Heuristic Challenges for Science and Mathematics to Modeling Nature Donald G. Palmer Abstract

Science is about developing theories of nature based upon evidence, in particular experimental evidence. Scientific theories use mathematical tools to measure, explain, and predict natural and human-made processes. Over the last few centuries, we have begun to perceive objects across many levels of scale. We have come to understand that we live on a spinning earth, which is circling a sun, which is moving within the Milky Way galaxy, which is moving in a cluster of galaxies. And we have come to understand that we are comprised of organs and blood vessels that continually pump blood, which moves cells made of proteins that supply oxygen and nutrients to other cells. And all these cells and proteins are made up of molecules and atoms and particles that are constantly moving on their levels. Scientific theories and disciplines tend to address one or another level of scale e.g., quantum physics, molecular chemistry, human medicine, planetary ecology, stellar systems, galaxy clusters. Some address multiple levels of scale. However, nature includes all these levels operating together, in some interconnected fashion.

Introduction

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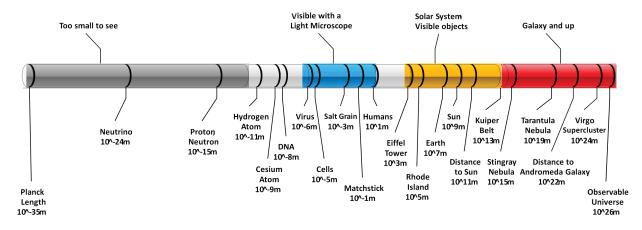
Heuristic Challenge for Science

Science is about developing theories of nature based upon evidence, in particular experimental evidence. When we touch our finger to a pane of glass, the direct evidence is of our finger touching the glass. If we perceive the action with a magnifying glass we would see specific ridges of our skin touching the less than smooth surface of the glass. If we perceive the action with a microscope we would see cells touching the rough surface of the glass. We can continue indirect observations using different magnifying tools down to the protein, molecular and atomic scale levels. We could setup multiple observational tools to observe different scale levels during the same action and we would gather the observational evidence that the action occurs at all these levels simultaneously, not one or the other.

Some scientists have suggested that nature exists only at the lowest level and such an action only occurs on the atomic or sub-atomic level where we never actually touch the glass and that all but the lowest levels are illusory¹. However, to base this explanation only on what we believe occurs on the indirectly observed molecular or atomic level is to subvert the direct observational evidence of our eyes. This would seem at odds with the study of science. It would seem more appropriate to consider nature as existing at all levels together.

Another concerning item is that we have different models of nature, depending upon which level you are concerned with. There is the 'classical' model of what we see around us. We have medical models of our bodies, organs, and cells. Then there are the models of proteins and macromolecules that are built up from models of molecules and atoms, which build upon models of particles. Multi-Scale Modeling attempts to bridge adjacent models, and is currently an area in expansion, however to model nature we should be attempting to describe all levels using a single model.

If we are to efficiently model this one action of touching a pane of glass at all levels, we will need to be able to specify the actions at every level and then combine them across levels. Each level exists at a certain scale, with differing modeled objects at different scales. Since we model each level as actions in a three dimensional space, combining them would most effectively use a four dimensional model, identifying the scale level as one of the locators in the modeled space. This heuristic model would provide a means of locating a pen on a table, an atom of the pen, and a star in a different galaxy all in one model. It would not be limited by individual models at each scale, nor require multiple models to be integrated together across scale. Such a model would allow for actions between levels, both upward in scale as well as downward in scale.



This does present an issue with current conceptions of nature as using a model with 'space-time' as four dimensional. The above discussion suggests modeling the many levels of nature using four dimensions is more direct and potentially better than our traditional three dimensional space that still needs to account for objects at different scales. This would suggest that 'space-time' as four dimensional might not be a good model. Maybe we model 'space' as four dimensional and tack time onto these four spatial dimensions – conceivably modeled as a fifth dimensional 'space-time'.

This heuristic model could provide the means to account for actions on the molecular level to effect actions on the protein or mitochondrial levels. Since it provides a means of connecting activities at different levels, its usefulness in medicine could be powerful, directly connecting actions on the chemical and protein level to those on the cellular and even organ levels. It also provides the capability for actions on a larger level, say our human scale, to affect actions on smaller scales, such as humans building the Large Hadron Collider (LHC) to guide sub-atomic particles into collisions with each other.

