to $x\#_{22} = x_{22}$.
The value $4:\sqrt{\Phi}$ (=3.14461) is close to the value of the Circle Figure π (=3.14159), so one is not able to distinguish the lines of the two constellations at Sketch 3.

Starting values of first constellation:

1) $L_{Sq2} = \pi$ and
2) gradient of $g_2(x)$ is $\sqrt{\Phi}$

$D_1 = 4 * \pi * (\Phi^{1.5} - \Phi) * (\Phi^2 - \Phi^{1.5})$;
$L_{Sq1} = 2 * \pi * (\Phi^{1.5} - \Phi)$;

$D_2 = 2 * \pi * (\Phi^2 - \Phi^{1.5})$
$L_{Sq2} = \pi$

$g_1(x) = x : (\Phi - \sqrt{\Phi})$
$g_2(x) = 0.5*D_2 + \sqrt{\Phi}*x$
$\varphi = \arctan(\sqrt{\Phi}) = 51.8273°$
$x_{11} = 2 * \pi * (\Phi - \sqrt{\Phi}) * (\Phi^{1.5} - \Phi)$
$x_{22} = \pi * (\Phi - \sqrt{\Phi})$

$D_{2\_0} = L_{Sq1}$
$L_{Sq2\_0} = D_{2\_0} : F_{LdV}$
$F_{LdV} = D_1 : L_{Sq1} = D_2 : L_{Sq2} = 2 * (\Phi^2 - \Phi^{1.5})$

$D_{LdV} = D_2$
$L_{LdV} = L_{Sq1}$

Starting values of second constell. marked with #:

1) $L\#_{Sq2} = 4:\sqrt{\Phi}$ and
2) $x\#_{22} = x_{22} = \pi * (\Phi - \sqrt{\Phi})$

$D\#_2 = (x_{22}^2 + L\#_{Sq2}^2) : L\#_{Sq2}$
$F\#_{LdV} = D\#_2 : L\#_{Sq2} = (x_{22}^2 + L\#_{Sq2}^2) : L\#_{Sq2}^2$

$L\#_{Sq1} = D\#_2 * (F\#_{LdV} - 1)^{0.5} : (1 - 0.5 * F\#_{LdV})$
$D\#_1 = F\#_{LdV} * L\#_{Sq1}$

$x\#_{11} = [(0.5*D\#_1)^2 - (L\#_{Sq1} - 0.5*D\#_1)^2]^{0.5}$
$g\#_1(x) = L\#_{Sq1} * x : x\#_{11}$
$g\#_2(x) = 0.5*D\#_2 + D\#_2 : L\#_{Sq1} * x$
$\varphi\# = \arctan(D\#_2 : L\#_{Sq1}) = 51.8612°$

In the next step the intersection at coordinate (x_S,y_S) of the both straight line functions $g_2(x)$ and $g\#_2(x)$ is presented. The mathematical derivation is:

$g_2(x) = g\#_2(x)$ or $0.5 * D_2 + D_2 : L_{Sq1} * x = 0.5 * D\#_2 + D\#_2 : L\#_{Sq1} * x$

$x_S = 0.5 * (D_2 - D\#_2) : (D\#_2 : L\#_{Sq1} - D_2 : L_{Sq1}) = -0.85477386$
$y_S = g_2(x_S) = g\#_2(x_S) = 0.67157222$

In the following the absolute value is used for x_S, which is represented by the intersection IS on the right side of Sketch 3:

$x_S = +0.85477386$

The distance dS from the coordinate system at x=0,y=0 to the intersection IS is:

$d_S = (x_S^2 + y_S^2)^{0.5} = 1.08703615$

This value is pretty near to the x-value x_{22} (= 1.08703611). The deviation is relatively small:

$d_S - x_{22} = 4.765E-08$

The value x_S of the intersection IS divided by the Circle Figure π is about the term "$\sqrt{\Phi}$-1":

