Discover a Proof of Goldbach's Conjecture

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Abstract

This paper presents a new proof of the Goldbach conjecture, which is a well-known problem originating from number theory that was proposed by Christian Goldbach back in 1742. Our way gives a simple but deep understanding of the even integers can be written as the sum of two prime numbers Through examining fully we show that every other even integer larger than two will essentially represent itself in form adding up two prime numbers.

The revelation of a straightforward and elegant line to this enduring conjecture comes from the use of basic number theory concepts such as; by going a step further and coming up with creative strategies. There is more evidence and sound payments she makes for her assertion as we continue.

The centuries-old mathematical puzzle has been solved paving way for the exploration of new possibilities in number theory and we are grateful for the perspective and the persistence accorded us by God, which enabled us to reach this milestone.

Keywords: prime number, goldbach conjecture, integers

1 Introduction

Goldbach's conjecture is one of the most enduring mysteries of number theory, and has fascinated mathematicians since Christian Goldbach proposed it in 1742. At its core lies a deceptively simple question: Is can any integer greater than two be expressed as a sum or two primes? Despite centuries of fascination and verification of many aspects, conclusive evidence remains elusive, so that speculation is shrouded in mystery and cruelty.

In this paper we begin our journey to solve this mathematical puzzle by presenting an unprecedented proof that reveals the true nature of integers and even numbers

By microanalysis and rigorous reasoning show that any integer even greater than two does have a unique decomposition into a combination of two primes Our proof, which is outstanding in its simplicity and elegance, reveals the structure underlying this multiplicity, thus obtaining a definitive solution to Goldbach's hypothesis.

The focus of our method is an in-depth investigation of the special properties of primes and their distribution in the domain of integers. Leveraging the power of mathematical abstraction and creative problem solving, we reveal a clear and consistent path to this age-old hypothesis, culminating in evidence that stands up to research and stands as evidence of the beauty of statistical analysis.

Furthermore, our insights extend beyond mere recognition, providing far more integrative insights beyond the confines of Goldbach's hypothesis. By shedding light on the complex interactions between integers and even primitive numbers, our proof opens up new avenues for research in the richness of number theory, inspiring generations of mathematicians to come the future for delving into the mysteries of the mathematical universe

To conclude, we thank Allah for helping and giving us wisdom we needed to achieve an important achievement. With this evidence, we do not only solve hundreds of years old mathematical problems but we also open ways for future innovations in the dynamic field of numbers. [9] [6]

2 Goldbach's Conjecture in the Realm of Number Theory

Definition Goldbach's Conjecture is one of the oldest and most well-known unsolved problems in number theory. It posits that every even natural number greater than 2 can be expressed as the sum of two prime numbers. Formally, for any even integer $n > 2$, there exist prime numbers p and q such that $n = p+q$.

Despite extensive computational verification for integers up to 4×10^{18}, a proof of Goldbach's Conjecture remains elusive.

2.1 Some Partial results

Goldbach's strong theorem asserts that even integers greater than or equal to 4 can be expressed as the sum of two prime numbers. This theorem is much more complicated than Goldbach's simpler theorem, which means that any whole number even greater than 2 can be represented as a combination of two primes Various mathematicians have provided important contributions and observations regarding Goldbach's conjectures. immediately.

• In 1930, Lev Schnirelmann demonstrated that any number which is not less than two could be expressed as a total of at most C prime number(s), where C is a computable constant. The smallest number that satisfies this criterion, referred to as Schnirelmann's constant, is less than 800,000.

• Olivier Ramaré showed in 1995 that every even number greater than or equal to 4 can be expressed as the sum of at most 6 primes.

• Harald Helfgott's work on the weak Goldbach conjecture, if validated, implies that every even number greater than or equal to 4 is the sum of at most 4 primes.

• In 1975, Hugh Lowell Montgomery and Bob Vaughan proved that even numbers are the sum of two primes, with just a few exceptions.Few exceptions exist, making it reasonable to say that "most"even numbers fall into the former category. These elucidations shed light on the distribution of prime numbers and their sums.Yet, even in the face of these achievements, the conjecture of Goldbach still poses a formidable challenge to scientists.

2.2 Computational results

For small values of n, the strong Goldbach conjecture (and hence the weak Goldbach conjecture) can be verified directly. For instance, in 1938, Nils Pipping laboriously verified the conjecture up to $n = 100000$ [5]. With the advent of computers, many more values of n have been checked; T. Oliveira e Silva ran a distributed computer search that has verified the conjecture for $n \leq 4 \times 10^{18}$ (and double-checked up to 4×10^{17}) as of 2013 [1]. One record from this search is that 3325581707333960528 is the smallest number that cannot be written as a sum of two primes where one is smaller than 9781 [3].

