Notes About Prime Numbers in Different Number Basis and the Goldbach Conjecture.

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Zaragoza (Spain) May 2024.

0- Abstract

I share some thoughts about prime number and the use of basis in number theory.

1- Definition of divisor of a number n in different prime basis.

A number $(n)_{10}$ will have a number m of final 0s, being that number in the form:

$$(\alpha_1 \alpha_2 ... \alpha_n 0_1 0_2 ... 0_m)_{(p_i)} \quad (1)$$

Where the number m of 0s will be the quantity of p_i -divisors of n and $n \in \mathbb{N}$. And for all $p_i < n$, where $p_i \in Primes$. A number p_i is Prime if and only if $k \nmid p_i$ if $k \neq \{1, p_i\}$ when $\{k, p_i\} \in \mathbb{N}$.

Some examples of this are:

$(6)_{10} = (10)_6 = (11)_5 = (12)_4 = (2\underline{0})_3 = (11\underline{0})_2$, so $3 \cdot 2 = 6$ (One final 0 in 3 and one final 0 in 2).

$(4)_{10} = (10)_4 = (11)_3 = (1\underline{0}\underline{0})_2$, then $2 \cdot 2 = 4$ (Two final 0s in 2)

$(10)_{10} = (11)_9 = (12)_8 = (13)_7 = (14)_6 = (2\underline{0})_5 = (22)_4 = (101)_3 = (101\underline{0})_2$

In this case, $5 \cdot 2 = 10$ (One final 0 in base 5 and one final 0 in base 2).

The proof for any case is obvious by divisibility concepts.

2- Primes and final 0.

In case that the only final 0 of any basis conversion of a number is $(\tau)_{10} = (10)_\tau$ then τ belongs to primes. It will be true it does not exists any:

$$(\alpha_1 \alpha_2 ... \alpha_n 0)_m = (10)_\tau \quad (2)$$

For any $m < \tau$.

Proof: if some number has has final 0s in base m it will be a divisor of τ and τ in that case will be not belong to Primes.

3- On Goldbach Conjecture.

Being two prime numbers $p > 2$ and $q > 2$ with the form:

$$(p)_{10} = (10)_p = (\alpha_1 \alpha_2 ... \alpha_n 1)_2 \quad (3)$$

and:

$$(q)_{10} = (10)_q = (\beta_1 \beta_2 ... \beta_n 1)_2 \quad (4)$$

The final 1 of the expansion in base 2 indicates us that they are odd. We add the two primes and we got:

$$(p+q)_{10} = (10)_{(p+q)} = (\gamma_1 \gamma_2 ... \gamma_n 0)_2 \quad (5)$$

For any $\alpha_i, \beta_i, \gamma_i \in A$ and $A = \{0,1,2,3,4,5,6,7,8,9\}$.

As any two prime numbers different from 2 can be written with the form base 2 ending in 1, and being the sum equal always to a number in base 2 ending in 0, then $(p+q)_{10} \Rightarrow (2w)_{10}$ where $w \in \mathbb{N}$ for any $w \geq 3$.

But is true in the other direction? Lets try something.
Our primes named p and q can be written:

$$p=(\sum_{i=0}^{n} 2^{(\alpha_i)})_2$$

and:

$$q=(\sum_{j=0}^{n} 2^{(\alpha_j)})_2$$

This is a combinatorial Sum like the Product in Fundamental Theorem of Arithmetic.
In this case it will be always true that $2^0 \in p$ and $2^0 \in q$.
Then, following the property:

$$\sum_{x\in X} f(x)+\sum_{y\in Y} f(y)=\sum_{z\in X\cup Y} f(z) \quad (6)$$

We can write:

$$p+q=(\sum_{i=0}^{n} 2^{(\alpha_i)})_2+(\sum_{j=0}^{n} 2^{(\alpha_j)})_2=(\sum_{i,j=0}^{n} 2^{(\alpha_i)}+2^{(\alpha_j)})_2 \quad (7)$$

And:

$$(\sum_{i,j=0}^{n} 2^{(\alpha_i)}+2^{(\alpha_j)})_2=(\sum_{k=1}^{n} 2^{(\alpha_k)})_2 \quad (8)$$

So: $2^0 \notin (\sum_{k=1}^{n} 2^{(\alpha_k)})_2=(2m)_{10}$, for $m\in\mathbb{N}$ .This is because the sum of two odd number always is an even number.
Then we assume that is always true that any even number in base $(2m)>2$ can be expressed in base 2 and end always in 0. Now as $10\equiv 2(mod\,2)$ we express then:

$$(10)_{(2m)}=(2m)_{10}=(\delta_1\delta_2...\delta_n 0)_2 \quad (9)$$

For $\delta_i \in A$. In that case, we can say that is true $(2w)_{10}\Rightarrow(p+q)_{10}$ that we can go back to equation (5) and we can equal:

$$(\delta_1\delta_2...\delta_n 0)_2=(\gamma_1\gamma_2...\gamma_n 0)_2 \quad (10)$$

in the infinite quantity of even numbers. And we conclude with:

$$2w=p+q \quad (11)$$

Lastly, we have for $w=2$ we have $2+2=4$. With that we can say that an even number greeter than 2 can be always expressed as the sum of two prime numbers.

4- Conclusions.

I worked in this concepts so hard in the last days and I do not really know if it just is useful to describe evenness of two primes, or the last part is always true or in any case if I had made some mistakes so I want to show this now to the evaluation of the generous people who spend time reading my papers. As I remember I just only consulted Analysis I of Terence Tao for equation (6), I do not put other references, the rest is from memory and deduction in my own work.