$x_S : \pi = 0.272083 \approx \sqrt{\Phi} - 1 = 1.272020 - 1 = 0.272020$

Four straight line functions are additionally drafted at Sketch 3 with the intention to find common grounds between the quantities L_{Sq2} and $L\#_{Sq2}$:

1) $k(x) = y_S : x_S * x$
   $k(x_S) = y_S$
   $k(x_{22}) = y_S : x_S * x_{22} = 0.85405425$ - corresponds to Intersection IS_{x22}
   $k(x_{ges}) = \sqrt{\Phi}$ ---> $x_{ges} = \sqrt{\Phi} * x_S : y_S = 1.619020$
   Circle with diameter D_x at point (x_{ges}; $\sqrt{\Phi}$) is: $D_x = (x_{ges}^2 + \sqrt{\Phi}^2) : \sqrt{\Phi} = 3.332701$ [≈ 10:3]
   Circle is not drawn at Sketch 3, but is similar generated to other circles shown at Sketch 3.

2) $k_1(x) = y_S : (x_{22} - x_S) * (x_{22} - x)$
   $k_1(0) = 0.5 * (L_{Sq2} + L\#_{Sq2})$ - corresponds to the Intersection IS_1
   $k_1(x_S) = y_S$ - corresponds to the Intersection IS
   $k_1(x_{22}) = 0$

3) straight line function $k_2(x)$ is orthogonal to straight line function $k(x)$ and runs through Intersection IS:
   $k_2(x) = y_S + x_S^2 : y_S - x_S : y_S * x$
   $k_2(0) = y_S + x_S^2 : y_S = 1.7595242$ - corresponds to the Intersection IS_2
   $k_2(x_S) = y_S$ - corresponds to the Intersection IS

   the value $k_2(0) = 1.7595242$ is near to the value of Radius R_2 (=0.5*D_2=1.758861) or to the one of $R\#_2$ (=0.5*$D\#_2$=1.760188)

4) straight line function $k_3(x)$ is orthogonal to straight line function $k(x)$ and runs through Intersection IS_{x22}:
   $k_3(x) = x_{22} * (y_S : x_S + x_S : y_S) - x_S : y_S * x$
   $k_3(0) = x_{22} * (y_S : x_S + x_S : y_S) = 2.2376285$ - corresponds to the Intersection IS_3
   $k_3(x_{22}) = y_S : x_S * x_{22} = 0.85405425$ - corresponds to the Intersection IS_{x22}

   The value $k_3(0)$ (=2.2376285) is close to the radius R_C (=0.5*D_C=0.5*F_{LdV}*L_{Sq}=2.2373062), which corresponds to the square side line $L_{Sq} = \pi * \sqrt{\Phi} = 3.99617$.

Question:

Is it necessary to find another geometric constellation, which is more appropriate to gain important informations?

6) Conclusion:

Are the formulas of Chapter 2 (eventually already known? The author did not see anything before in Literature or Internet) not simple, but interesting? The author hopes, that the topics of this report support also the affection of young people to the mathematics.

The Quadrature of Circle describes the two-dimensional case. If one undertakes researches for the Circle Figure π, one should include also the three-dimensional case. This inclusion does not make the examination more easy.
How could this three-dimensional case be named: the Quaderur of the Sphere? [Quadrat (means square) and Quader (means cuboid) are german expressions]

Hidden in the drawing *Vitruvian Man* of Leonardo da Vinci there are two pairs of square/circle, which each are equal in area. Only the smaller square Sq_1 and the bigger circle C_2 are drawn at the drawing, otherwise the drawing would be overloaded with geometry.
The german authors Klaus Schröer and Klaus Irle were the first one, who determined the Circle Diameter D_2 of Circle C_2 at the Proportion Study by application of an iteration method. The value of Diameter D_2 could be confirmed by the author with help of the simple method of equalization of two straight lines.