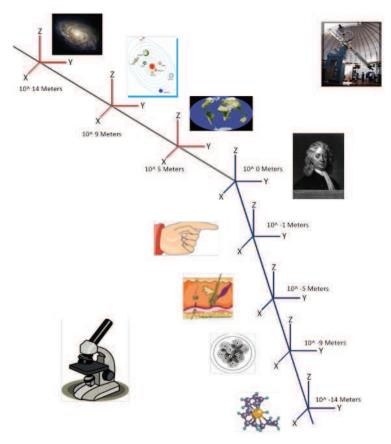
We could say we actually live in a four-dimensional world, not a three dimensional one — without changing anything we sense or measure about the world. It does give us a means to model all the scale levels of nature in a single model. To change an analogy from the story 'Flatland: A Romance of Many Dimensions' by Edwin A. Abbott²: What if we are the Flatlanders, however instead of us living on a two-dimensional plane, we only believe we live on a two-dimensional plane, yet we really exist with three-dimensions. We can perceive objects in

this other dimension with special tools, yet we believe everything that is 'below' us in scale is inside us. When the Sphere visits Square and pops him out of his plane, Square sees that all the people and objects of his world actually have a 'depth' to them filled with the other objects, like organs and cells and atoms, which they cannot see directly. And each Flatlander has additional surfaces to them that they do not understand as surfaces. This could be the more apt analogy (not considering the social discourse of Mr. Abbott).

Short Critique of Nature as Three dimensional

If we only lived in three dimensions, then we should expect higher precision of measurements to produce more accurate information about the objects we are making measurements about at the original scale. This would be the expectation of a mathematical space of three dimensions — more precise measurements of objects result in more accurate models. However, nature does not always mesh nicely with mathematics, since adding more and more precision to measurements at one scale shifts us into another scale with different objects, say from our bodily organs to cells or to molecules. More importantly, the objects we can measure at one level, say our body at our level, are not the same objects we can measure at other levels, say the cellular level. I believe there is a statement in physics that two objects cannot occupy the same physical space. So, how can these different objects exist in the same physical three dimensional space?

It appears that we have to 'travel' along scale to get to one or the other objects (see interactive website by Huang bothers⁸). What are we travelling through? How can we shift from the scale of our body to that of our cells and stay in the same physical location? If we believe we only exist in three dimensional space, how can these aspects work? If we use a four dimensional model, then this all makes sense – we do travel through a 'scale space' dimension to get from our level to that of proteins and molecules.



Consider a 3-dimensional cube with 1 cm sides at the cellular level and another cube with 1 cm sides at our level and connect the vertices of each cube. I do not think this is too difficult to imagine. This figure would constitute a 4-dimensional hypercube, with a 3-dimensional cube at each end and an enclosed space through the intervening scales including the objects at these intermediate scales. You have just visualized a 4-dimensional hypercube or tesseract. This should be a clue that our world may not be properly modeled in three dimensions since we actually observe different objects at different scales. It also means we need to identify the scale of the objects we measure in order to properly locate an object in our world - constituting a fourth location axis.

(see Figure below from Wikipedia http://en.wikipedia.org/wiki/Tesseract):



Potential Implications of a Four Dimensional Scale Space

Consider that a fourth scale dimension would suggest reasons for non-interaction of objects and particles at different scales – say a neutrino and a cell (or our body). Rather than passing 'through' the cell or body (missing all objects of the cell or body), it actually passes 'below' it at a smaller scale. The Flatland analogy would even suggest there is a 'lower scale' boundary to ourselves – under which other particles could pass. For two objects to 'hit' each other, they would need to be at the same level, or have objects at the same common level that could interact. In fact they would only 'hit' each other if they have objects at common scale levels. This goes to the initial example of touching our finger to a pane of glass.

Observationally, it would appear that most objects we know of in our immediate neighborhood exist with a number of essentially the same scale levels. Our bodies, the table, the book, or computer all have a surface at our level with objects at the molecular and atomic levels and structures of objects in between. Stars have a somewhat indefinite surface at a much larger scale than ourselves, yet also have atoms and molecules. What goes on at those in-between scale levels?

Particles could move in scale, potentially revolving around a nucleus, not just at different 'orbits' but at different scales as well. How might we measure an object that shifts in scale? Could we pin-point its location in a 3-dimensional model of space or would we have difficulty with its location? If we shower the particle with photons or other particles and it shifts in scale, would we be less likely to determine its location? We might predict the more energy we hit it with, the more it would shift in scale and the harder it would be to locate in our three dimensional model.

How would an object that moves in scale appear to us? It may shrink or expand in size. Since scale is only one dimension, it might not do this equally along all 3 other axes. So it might shrink in length but not width or depth. This might be the situation where the object is moving in one of our normal 3 dimensions at a certain velocity and also moves in scale. It might then appear to shrink (or expand) in the direction of motion. This would be a relative appearance, as the object would not actually have shrunk – it would only appear to us that way.

