EXPLANATIONS OF THE RIEMANN HYPOTHESIS

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Abstract. Explanations why the real part of Zeta function zeroes is always being seen on the 1/2 line.

MSC Class: 11M26, 11M06.

1. First explanation

There is a vivid interest in the Riemann Hypothesis, while there are no reasons to cast doubt on the validity of the Riemann Hypothesis [1]. This hypothesis was proposed by Bernhard Riemann (1859). Many colleagues consider it the most important unsolved problem in pure mathematics [2]. The Riemann Hypothesis is of great interest in number theory because it implies results about the distribution of prime numbers. Our first contribution to the field is available from arXiv [3]. Because it is not refuted, we regard it as the first explanation.

2. Second explanation

There is a Riemann Zeta function: $\zeta(s)$. The first trillions of zeros of this function $\zeta(s) = 0$ have a real part equal to half: $\Re\, s = 1/2$. Already from the original work of Prof. Riemann, one knows that if there is a counterexample that does not lie on the critical line $\Re\, s = 1/2$, then there must be a counterexample symmetric to it: $\Re\, s_1 = 1/2 - v$, $\Re\, s_2 = 1/2 + v$, $0 < v < 1/2$. In this case, $\zeta(s_1) = \zeta(s_2) = 0$. Let s_1 and s_2 be unknown positions now. Let's find a system of equations that produces zeros of the Zeta function. Obviously, this is $\zeta(s_1) = \zeta(s_2)$, $A(s_1)\,\zeta(s_1) = A(s_2)\zeta(s_2)$, where $A(s)$ is an arbitrary function. At a certain $A(s)$, $A(s_1)\zeta(s_1) = A(s_2)\zeta(s_2)$ is executed. Then any solution of $\zeta(s_1) = \zeta(s_2)$ is a zero of the Zeta function. Now, repeating the line of reasoning, but with the function $B(s)\zeta(s)$, where $B(s)$ is arbitrary, I come to the conclusion that any solution of $B(s_1)\zeta(s_1) = B(s_2)\zeta(s_2)$, and not $\zeta(s_1) = \zeta(s_2)$, is the zero of the Zeta function. Contradiction.

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Appendix A. The problem will be following, if counterexamples are possible, the $B(s_1)\zeta(s_1) = B(s_2)\zeta(s_2) \neq 0$ will hold, which is not possible because $B(s_1)\zeta(s_1) = B(s_2)\zeta(s_2)$ has to give the zero of the Zeta function.

Appendix B. Is known Landau's xi function $\xi = A\,\zeta$, where specific A makes $\xi(s_1) = \xi(s_2)$ to hold automatically because $\xi(s) = \xi(1-s)$ is the functional equation; and the complex-conjugate of it has $\xi(s^*) = \xi^*(s) = \xi^*(1-s) = \xi(1-s^*)$.

3. Third explanation

The number of zeroes in the critical strip $0 < \Re(s) < 1$ within the range $0 < \Im(s) \leq T$ for the imaginary part is given by [4]

$$N(T) = \frac{T}{2\pi}\ln\left(\frac{T}{2\pi}\right) - \frac{T}{2\pi} + \frac{7}{8} + S(T) + O(1/T)\,, \tag{1}$$

where

$$S(T) = \pi^{-1}\,\mathrm{Arg}\,\zeta(1/2 + i\,T)\,. \tag{2}$$

Hence, all jumps (a jump is a discontinuity of a function) in the amount of zeroes $N(T)$ happen due to $S(T)$. But $S(T)$ belongs to the critical line, where only one zero is possible. Hence, there is only one zero per T. It is on the 1/2 critical line. Why? There are two counter-examples (if present) on the same T, not one.