At the symmetric geometric constellation of two pairs of circle/square, whereby the circles lie tangentially at the lower lines of the squares, and under consideration of the described geometric conditions at Chapter 4 the quantity x_{22} is an important output and solving quantity of the Vitruvian Man-System.

By the comparison of two geometric constellations, which underlie the Vitruvian Man-System and which are connected with each other by the relation $x\#_{22} = x_{22}$, an interesting relation between two figures (which may be arbitrary chosen starting values) can be observed.
The author hopes, that Profi-Mathematician undertake more thoroughly investigations referring this quantity x_{22} and the geometric system behind the Vitruvian Man.

Only a genius person as the artist and mathematician Leonardo da Vinci was able to perform the art work *Vitruvian Man*, which gives an extraordinary harmony to the observer and which contains also (hidden) mathematical elements.

7) Literature:

[1] www.pimath.de/pi.html

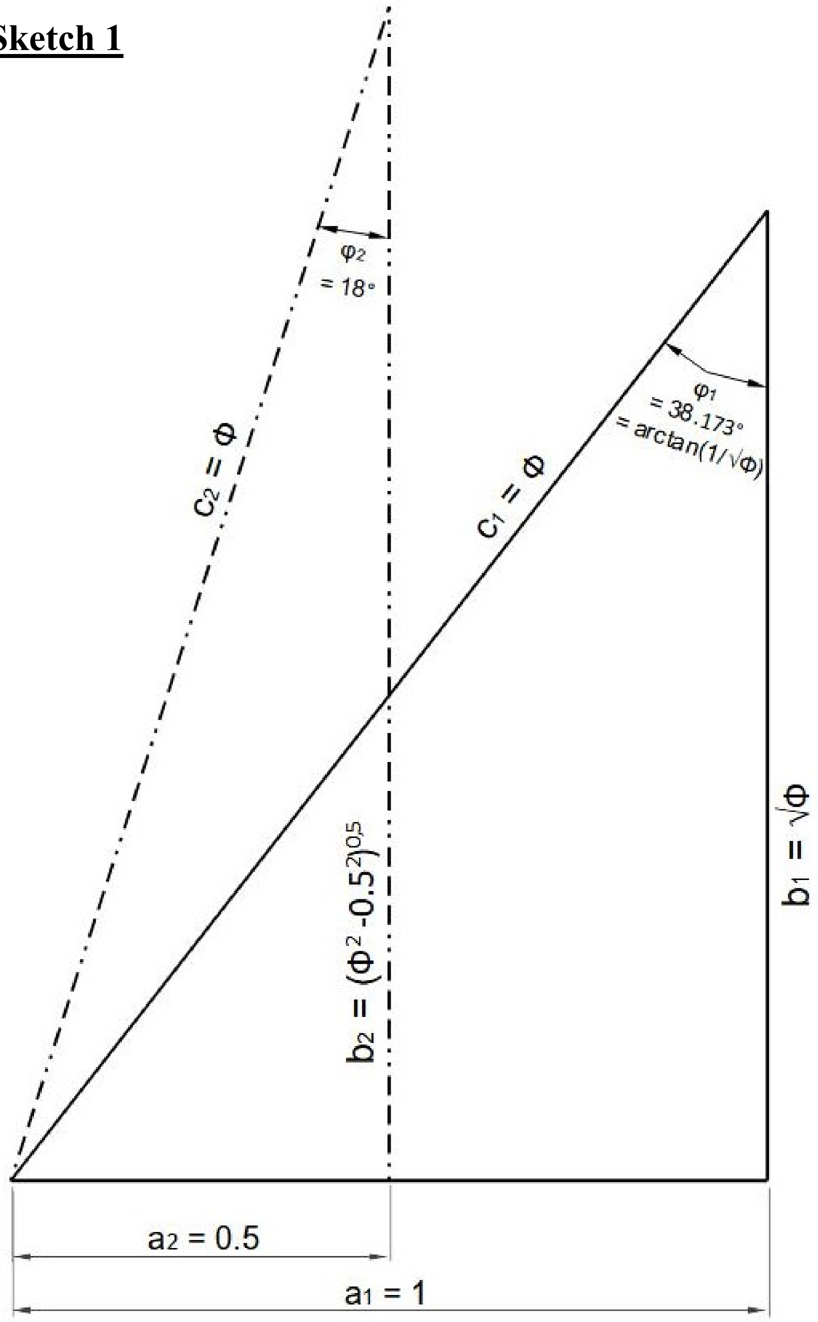
[2] Images of the Vitruvian Man in comparison with the additional drawn Square and Circle at the web page *www.klaus-schroeer.com* of Klaus Schröer