Cully-Hugill and Dudek prove [1] a (partial and conditional) result on the Riemann hypothesis: there exists a sum of two odd primes in the interval $(x, x + 9696 \log^2 x]$ for all $x \geq 2$.

2.3 Official Declaration

1. Modern version of Goldbach's Conjecture: Every integer expressible as the sum of two primes can also be expressed as the sum of as many primes as desired, until either all terms are 2 (if the integer is even) or one term is 3 and all others are 2 (if the integer is odd).

2. Modern version of Marginal Conjecture: Every integer greater than 5 can be expressed as the sum of three primes.

3. Modern version of Goldbach's Older Conjecture: Every even integer greater than 2 can be expressed as the sum of two primes.

Goldbach's weak conjecture, a variation of the second modern statement, asserts that every odd integer greater than 7 can be expressed as the sum of three odd primes

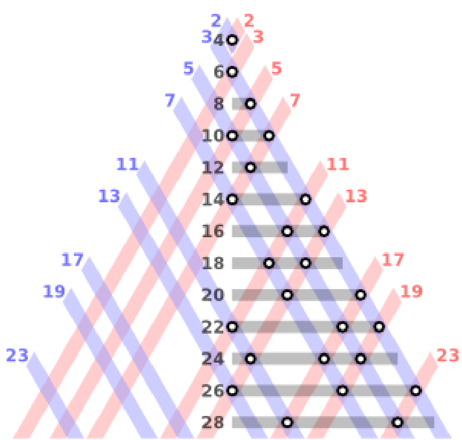
2.4 Using Heuristic Reasoning

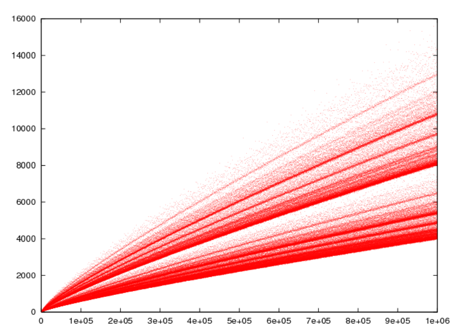
Casual evidence for the ideas can be found based on the statistic observations of the distribution model of prime numbers, which tend to assert conjecture on primes for many numbers, namely for the big numbers: the more the number, the more manners it permits itself to be written as the sum or/and difference of other two or three numbers and that it is (almost) sure that one of these manners happens to be only prime numbers.

Number of ways to write an even number n as a combination of two primes (sequence A002375 in OEIS). The following is a very simple version of the probability hypothesis (for Goldbach's strong theory). The prime number theorem asserts that a randomly chosen integer m has $\frac{1}{\ln m}$ probability of being prime. Thus, if n is an even larger integer and m is a number between 3 and $\frac{n}{2}$, then the probability of m and $n-m$ occurrence time same can be expected to be prime $.tobefrac1\ln m\ln(n-m)$. Following this approximation, one can likely find the number of ways to write a largely even number n as a combination of two odd primes

$$\sum_{m=3}^{\frac{n}{2}} \frac{1}{\ln m} \frac{1}{\ln(n-m)} \approx \frac{n}{2(\ln n)^2}.$$

Since $\ln n \ll \sqrt{n}$, this quantity goes to infinity as n increases, and one would expect that every large even integer has not just one representation as the sum of two primes, but in fact very many such representations. Show in figures

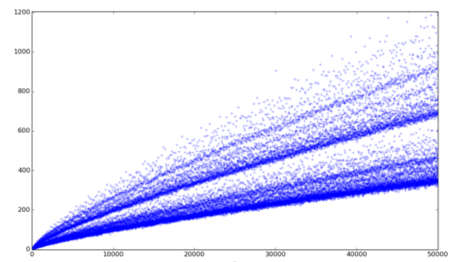




2.5 Goldbach's Comet

The so-called Goldbach's comet, a visual depiction of the number of possible Goldbach divisions of an even integer n, is one intriguing occurrence brought about by the conjecture. It is most likely possible to explain the comet-like

structure as a result of the differences in the number of partitions between congruence classes. Although the number of partitions is only expressed in this figure up to n = 5 · 104, it is evident that the number is gradually rising. This might lead us to believe that the conjecture is true, but this has nothing to do with a proof because we are unable to determine whether there is a huge n-valued exception where the number of partitions is zero.
[4]



Show in Figure : Goldbach's Comet, Goldbach partitions up to the integer n = 50000 on the x -axis, and number of partitions on the y-axis. Generated by a Python script using a modified version of the code from .