What if an object is stationary in our three dimensions yet only moving in scale at a constant velocity? If we were all in free space, we could all consider ourselves at rest and feel we are not moving. If we were moving in the same scale direction at the same velocity as the object, then it should not change in any of our three dimensions – in volume.

Now, if we are all relatively stationary in our standard three dimensions on the earth, yet all moving in scale then we should still not observe a change in volume. There should still be some indication of the motion in scale. Constant movement in scale would appear to be moving first 1 unit, then 10 units, then 100 units. In other words, the objects in motion in scale would appear to be accelerating. This is an important consideration, as a constant velocity in scale would appear to an object at rest (in our standard three dimensions) as a constant acceleration.

On the earth we and other objects would not appear to be moving relative to each other, yet that constant velocity in scale, that apparent acceleration, should show up in some fashion. In a typical F = ma way, it should appear as some force on us. We might not initially understand the velocity in scale as movement (especially if we do not consider scale a direction), since objects around us would not appear to change. However we would certainly notice the associated force from this apparent constant acceleration. Even though our F = ma law would imply the force we notice should be attributable to some "a" as other accelerations, we might consider this masked acceleration somehow different than other accelerations (we might call it gravity). Then it would be somewhat unobvious that the typical accelerations in our three dimensions would be somehow equivalent to the masked one.

Note that it would appear to require 'energy' to account for this ongoing force. Not a typical energy where we can measure the movement and force over time.

Heuristic Challenge for Mathematics

While the above suggestion might seem a simple expansion of space to four dimensions, there is a key difference between our traditional 3 dimensions and that of scale. The measurements in our traditional dimensions are all equivalent – 1 unit (e.g., one centimeter) in any direction equates to the same unit in any other dimension. These are linearly equivalent related dimensions. This is not the case with scale, which has a different measurement or metric. Relative to the units we use in our traditional dimensions, scale has an exponential measurement. One traditional unit converts to a unit with an exponent (> 1) along the dimension of scale. Several people have made visualizations of this exponential metric as traveling up or down in scale, usually using powers of ten as the unit^{3,4,5,6,7,8}.

This introduces an additional challenge as such a model has 3 equivalent linear dimensions plus 1 that is exponential in extent (when related to the other three). This last is a challenge for Mathematics rather than Science. Traditional geometry assumes all dimensions of a space have the same units – the distance between any two points in space uses measurements. However, to model nature, we may need a geometry that does not have the same distance measurement, or metric, in all spatial dimensions.

To address geometric spaces with different metrics for different dimensions would be a new direction for Mathematics. If, in addition, we need to enhance our current measuring tools so as to be able to relate measurements, represented by linearly defined numbers, with other measurements across non-linear scales these new areas could have a direct impact upon what and how Science measures the universe.

Recommendations & Conclusion

As we move toward digitally modeling entire bodies in space, we will find the need for locating objects in scale, as well as in 3-D space. This will require a 4-dimensional scientific model. This model will require some distance measure and units that cross scale. Since the distance metric of scale is different than our traditional 3-dimensional space, we will need to develop new mathematics to understand the characteristics of such a space.

We could consider a simple model that collapses all three 'standard' dimensions onto a single dimension and then models scale along a separate axis. Since we do not know the conversion value between these two, we could consider the conversion value to be an unknown value. This would be similar to using complex numbers and the complex plane as our simple model. This might suggest that we are already making use of mathematics that differentiates our standard dimensions and measurements with a dimension that acts differently. We just have not interpreted the mathematics appropriately for the model, nor have we identified that unknown value.

Bibliography

- 1. Trosper, Jaime; 2014; Why Physics Says You Can Never Actually Touch Anything; Futurism; June 17, 2014
- 2. Abbott, Edwin A.; 1884; Flatland: A Romance of Many Dimensions; Seeley & Co.; London

Scale, incl. Videos/Websites

- 3. Boeke, Kees; 1957; COSMIC VIEW: The Universe in 40 Jumps; John Day Co.; NYC
- 4. Gott, J. Richard and Vanderbei, Robert J.; 2010; Sizing Up the Universe: The Cosmos in Perspective; National Geographic
- 5. National Film Board of Canada (Illustr. Szasz, Eva); 1968; Cosmic Zoom; Canada
- 6. Eames, Charles; 1977; Powers of Ten;
- 7. Huang, Cary & Michael; 2010; The Scale of the Universe
- 8. Huang, Cary & Michael; 2012; The Scale of the Universe 2