[3] “Ich aber quadriere den Kreis“ of Klaus Schröer and Klaus Irle (Year 2017)
[“But me I square the circle“: translation of the title by the author may not be correct]

Online-Report [4]: “Leonardo’s Vitruvian Man Drawing“ of Vitor Murtinho (Year 2015)

Just for information please see some well-known special features of the Quotient Φ of the Golden Section illustrated by Sketch 1:

Sketch 1



Following Equation **4** is only valid by use of the Quotient Φ of the Golden Section referring to a right-angled triangle with side lengthes a_1 (=1), b_1 (=√Φ) and c_1 (=Φ):

$$(c_1^1)^2 = (c_1^0)^2 + (c_1^{0.5})^2 \qquad \textbf{(4)}$$

or

$$c_1^2 = a_1^2 + b_1^2$$

With side lengthes a_2=0.5 and c_2 (=Φ) one yields the angle φ_2:

φ_2 = 18°

One takes for granted, that the angle is a full number. But is more behind that?
Following simple relation contains the figure 5:

(90° - 18°) ∗ 5 = 360°

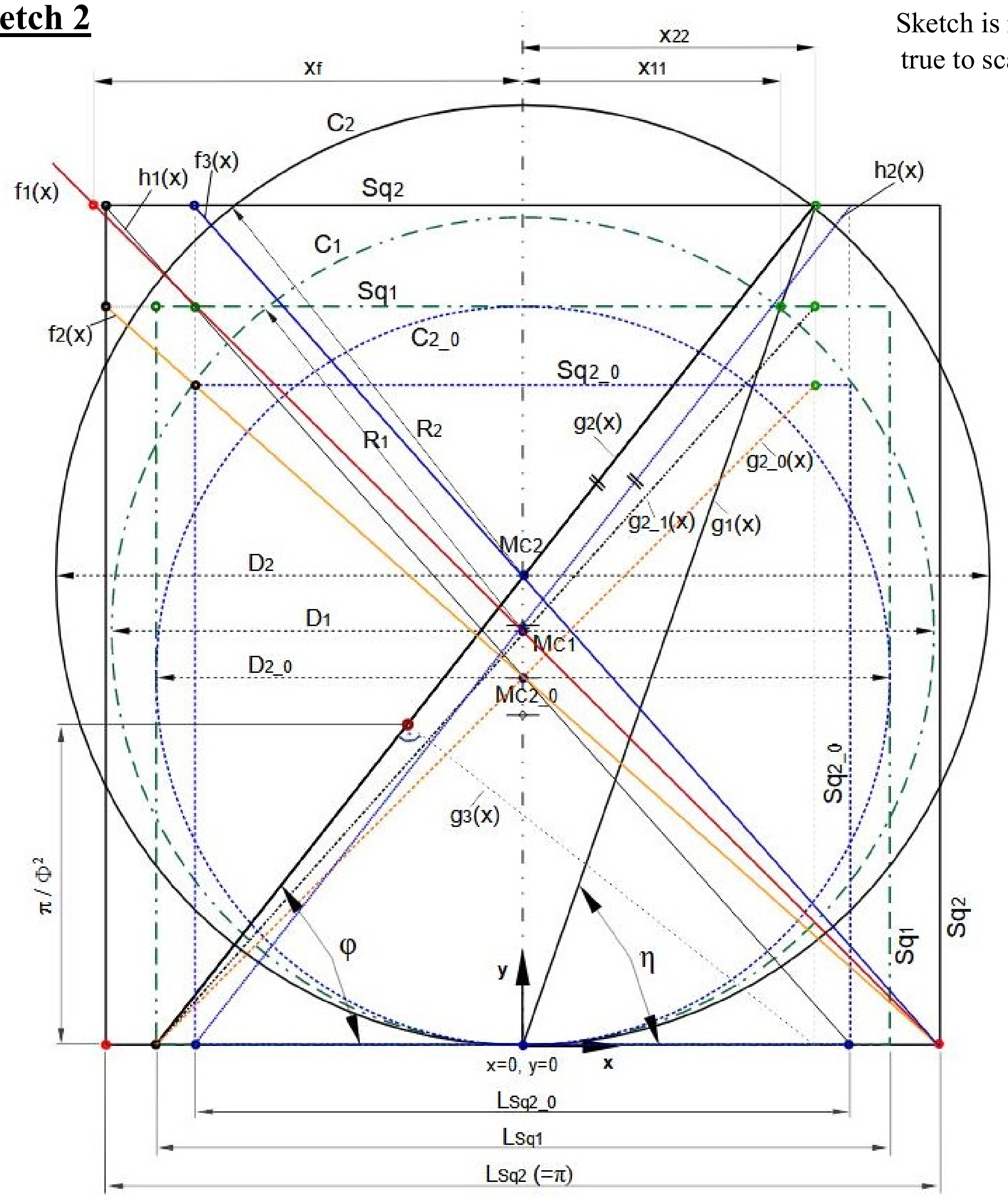
Another style for Φ with the figure 5:

