Quadrature of the Circle with Compass and Straightedge and a Surprising Result for the Value of in π in π r^2

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Abstract

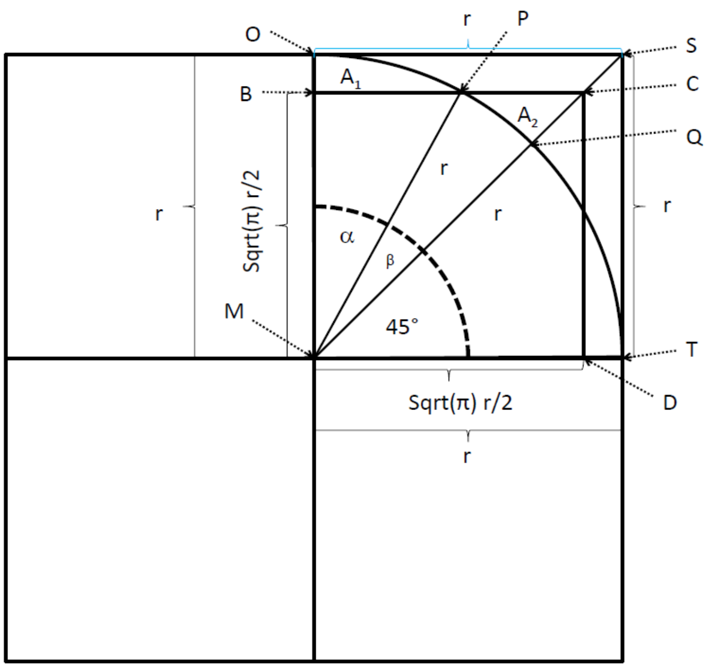
It is general believe and deemd to be proven that the value of π in the formula for calculating the area of a circle; i.e. π r^2, is identical to the value of π in the formula for calculating the circumference of a circle; i.e. 2π r, which is irrational. Therefore, quadrature (or squaring) of the circle with compass and straightedge (or ruler) has been deemed to be impossible. We show that this was a prejudice and proof that quadrature is possible and clearly delivers π = 3 in the formula π r^2 for calculating the area of the circle. We also show a physical experiment that unambiguously proofs this result.

Proof

For millennia, mathematicians have hold the firm believe that there is only one irrational value for π in the formula for calculating the circumference of a circle, i.e. 2π r, and the formula for calculating the area of the circle, i.e. π r^2, and that therefore the quadrature (squaring) of the circle with compass and straightedge (ruler) is impossible.

Fig.1 shows a square that is a quarter of a square which is inserted in a symmetrical way into a circle which itself is surrounded by a square with a side of twice the radius of the circle.

Fig.1



As shown in Fig. 1, the prerequisite for the squaring of the circle would be that area A1 (surplus area of the circle) is equal to area A2 (surplus area to the square). Then the square $(\mathrm{sqrt}(\pi)/2)^2$ would be equal to the area of one fourth of the circle with radius r. There are two crucial points in this drawing. One is point B, which is the endpoint of line MB and cannot be drawn exactly, if π is irrational, since there is no way of constructing the square root (sqrt) of such a π. The second crucial point is point P. which is the point of intersection of the radius and the circumference of the circle, the situation of which is crucial for the size of area A1 and area A2.

Note: It is interesting to note that $\sqrt{/\pi)}/2 \propto 0{,}886$ is quite similar to $\sqrt{(3)}/2 \propto 0{,}866$ and can hardly be differentiated in a drawing at the above scale.

The visual impression of the drawing is that the angle α looks very much like angle 2β, that is like 30°.

Since nature is often highly symmetric, the drawing suggests to simply try a calculation with α = 30°.