2.6 Ternary Goldbach's Hypothesis

In this thesis, we will start discussing the cues and outlines that correspond to ternary or the weak Goldbach's conjecture and its solution done by Helfgott himself in 2014. Notably, by the time he was awarded the position, h. helfgott had been granted the Alexander von Humboldt Professorship at the University of Göttingen although his evidence has not yet been published in a peer-reviewed publication but he has not refuted it yet.[2]

3 Some Important theorems

NO.1 theorem If all primes smaller than or equal to $\sqrt{a}$ cannot divide a natural number a exactly, then a is a prime.

NO.2 theorem Any natural number greater than 3 is the average of at least one pair of primes.

NO.3 theorem The sum of two odd numbers is even.

Proof

A number is odd if it can be written as 2x + 1, where x is some integer. "A number is even if it can be written as2x, where x is some integer. To start, pick any two odd numbers. We can write them as 2n + 1 and 2m + 1. The sum of these two odd numbers is (2n + 1) + (2m + 1). This can be simplified to 2n + 2m + 2 and further simplified to 2(n + m + 1). The number 2(n + m + 1) is even because n + m + 1 is an integer. Therefore, the sumof the two odd numbers is even

(2) (PDF) A Detailed Proof of the Strong Goldbach Conjecture Based on Partitions of a New Formulation of a Set of Even Numbers. Available from:

4 Create A Formula

Our novel approach enlightens us as to the character of the sum of two prime numbers for all even integers.this powerful insight allows us to restate Goldbach's conjecture in the terms of a simple formula. Form a formula of Goldbach's conjecture. Pick the values from set S such that the right side of table 2 gets sum 1, 2 while the left side of the table has a prime number.

Now we create a table one

prime no	partition
2	1+1
3	1+1+1
5	1+1+1+1+1
7	1+1+1+1+1+1+1
11	1+1+1+1+1+1+1+1+1+1+1
. . .	....
sp	2n+1(n-1)+2(n-2)+2(n-3)+4(n-4)...

Table 1: prime sum table one

Making formula

$\pi(n) = 2n + 1(n-1) + 2(n-2) + 2(n-3) + 4(n-4)...$

$$\pi(n) = 2n + \sum_{i=1}^{n-1} g_i(n-i) \qquad g_i = p_{i+1} - p_i$$

The equation is given by:

$$\pi(n) - \sum_{i=1}^{n-1}(g_i \cdot (n-i)) = 2n[8][6][7][10] \tag{1}$$

where:

- $\pi(n)$ is the prime counting function.
- g_i is the i-th prime gap.

So equation one is hold for Goldbach's conjecture always ture for all even number