Φ = (5 + 5∗√5) : 10

18° corresponds to: π : 10 rad

Sketch 2

Sketch is not true to scale



Start values: 1) $L_{Sq2} = \pi$ and

2) gradient of $g_2(x)$ is $\sqrt{\Phi}$

$D_1 = 4 * \pi * (\Phi^{1.5} - \Phi) * (\Phi^2 - \Phi^{1.5})$;
$L_{Sq1} = 2 * \pi * (\Phi^{1.5} - \Phi)$;

$D_2 = 2 * \pi * (\Phi^2 - \Phi^{1.5})$
$L_{Sq2} = \pi$

$g_1(x) = x : (\Phi - \sqrt{\Phi})$
$g_2(x) = 0.5 * D_2 + \sqrt{\Phi} * x$
$\varphi = \arctan(\sqrt{\Phi}) = 51.8273°$
$x_{11} = 2 * \pi * (\Phi - \sqrt{\Phi}) * (\Phi^{1.5} - \Phi)$
$x_{22} = \pi * (\Phi - \sqrt{\Phi})$

$D_{2\_0} = L_{Sq1}$
$L_{Sq2\_0} = D_{2\_0} : F_{LdV}$
$F_{LdV} = D_1 : L_{Sq1} = D_2 : L_{Sq2} = 2 * (\Phi^2 - \Phi^{1.5})$

