A New Approach to the Goldbach Conjecture

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Abstract

In 1742 Christian Goldbach suggested that any even number four or greater is the sum of two primes. The Goldbach conjecture remains unproven to the present day, although it has been verified for all even numbers up to 4×10^{18}. Previously this problem has been attacked using deep analytical methods and with complicated integer sieves. This paper takes an entirely new approach to the Goldbach conjecture using pairs of composite integers (composite pairs) that are used to find pairs of prime numbers (prime pairs) that sum to the same even natural number.

Key Words: prime pairs, composite pairs, Goldbach conjecture

1 Introduction

In the past hundred years more and more complicated analytical and sieve methods have been used to find partial answers to the Goldbach conjecture. In 1924 assuming the Riemann Hypothesis, Hardy and Littlewood showed the Goldbach conjecture was true for all even numbers up to n for all but $n^{1/2+c}$ for small c [1]. In 1948 with sieve theory Alfred Renyi showed that every sufficiently large even number is the sum of a prime and a composite number having K factors or less [2]. J. Pintz and I. Z. Ruzsa set K at *8* in 2020 [3]. In 1973 Chen Jingrum used sieve theory to prove that every sufficiently large even number is the sum of two primes or a prime and a composite number with two factors [4].

We take an altogether different approach to proving the Goldbach conjecture by using pairs of composite integers (composite pairs) for finding pairs of primes (prime pairs), where both sets of pairs sum to the same even natural number. For an even number n the number of composites c that can be paired with primes p with $c + p = n$ can be no more than the total number of primes. We remove composite pairs from the set of composite numbers until the number of composites remaining equals the number of primes. After that each composite pair removed will be matched by a prime pair. The number of composite pairs is directly correlated to the number of prime pairs.

2 Using composite pairs

For an even number n let there be $2z$ odd numbers between 3 and $n - 3$. Note: for $n = 4m + 2$ a prime or composite $2m + 1$ is counted twice. Since $c + p = 2z$, the number of composites and primes must both be either even or odd. Thus $c - p = 2a$ for some integer a. $(c + p)$ + $(c - p)$ = $2z + 2a$ giving $z + a$ composite numbers. $(c + p) - (c - p)$ = $2z - 2a$ giving $z - a$ prime numbers. If $a < 0$, there are at least two more prime numbers than composite numbers, and so there would have to be at least one prime pair.

If $a > 0$, we match all composite x with primes $3 \leq p \leq (n - x)/3$. If x leaves the same remainder as n when they both are divided by p, then $n - x$ is divisible by p, making $(x, n - x)$ a composite pair. We can always remove a composite pairs, bringing the number of composites remaining to $z - a$, the number of primes. After that the number of additional composite pairs removed will be matched by the same number of prime pairs, adding up to n.

For $n = 200$ there are 54 odd composite numbers and 44 primes between 3 and 197. There are 13 pairs with two odd composite integers that sum to 200. Subtracting 26 from 54 leaves 28 pairs with one composite integer. There are 49 pairs of odd integers that sum to 200. Subtracting 41 (28+13) from 49 leaves 8 prime pairs: (3, 197), (7, 193), (19, 181), (37, 163), (43, 157), (61, 139), (73, 127), (97, 103).

For $n = 202$ there are 54 odd composite numbers and 45 primes between 3 and 199. There are 13 pairs with two odd composite integers that sum to 202. Subtracting 26 from 54 leaves 28 pairs with one composite integer. There are 50 pairs of odd integers that sum to 202. Subtracting 41 (28+13) from 50 leaves 9 prime pairs: (3, 199), (5, 197), (11, 191), (23, 179), (29, 173), (53, 149), (71, 131), (89, 113), (101, 101).

There is a relationship between total primes ($tprime$), total odd composites ($tcomp$), total composite pairs ($tcomp2$), and total prime pairs ($pripair$) for odd numbers between 3 and $n-3$, and $n \geq 6$.

$$tcomp2 - pripair = a.$$
$$tcomp - tprime = 2a.$$
$$tcomp^2 - tprime^2 = (tcomp - tprime)(tcomp + tprime).$$
$$(2)(tcomp2 - pripair)(tcomp + tprime) = tcomp^2 - tprime^2.$$

For $n = 4m + 2$ a prime (101) or composite $2m + 1$ is counted twice.

For $n = 200$ $(2)(13 - 8)(54 + 44) = 54^2 - 44^2$.

For $n = 202$ $(2)(13 - 9)(54 + 46) = 54^2 - 46^2$.

The ratio of (prime pairs / total prime count) / (total prime count / odd numbers) remains close to 33% for large powers of two.

number	pripair	tprime	odd 3 to n-3	pp/tp	tp/odd	(pp/tp)/(tp/odd)
134217728	283746	7603552	67108862	3.7%	11.3%	32.9%
67108864	153850	3957808	33554430	3.9%	11.8%	33.0%
33554432	83467	2063688	16777214	4.0%	12.3%	32.9%
16777216	45746	1077870	8388606	4.2%	12.8%	33.0%
8388608	24928	564162	4194302	4.4%	13.5%	32.9%

Figure 1:

Powers of two contain plenty of prime pairs, as indicated in Figure 1. As shown in the equation above, the number of prime pairs and composite pairs increase together. With even numbers containing odd divisors the proportion of composite pairs in the composite sets will never be less than it is in the powers of two composite sets. For an odd c with $n = (c)(2^m)$ and all odd a, b with $a + b = 2^m$ there are generally a larger proportion of composite pairs (ca, cb) than there are for $n = l + m = 2^k$, where both l and m must be composites. **(pp/tp)/(tp/odd) will never be less than it is for the powers of two.**

When n is increased from 2^{23} to 2^{27} **(pp/tp)/(tp/odd)** remains proportionally the same. The 2^{23} to 2^{27}proportion increase for prime pairs (pp) = 11.3826, primes (tp) = 13.4776, and total odd numbers between 3 and $n - 3$ = 16.0. (11.3826 / 13.4776) / (13.4776 / 16.0) = 1.0026. The **(pp/tp)/(tp/odd)** proportionality factor between all large powers of two will remain close to 1.00. Thus we believe **(pp/tp)/(tp/odd) will remain approximately 33% for all large powers of two and the same or greater for all large *n*. We believe there will always be prime pairs associated with every even** $n \geq 4$.

3 Conclusion

It's generally believed the Goldbach conjecture is true. The numerical evidence for it is overwhelming. With the relationship between prime pairs and composite pairs we have a solid reason why the Goldbach conjecture is true.

Conflicts of Interest: The author declares no conflicts of interest.

References

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