Bridging the Boundary of Applied and Pure Mathematics: A Philosophic Argument for Expanding Mathematics and Scientific Disciplines via a New Numeric System

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ABSTRACT

This presentation is about new directions of study and discovery in both Mathematics and Science. Over the last several centuries, science has discovered objects in the world along a continuum of scale. In one direction, we have found planets and stars, galaxies and galaxy clusters. In the other direction we have found cells and proteins, atoms and neutrinos. In order to locate and model this world, we use the 3 traditional directions of length, width and height. However inherent in all our measurements is the scale of the objects we are measuring – a continuum we do not directly see with our eyes. A key reason we do not include this direction as part of our scientific models is that we do not have the appropriate mathematical tools to take measurements along this continuum. New mathematical tools may require a numeric representational system with more power than our traditional decimal or positional based numerals. Such a more powerful system may provide a single value for complex numbers able to measure across scale and adds to its structure the reversing operations of integration and differentiation. The ability to calculate integration and differentiation results would be a powerful mechanism for applied and for pure mathematics.

The author presents some opening remarks on what is anticipated to be a much larger discussion, looking at a model of reality where objects at all levels of scale can be located and considers directions for generating more powerful mathematical tools than we have today.

Presentation

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In the past new directions have involved a review and challenging of certain assumptions we make that underlie our theories and concepts. I will suggest we reconsider two underlying assumptions – one in mathematics and one in science. Then we will challenge them by asking questions about how we represent numbers and how we model our world. The assumption in mathematics is that we already have the most advanced means of representing numbers. This leads to the belief that we can, today, represent any numeric value possible. The assumption in science is that we already have all tools to make all measurements in the universe. This assumption allows us to believe we can, today, measure any event or action in the universe. This discussion will consider the possibility that both assumptions are incorrect, indicating new numeric values can provide new measurements. I conclude that a new numeric system can lead to making measurements we currently cannot make today.

Consider that current science would not be possible without the decimal (and positional) numeric system we use to represent measurements. I submit that a system such as Roman numerals is completely inadequate for the measurements of current science. Fractions are, likewise, not up to the task of capturing measurements and providing the arithmetic for equations defining scientific laws today. Measurements on the quantum scale would not be possible using fractions or other limited numeric representational systems. Without our current method of representing numeric values – particularly for measurements – we would not have the science of today.

The decimal numeric system became the defining means of representing numbers and measurements less than a thousand years ago. Its use predates the explosion of science in the last 400 years[1], lending support to the idea that current science needs such a representational system to manage the measurements of today. Note that this discussion is not about the type of number we refer to – such as Integers, Rationals, Reals, or Complex. It is about how these numbers are represented as values and therefore measurements. Because numbers and number values are critical in both pure and applied areas of mathematics, numeric representational systems lie on the boundary between these areas. If we are unable to represent certain mathematical values, then we will also be unable to represent certain measurements in the universe and therefore could be missing aspects of the universe we endeavor to study. There is an inherent suggestion, here, that a more powerful numeric system could provide more advanced scientific theories than we have today.

While an experiment can involve many measurements, each measurement results in a value with (or sometimes without) units attached[2,3]. Therefore, a single measurement equates to a single numeric value. A physics equation can refer to a variable 'x', however the results of an experiment need to put values in for the variable. This is the importance of numeric systems for science – measurements require values. The assumption that we can represent any number – and therefore any measurement – is what we are concerned with first. A particular concern in science is the use of Complex numbers, as they are not represented as a single value needed for a measurement or functional value. Complex numbers are represented as $z = x + iy$, where 'z' is considered the single complex value while 'x' and 'y' are represented by decimals and require two values since we have to accommodate an undefined value for sqrt(-1) – represented as 'i'. Our current numeric representation of a complex number is, therefore, not a single value and therefore we cannot represent a true complex value that could be a scientific measurement. We are always left with two numeric values and sometimes we just drop the one we cannot define, using only the 'real' part of the complex number for a measurement. From a philosophic perspective, this seems unsatisfactory: We should be able to represent a complex value as a single value such that it can correspond with a measurement in the world. This indicates our mathematical assumption is incorrect as we cannot represent a complex number as a single value 'z', we can only represent it using two values 'x + iy'.

On to the challenging science question: What would it take to model everything we know of in the universe, from the smallest particles to the largest globular clusters? In considering such a model, we want to determine if there are measurements we currently do not (or cannot) make in our study of the world. We presume a 3-dimensional space model can capture objects at all scales, but can it? How are we to model our bodies, organs, cells, proteins, molecules, atoms, and sub-atomic particles that ostensibly fill the same three-dimensional volume? When we touch our finger to a pane of glass, the direct evidence is of our finger touching the glass. If we perceive the action with a magnifying glass we would see specific ridges of our skin touching the less than smooth surface of the glass. If we perceive the action with a microscope we would see cells touching the rough surface of the glass. We can continue indirect observations using different magnifying tools down to the protein, molecular and atomic scale levels. We could setup multiple observational tools to observe different scale levels during the same action and we would gather the observational evidence that the action occurs on different objects at all these levels simultaneously, not at any one or another level. If science is about observations, then we should be able to include such observations in our model.

Now, within any scale level, we use a three-dimensional space in which to study objects of that level. To model all objects at all levels of scale, we need to combine all these three-dimensional levels along the continuum of scale. We will then need to identify the scale of the objects under observation in addition to our traditional three. This will require a four-dimensional model. Even if we are pure reductionists and say that only the smallest level is reality and all others are illusion, we will need the scale continuum model to demonstrate that particle actions represent the larger scale actions. We need a four-dimensional space in which to build our model of all objects in the universe we know today.