$D_{LdV} = D_2$
$L_{LdV} = L_{Sq1}$

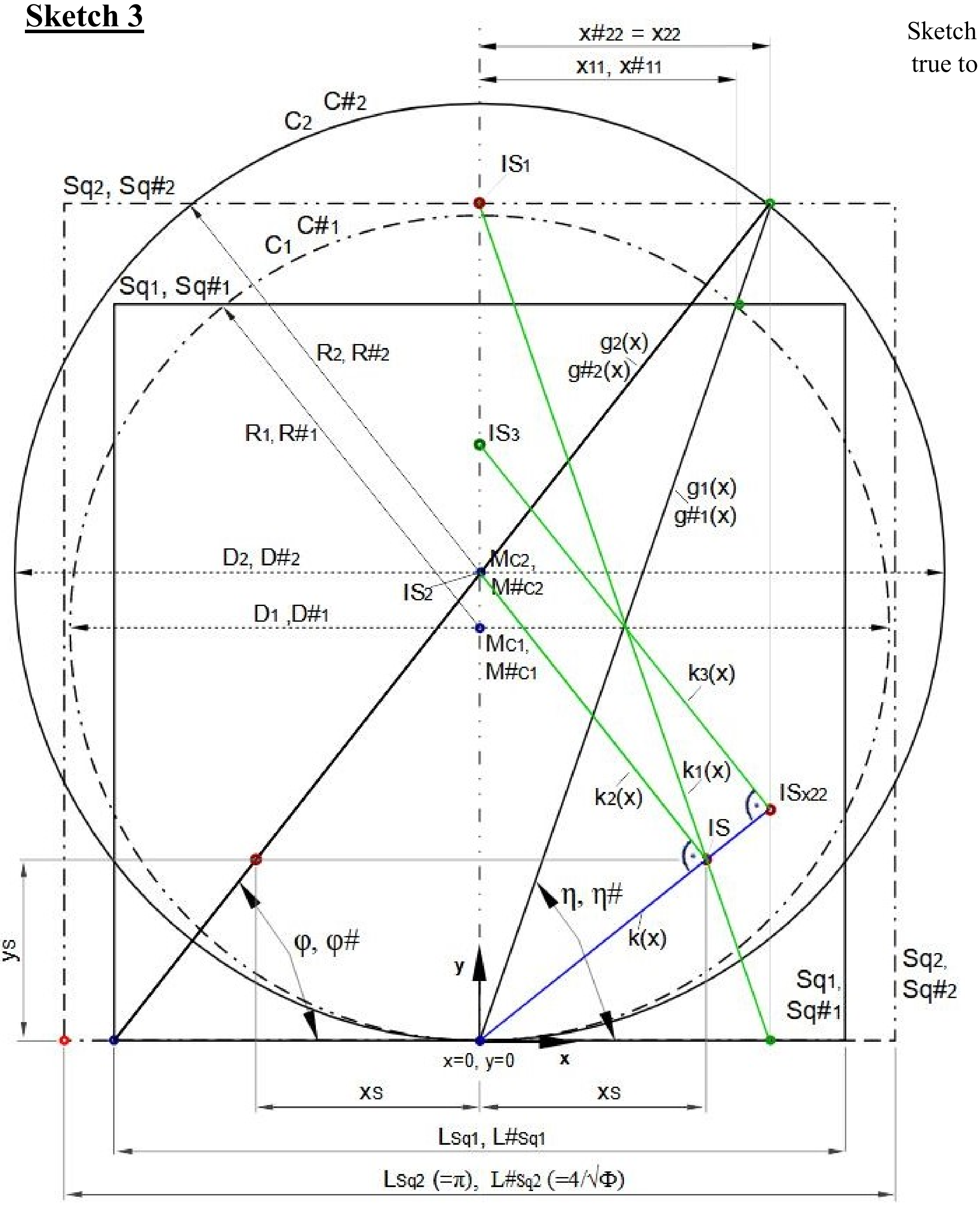
The following results **(R)** are only possible by use of the gradient $\sqrt{\Phi}$ of the straight line function $g_2(x)$

$L_{Sq2} = 0.5*(D_2 + L_{Sq1})$ **(R)**

$f_1(x) = 0.5*D_1 - (D_1 : L_{Sq2})*x$;
$f_1(x_f = -\Phi*\pi*\sqrt{\Phi}:4) = L_{Sq2}$ **(R)**
$f_1(-0.5*L_{Sq2\_0}) = L_{Sq1}$ **(R)**
$f_2(x) = 0.5*D_{2\_0} - (D_{2\_0} : L_{Sq2})*x$;
$f_2(-0.5*L_{Sq2}) = L_{Sq1}$
$f_3(x) = 0.5*D_2 - (D_2 : L_{Sq2})*x$;
$f_3(-0.5*L_{Sq2\_0}) = L_{Sq2}$