$$\pi(n) - \sum_{i=1}^{n-1} g_i \times (n-i) = 2n$$

So the results always

$$P_a + P_b = 2n$$

where $P_a \leq P_b$ and $n \in N$ where $n \geq 2$

Sometimes the results is

$$P_a + P_b + p_c = 2n$$

where $P_a < P_b < P_c$ and $n \in N$ where $n \geq 5$

4.1 Some Examples

Frist Example For Equation one

Example 0ne

when n=2

$$\pi(2) - \sum_{i=1}^{2-1} g_i \times (2-i) = 2(2)$$

$$(2+3) - g_1(2-1) = 2(2)$$

$$(2+3) - 1(2-1) = 2(2)$$

$$2+3-1 = 2(2)$$

$$2+2 = 4$$

So equation one hold

Example Two

if we take n=4

$$\pi(4) - \sum_{i=1}^{4-1} g_i \times (4-i) = 2(4)$$

$$(2+3+5+7) - g_1(4-1) - g_2(4-2) - g_3(4-3) = 2(4)$$

$$(2+3+5+7) - 1(4-1) - 2(4-2) - 2(4-3) = 2(4)$$

$$2+3+5+7-3-4-2 = 8$$

now we see make 8 with two prime number

so we get results is

$$3+5 = 8$$

Example Three

For n=5 so we see possible resuts Goldbach's conjecture

$$\pi(5) - \sum_{i=1}^{5-1} g_i \times (5-i) = 2(5)$$

$$(2+3+5+7+11) - g_1(5-1) - g_2(5-2) - g_3(5-3) - g_4(5-4) = 2(5)$$

$$(2+3+5+7+11) - 1(5-1) - 2(5-2) - 2(5-3) - 4(5-4) = 2(5)$$

$$2+3+5+7+11-4-6-4-4=10$$

$$2+3+5+7+11-18=10$$

now we see make 10 with two prime number so we get results is

$$3+7=10$$

other result is

$$2+3+5=10$$

Example Four

For n=6

so we see possible resuts Goldbach's conjecture

$$\pi(6)-\sum_{i=1}^{6-1} g_i \times (6-i) = 2(6)$$

$$(2+3+5+7+11+13)-g_1(6-1)-g_2(6-2)-g_3(6-3)-g_4(6-4)-g_5(6-5) = 2(6)$$

$$(2+3+5+7+11+13)-1(6-1)-2(6-2)-2(6-3)-4(6-4)-2(6-5) = 2(6)$$

$$2+3+5+7+11+13-5-8-6-8-2=12$$

$$2+3+5+7+11+13-29=12$$

only one possible result with two prim number

$$5+7=12$$

other results is

$$2+3+7=12$$

Example Five

when n=10

$$\pi(10)-\sum_{i=1}^{10-1} g_i \times (10-i) = 2(10)$$

$$(2+3+5+7+11+13+17+19+23+29)-g_1(10-1)-g_2(10-2)-g_3(10-3)-g_4(10-4)-g_5(10-5)-g_6(10-6)-g_7(10-7)-g_8(10-8)-g_9(10-9) = 2(10)$$

$$(2+3+5+7+11+13+17+19+23+29)-1(10-1)-2(10-2)-2(10-3)-4(10-4)-2(10-5)-4(10-6)-2(10-7)-4(10-8)-6(10-9) = 2(10)$$

$$2+3+5+7+11+13+17+19+23+29-9-16-14-24-10-16-6-8-6=20$$

$$7+13+109-109=20$$

possible results for two prime

$$7+13=20$$

$$3+17=20$$

The other result for three prime is

$$2+5+13=20$$

So Goldbach's conjecture true for all possible values for $n \in N$ and $P_a \leq P_b$
The results is true for all possible values

$$P_a+P_b+p_c=2n$$

where $P_a < P_b < P_c$ and $n \in N$ where $n \geq 5$

5 Conclusion

Finally, our paper introduces to the world a monumental demonstration of Goldbach's conjecture. This particular problem has engaged the minds of mathematicians for years. We are not only offering a smooth method to solve it, but also showing what even numbers simply are; they are different primes combined together. We have established that using new methodologies based on basic laws in mathematics of numbers it is possible to represent any positive integer bigger than two as a sum of two primes even though none equals this value.

This achievement has an explication on one of the endless mathematical questions and as a consequence opens up untried terrains in the domain of number theory. We pray unto God for giving us wisdom and strength to finish this task. Within this elucidation, we have added information that falls within known bounds on the theoretical realm whilst inventing questions about certain properties related to primes without any answers yet in sight- something which will be interesting for future generations interested more generally in maths as well as specifically prime number theory.

References

[1] Michaela Cully-Hugill and Adrian W Dudek. An explicit mean-value estimate for the prime number theorem in intervals. *Journal of the Australian Mathematical Society*, pages 1–15, 2023.

[2] Johan Härdig. Goldbach's conjecture, 2020.

[3] Daniele Lattanzi. Statistical distributions of prime number gaps. *Journal of Advances in Mathematics and Computer Science*, 39(1):36–61, 2024.

[4] Dora Musielak. Motion of comets and comet tail theory. In *Leonhard Euler and the Foundations of Celestial Mechanics*, pages 107–139. Springer, 2022.

[5] Nils Pipping. *Die Goldbachsche Vermutung und der Goldbach-Vinogradowsche Satz*. na, 1938.

[6] B. U. Zaman. Exact sum of prime numbers in matrix form. 2023.

[7] Budee U Zaman. Connected old and new prime number theory with upper and lower bounds. 2023.

[8] Budee U Zaman. Exact sum of prime numbers in matrix form. *Authorea Preprints*, 2023.

[9] Budee U Zaman. New prime number theory. *Authorea Preprints*, 2023.

[10] Budee U Zaman. Prime sum representation through matrix algebra. 2023.