# ENERGIES OF VACUUM. ENERGY-MOMENT PSEUDOTENSOR AND THE LAMBDA. LISA CAN NOT WORK.

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## ABSTRACT

Since the formation of the general theory of relativity, there are disputed questions. Levi-Civita and Schrödinger also criticized the non covariant energy-moment pseudotensor. The cosmological constant was known as a beauty flaw in theory. In both case the empty space contents energy. The origin and role of these concepts and quantities are not well known and are the source of many misconceptions. Through a few simple examples, we can see why.

## Content



Einstein and Grossmann defined the equations of gravitational field in 1913 [1] in the following covariant form:

$$G_{\mu\nu} = -\varkappa T_{\mu\nu},$$

$$G_{\mu\nu} = 0$$

Here G stands for the Ricci tensor. The

$$^1/_2\, g_{\mu\nu} T$$

on the right of the first equation is missing here.
Due to various difficulties [1...8] [10], they abandoned general covariance and arrived at the following field equations:

$$\sum_{\alpha\beta\mu} \frac{\partial}{\partial x_\alpha}\left(\sqrt{-g}\,\gamma_{\alpha\beta} g_{\sigma\mu} \frac{\partial \gamma_{\mu\nu}}{\partial x_\beta}\right) = \varkappa (T_{\sigma\nu} + t_{\sigma\nu}),$$

$$T_{\sigma\nu} = \sum_\mu \sqrt{-g}\, g_{\sigma\mu} \Theta_{\mu\nu}, \quad t_{\sigma\nu} = \sum_\mu \sqrt{-g}\, g_{\sigma\mu} \vartheta_{\mu\nu}$$

Here the Greek letters mean contravariant tensors.

This is where $t_{ik}$ the energy-moment pseudotensor of the gravitational field, appears for the first time. [1] This pseudotensor is not generally covariant, is not symmetric, rotation is not included.
(No objection can be raised to this because the equations are not general covariant.)

Field equations as we know them today were invented by Einstein in 25 November 1915. [6]

$$G_{\mu\nu} = -\varkappa \left( T_{\mu\nu} - \frac{1}{2} g_{\mu\nu} T \right).$$

The more common form:

$$R_{ik} - \frac{1}{2} g_{ik} R = \frac{8\pi k}{c^4} T_{ik}$$

The non covariant pseudotensor is not included here.
Divergence of the right side

$$\frac{\partial T_i^k}{\partial x^k} = T_{i,k}^k = 0$$

expresses the conservation of energy of matter when no gravitational field is present. Einstein's pseudotensor with mixed indices:

$$t_\mu{}^\nu = \frac{c^4}{16\pi G\sqrt{-g}} \left( \left( g^{\alpha\beta}\sqrt{-g} \right)_{,\mu} \left( \Gamma^\nu_{\alpha\beta} - \delta^\nu_\beta \Gamma^\sigma_{\alpha\sigma} \right) - \delta^\nu_\mu g^{\alpha\beta} \left( \Gamma^\sigma_{\alpha\beta}\Gamma^\rho_{\sigma\rho} - \Gamma^\rho_{\alpha\sigma}\Gamma^\sigma_{\beta\rho} \right) \sqrt{-g} \right)$$

$$\left( \left( T_\mu{}^\nu + t_\mu{}^\nu \right) \sqrt{-g} \right)_{,\nu} = 0.$$

Einstein's pseudotensor is derived from the left side of the field equations and is not symmetrical. [11] In the case of the Landau-Lifshitz pseudotensor, the basic condition is that in the center of the local inertial system $t_{ik}$ must be equal to 0. [12] The coordinate system is chosen so that the first derivative of $g_{ik}$ according to coordinates is 0. $g_{ik}$ doesn't have to be Galileo like. These conditions lead to the following pseudotensor:

$$\begin{aligned} t^{ik} = \frac{c^4}{16\pi k} \{ & (2\Gamma^n_{lm}\Gamma^p_{np} - \Gamma^n_{lp}\Gamma^p_{mn} - \Gamma^n_{ln}\Gamma^p_{mp})(g^{il}g^{km} - g^{ik}g^{lm}) + \\ & + g^{il}g^{mn}(\Gamma^k_{lp}\Gamma^p_{mn} + \Gamma^k_{mn}\Gamma^p_{lp} - \Gamma^k_{np}\Gamma^p_{lm} - \Gamma^k_{lm}\Gamma^p_{np}) + \\ & + g^{kl}g^{mn}(\Gamma^i_{lp}\Gamma^p_{mn} + \Gamma^i_{mn}\Gamma^p_{lp} - \Gamma^i_{np}\Gamma^p_{lm} - \Gamma^i_{lm}\Gamma^p_{np}) + \\ & + g^{lm}g^{np}(\Gamma^i_{ln}\Gamma^k_{mp} - \Gamma^i_{lm}\Gamma^k_{np})\}, \end{aligned}$$