Note that such a model doesn't change theories at any level, however it does provide for potentially new relationships between and across levels. Considering a four-dimensional Model incorporating all known objects, we now challenge our assumption that we can, today, make all needed measurements for science. So, how should we measure the distance between objects at different levels of scale, say between a pen and an atom of the desk? If we live in a three dimensional universe this should be a straight-forward measurement, like from the pen to the edge of the desk. However, I am not aware of any such measurements in the study of our world and the universe today that measures across multiple levels of scale. Multiscale modeling attempts to connect models at near-by levels of scale, however crossing many levels of scale – such as atoms to proteins to cells, to sinews, to organs – is an over-extension of such modeling. I will suggest that we currently do not include the scale continuum as a physical aspect of reality because we are not able to measure across it.

There are a number of books, videos, and websites that provide a journey through scales. Possibly the earliest is Kees Boeke's 1957 book *Cosmic View: The Universe in 40 Jumps*. A well known video (inspired by Boeke's book) is *Powers of Ten* by Charles and Ray Eames[6]. Two things to note in all these videos (and books): 1) As we progress up or down in scale, we see different objects. This is characteristic of travel in our normal three dimensions – we see different objects as we travel. The books and videos explicitly describe such movement in scale as a journey and that we travel through scale. 2) From a standard unit of length perspective, travel up or down in scale involves traveling in exponential units – 'Powers of Ten'. One unit upward would be an increase of 10 of our 'standard units' and two units upward would be an increase of 100 of our 'standard units'. This means a linear movement in the scale direction would appear as a power (or exponential) change in the lengths we measure at our scale. This could mean a constant velocity in scale (relative to ourselves) would appear to us as a constant acceleration, even that our universe appears to be expanding. There are many possible directions to consider just from this relative perspective, however I will not do so here.

This situation complicates a spatial metric, since all dimensions are not equivalent. Such a complication will impact any measurements in our model of the universe and could be an example of measurements we do not currently account for in science. Examples of where such measurements across scale are attempted today would be simulating the history of the universe and predicting our weather. Both examples attempt to consider actions across multiple levels of scale but have to limit their calculations to certain 'resolutions'. Any action that falls below a limiting resolution defaults to a 'subgrid scale' that is a sort-of pre-defined space that represents all smaller resolutions and actions with objects. I submit that while we think of 'resolution' as limiting our ability to calculate results, what is really the issue is the inability of our numeric measurement tools to cross multiple levels of 'resolution', to address measurements across scale. This indicates our science assumption is incorrect as we cannot properly measure across scale.

To return to numeric systems, there is a pattern that can be seen in the evolution of numeric representational systems. Whole numbers can be easily represented by one mark, then add another mark (for two marks), then add another for three marks, etc. By representing groups of marks with other symbols, we can get a numeric system such as Roman numerals. We can extrapolate such a symbol system to representing the Integers, using addition and the reversing operation of subtraction to achieve this. Next we have fractions, which adds the reversing operations of multiplication and division to the symbols and can represent the Rationals. Today we have decimals, which adds the use of exponents for numeric representations, along with the reverse operation of logarithms, and can represent the Real numbers.

Challenging the mathematical assumption that our current tools are the end-all of numeric representational systems, what might a next numeric system involve? Extrapolating beyond the reversing operations of addition-subtraction, multiplication-division, and exponentiation-logarithms, we might consider adding the reversing operations of integration-differentiation into the definition of a new representational system. Such operations, built into our numeric system, might make many calculations involving integration and/or differentiation significantly easier. It could address MultiScale modeling issues of integrating across scale and could provide for a single value representing complex numbers.

As noted at the beginning, this talk is an attempt at presenting arguments for stepping beyond what our current mathematics and our current science can address. It is far too large an endeavor to capture here or by one individual. Such directions will require scientists from multiple disciplines to work together, for mathematicians to create (not simply discover) new tools that bridge pure and applied mathematics. It is unlikely that such a new numeric system will be a 'paper and pencil' operation, but will require computers to utilize, maybe even a new generation of computers built using it.

In conclusion, we find that both the mathematics assumption that we can represent any number and the science assumption that we can make any measurement are not supported by the evidence. This leads to new directions in both areas – for a new numeric system and scale continuum model of the universe.

Finally, we could say we actually live in a four-dimensional world, not a three dimensional one without changing anything we sense or measure about the world. It gives us a means to model all the scale levels of nature in a single model. To change an analogy from the story 'Flatland: A Romance of Many Dimensions' by Edwin A. Abbott where a two dimensional being is shown a three-dimensional perspective: What if we are the Flatlanders, however instead of us living on a two-dimensional plane, we only believe we live on a two-dimensional plane, yet we really exist with three-dimensions. We can perceive objects in this other dimension with special tools, yet we believe everything that is 'below' us in scale is 'inside' us. When the Sphere visits Square and pops him out of his plane, Square sees that all the people and objects of his world actually have a 'depth' to them filled with the other objects, like organs and cells and atoms, which they do not see directly. This could be the more apt analogy (not considering the social discourse of Mr. Abbott).

Thank you for your time and patience.

Appendix:

Here are a couple possible hints for developing a new numeric system: 1) Variables represented using Euler's number 'e' and exponentiation or natural logarithms allow for the continual differentiation (and integration) of mathematical statements, without 'bottoming out' as other base systems do. This suggests using 'e' as a base could allow a new numeric system to integrate or differentiate across many levels, as suggested in crossing levels of scale. 2) Consider Euler's equation $e^{i\pi} + 1 = 0$, which can be re-written as $e^{i\pi} = -1$. We already are making use of 'e' as for 1) above. In addition, note that the left side is a positive value, while the right is a negative value. Can we swap a negative value for a positive one? Could this impact defining square roots of negative numbers?

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