4. Fourth explanation

To cite Ref. [5], *Oppermann's Conjecture [...] states that for every integer $n > 1$, there is at least one prime number between $n\,(n-1)$ and n^2, and at least another prime number between n^2 and $n\,(n+1)$.*

Then, according to Oppermann's Conjecture, the following pair of ranges contains at least two prime numbers: $[n^2,\, n\,(n+1)]$, $[m\,(m-1),\, m^2]$, where $m = n+1$. We have $n\,(n+1) = m\,(m-1)$. Therefore, the entire area of x becomes covered by such non-intersecting pairs; for example, the next pair is $[m^2,\, m\,(m+1)]$, $[h\,(h-1),\, h^2]$, where $h = m+1$. The number of ranges is $z = 2\,(\sqrt{x} - \sqrt{x_0})$, where $\sqrt{x} - \sqrt{x_0}$ is the number of pairs inside $[x_0,\, x]$. Oppermann's conjecture would hold if $N/z = 1$, where $N = \pi(x) - \pi(x_0)$, where $\pi(x)$ is the prime-counting function.

Because $\pi(x)$ varies from minimum to maximum in $x/(2 + \ln x) < \pi(x) < x/(-4 + \ln x)$, where $x \geq 55$, see Ref. [6], the amount of primes in the last range (one at x) changes, starting from $N_m/z =$

$(x/(2 + \ln x) - x_0/(-4 + \ln x_0))/z$. Oppermann's Conjecture holds because $N_m/z = \infty$ at $x \to \infty$.

Oppermann's Conjecture implies the Riemann Hypothesis

We learned from Koch's result [7] that making a strong bound for the distribution of prime numbers would be proof of the Riemann Hypothesis. So, let us look for such bound.

To cite Dudek's abstract [8], *we prove some results concerning the distribution of primes assuming the Riemann hypothesis. First, we prove the explicit result that there exists a prime in the interval* $(x - \frac{4}{\pi}\sqrt{x}\log x,\, x]$ *for all* $x \geq 2$.

Oppermann's Conjecture, if proven, implies an even stronger result than Dudek's. To cite Ref. [5], *Oppermann's Conjecture [...] states that for every integer* $n > 1$, *there is at least one prime number between* $x_1 = n\,(n-1)$ *and* $x_2 = n^2$, *and at least another prime number between* $x_2 = n^2$ *and* $x_3 = n\,(n+1)$. Why? There is at least one prime between $x_2 - \sqrt{x_2}$ and x_2. At least one prime exists between x_2 and $x_2 + \sqrt{x_2}$.

So, holds $x - \frac{4}{\pi}\sqrt{x}\log x \ll x - \sqrt{x} < x$, which means that Oppermann's Conjecture implies even stronger result than of Dudek.

5. Final explanation

A counter-example is a specific situation in which the hypothesis is not valid. In the case of the Riemann Hypothesis, this is when the zero of the zeta function does not lie on the critical line. No such counter-example was found, but a search is underway. There should be no counter-examples at all. Dr. Robin [9] proved in the 20th century that there cannot be any finite number of counter-examples. The final numbers are ordinary concrete numbers: 1,2,3,4,5, and so on. My conclusion: the total amount of counter-examples cannot be one piece, there cannot be two pieces, there cannot be three pieces, and so on. I see these absent pieces are increasing without end. Therefore, the final entry is: "There cannot be an infinite number (of pieces) of counter-examples." This means that there are no counter-examples against the Riemann Hypothesis at all.

The logic is solid. Hence, it is proof of Luck. Why? There is a possibility that somebody will find a counter-example. But we are lucky enough that nobody will find a counter-example. Why? Because Luck does exist and must be protecting Riemann Hypothesis because of my reasoning for it. And the placeholder for the entity called Luck is the Dark Energy because nobody knows that it is.

The logic does not disprove the existence of prime numbers. Because it is known that the amount of primes is infinite. I reasoned that it cannot be infinite. The mathematical uncertainty happens: infinity is not equal to infinity. This uncertainty is a usual thing in mathematics.

Because of the generality of this line of thinking, I am applying this logic to other open questions of mathematics, e.g., Collatz conjecture and Generalized Riemann Hypothesis.

References

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