$h_1(x) = 0.5*D_{2\_0} - (D_{2\_0} : L_{Sq2\_0})*x$;
$h_1(-0.5*L_{Sq2}) = L_{Sq2}$ **(R)**
$h_2(x) = 0.5*L_{Sq2} + (L_{Sq2} : L_{Sq2\_0})*x$;
$h_2(x)$ is parallel to $g_2(x)$

Sketch 3



The value 4:√Φ (=3.14461) is close to the value of the Circle Figure π (=3.14159), so one is not able to distinguish the lines of the two constellations at Sketch 3.

Worth for repeating:

$(xs^2 + ys^2)^{0.5} = 1.08703615 \approx x_{22} = 1.08703611$

$xs : L_{Sq2} = xs : \pi = 0.272083 \approx \sqrt{\Phi} - 1 = 1.272020 - 1 = 0.272020$

Please see the quantities and formulas given of this Sketch at pages 16 and 17.

Abbreviations of important Quantities:

C_{GS}: Circle C_{GS} is derived by the Quotient of the Golden Section and is related to the side length of the Square Sq_1

C_{VM}: Circle C_{VM} is the circle at the drawing *Vitruvian Man*

C_1: Circle C_1 is equal in area to the Square Sq_1

C_2: Calculated Circle C_2 is the comparative circle to the Circle C_{VM} at the drawing *Vitruvian Man*; Circle C_2 is equal in area to the Square Sq_2

C_{2_0}: Auxiliary Circle C_{2_0} is equal in area to the Square Sq_{2_0}

D_{GS}: Diameter D_{GS} is the diameter of Circle C_{GS}

D_{VM}: Diameter D_{VM} is the diameter of Circle C_{VM}

D_1: Diameter D_1 is the diameter of Circle C_1

D_2: Diameter D_2 is the diameter of Circle C_2

D_{2_0}: Diameter D_{2_0} is the diameter of Circle C_{2_0}; Diameter D_{2_0} is used as auxiliary quantity for the calculation of Diameter D_2

L_{SqVM}: Square Side Length L_{SqVM} is the side length of Square Sq_{VM}

L_{Sq1}: Square Side Length L_{Sq1} is the side length of Square Sq_1

L_{Sq2}: Square Side Length L_{Sq2} is the side length of Square Sq_2

L_{Sq2_0}: Auxiliary Square Side Length $L_{Sq2\text{-}0}$ is the side length of Square Sq_{2_0}

M_{C1}: Mid point of Circle C_1

M_{C2}: Mid point of Circle C_2

M_{Sq1}: Mid point of Square Sq_1

M_{Sq2}: Mid point of Square Sq_2

R_φ: Arm Radius R_φ corresponds to the calculated Diameter D_2 of Circle C_2

$R_{\varphi x}$: Arm Radius $R_{\varphi x}$ is the value of the arm radius measured at the drawing *Vitruvian Man.* Arm Radius $R_{\varphi x}$ according to [3] takes the value 43.6% of the *Vitruvian Man* width (=L_{SqVM})

Sq_{VM}: Square Sq_{VM} is the square at the drawing *Vitruvian Man*

Sq_1: Square Sq_1 is the comparative square to the Square Sq_{VM} at the drawing *Vitruvian Man*; Square Sq_1 is equal in area to the Circle C_1

Sq_2: Square Sq_2 is equal in area to the calculated Circle C_2

Sq_{2_0}: Square Sq_{2_0} is equal in area to the Circle C_{2_0}; Square Sq_{2_0} is used as an auxiliary quantity for the calculation of Diameter D_2

x_{11}: x-value of the Intersection of upper side line of Square Sq_1 and Circle C_1

x_{22}: x-value of the Intersection of upper side line of Square Sq_2 and Circle C_2