In the following, the calculation and its concomitant result with regard to π:

The area of the sector of the circle with angle α = 30° is $\pi r^2/12$, **(I)**
The area of the sector with angle β = 15° is $\pi r^2/24$. **(II)**
The area of triangle MBP is half the square $(\sqrt{(\pi)}r/2)(1/2)r$, i.e. $(1/8)r^2\sqrt{(\pi)}$ **(III)**
The area of the triangle MPC is
$(1/2)[r(1/2)\sqrt{(\pi)}]^2 - (1/8)r^2\sqrt{(\pi)} = (1/8)(\pi r^2) - (1/8)r^2\sqrt{(\pi)}$ **(IV)**

A_1 is **(I) – (III)**, i.e. $A_1 = \pi r^2/12 - (1/8)r^2\sqrt{(\pi)}$ **(V)**

A_2 is **(IV) – (II), i.e.** $(1/8)(\pi r^2) - (1/8)r^2\sqrt{(\pi)} - \pi r^2/24 = ((3-1)/24)\pi r^2 - (1/8)r^2\sqrt{(\pi)} =$
$\pi r^2/12 - (1/8)r^2\sqrt{(\pi)}$ **(VI)**

Accordingly $A_1 = A_2$, and therefore the square in the drawing is equal to $\pi\, r^2/4$.

All you have to do for squaring the circle is to create an angle of 30 ° at a radius at the centre of the circle and draw a line at 90° from the radius through the point of intersection of the leg of the 30° angle to the leg of an angle that extends at 45° from the radius and the centre of the circle and complete this to a square, which you have to do four times to get the square corresponding to a full circle.

According to the theorem of Pythagoras, this, however, also means that π in the formula for calculating the area of the circle, i.e. $\pi\, r^2$, is 3.

So there are two different constants π:

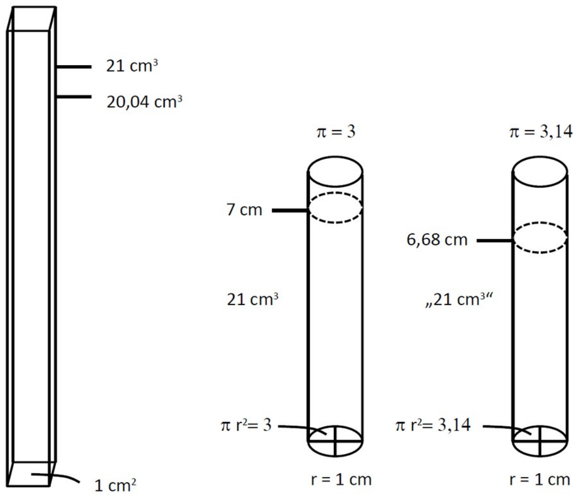
π_c = 3.1415...in the formula $2\pi\, r$ for calculating the circumference of the circle.
π_a = 3 in the formula $\pi\, r^2$ for calculating the area of the circle.

The consequences for the calculation for the volume of a sphere and its surface (a problem that is difficult to tackle) will be discussed in a later paper.

Physical Experiment

There also is a physical experiment that proves the above.

Fig. 2.



The well-known definition of 1 cm^3 (1 ml) is a cube with a side length of one. Fig. 2 shows a box-shaped container with a bottom area of exactly 1 cm^2. Therefore; any length of the container corresponds exactly to the amount of cm^3 (ml) to the amount in cm^3 (ml) of e.g. of a liquid contained therein:

Furthermore, there are shown two cylindrical containers with a circular bottom area having a radius of exactly 1. The left one shows the calibration mark of 21 cm^3, when you calculate with a bottom area of 3 cm^2 (π_a = 3). The right one shows the calibration mark of 21 cm^3, when you calculate with a bottom area of 3,14 cm^2 (π_a = 3.1415..).

When you fill the left cylindrical container with a liquid not having a high surface tension (e.g. ethanol) up to the mark at 7 cm and pour into the box-shaped container, you will experience that this container is filled up to the mark at 21 cm, e.g. the content of the left cylindrical container is exactly 21 cm^3 (ml).

When you fill the right cylindrical container with the same liquid up to the mark at 6,68 cm and pour into the box-shaped container, you will experience that this container is filled up to the mark at 20.04 cm, that is about 5% less than what would be expected, if π_a were 3.1415.. (Annotation: A wrong calibration of cylindrical containers for pressurized air might have cost the life of quite a few divers.)

Therefore, nature supports the theoretical result.