$$\begin{aligned} (-g)t^{ik} = \frac{c^4}{16\pi k} \Big\{ & \mathfrak{g}^{ik}{}_{,l}\,\mathfrak{g}^{lm}{}_{,m} - \mathfrak{g}^{il}{}_{,l}\,\mathfrak{g}^{km}{}_{,m} + \frac{1}{2} g^{ik}g_{lm}\mathfrak{g}^{ln}{}_{,p}\,\mathfrak{g}^{pm}{}_{,n} - \\ & - (g^{il}g_{mn}\mathfrak{g}^{kn}{}_{,p}\,\mathfrak{g}^{mp}{}_{,l} + g^{kl}g_{mn}\mathfrak{g}^{in}{}_{,p}\,\mathfrak{g}^{mp}{}_{,l}) + g_{lm}g^{np}\mathfrak{g}^{il}{}_{,n}\mathfrak{g}^{km}{}_{,p} + \\ & + \frac{1}{8}(2g^{il}g^{km} - g^{ik}g^{lm})(2g_{np}g_{qr} - g_{pq}g_{nr})\mathfrak{g}^{nr}{}_{,l}\,\mathfrak{g}^{pq}{}_{,m} \Big\}, \end{aligned}$$

The Landau-Lifshitz pseudotensor is symmetrical. Another important feature is that it contains only first-order derivatives of the metric tensor.

In gravitational field the covariant derivative of $T_{ik}$ is

$$T^k_{i;\,k} = \frac{1}{\sqrt{-g}}\,\frac{\partial(T^k_i\sqrt{-g})}{\partial x^k} - \frac{1}{2}\,\frac{\partial g_{kl}}{\partial x^i}\,T^{kl} = 0$$

With the help of the pseudotensor, on the other hand, integral conservation laws can be formulated:

$$P^i = \frac{1}{c}\int(-g)\,(T^{ik}+t^{ik})\,dS_k$$

It determines the resultant quadruple moment of gravitational matter and gravitational field. It is an integral of tensor density, which is not general covariant and not coordinate independent.

The Landau - Lifshitz pseudotensor can be derived from

$$(-g)\,(T^{ik}+t^{ik}) = \frac{\partial b^{ikl}}{\partial x^l}$$

$$(-g)\left\{\frac{c^4}{8\pi k}\left(R^{ik}-\frac{1}{2}\,g^{ik}R\right)+t^{ik}\right\} = \frac{\partial b^{ikl}}{\partial x^l}$$

On the right side the „superpotencial” and its derivate is not covariant. From this it can be seen that the $t_{ik}$ cannot be a covariant tensor. Since

$$\left((T_\mu{}^\nu + t_\mu{}^\nu)\,\sqrt{-g}\right)_{,\nu} = 0.$$

must be valid in all coordinate systems, this is Einstein's, and indeed all, not mentioned here applies to an energy-moment pseudotensor. If $t_{ik}$ were covariant, local conservation theorems would not be fulfilled.

From matter, its gravitational field is inseparable. $T_{ik}$ is localized, regardless of the coordinate system. The Schwartzschild metric describes gravitational field of a mass point. $t_{ik}$ can disappear at any point in gravitational space, but it can also be singular in an unsuitable coordinate system, in Minkowski space as well.

The energy of the gravitational field follows from the metric at a given distribution of matter, with given initial and boundary conditions. There are no universally covariant boundary conditions. In addition, the initial conditions are relate to the central of the gravitational mass, and the boundary conditions for its gravitational field, in vacuum.

$T_{ik}$ means energy density, it does not contain the energy of the gravitational field. At the centre of the spherically symmetric field there is the centre of the gravitational mass, but inside the body $T_{ik}$ is not equal to 0. So beneath the surface of the body we not find the Schartzschild metric as outside, where $T_{ik}$=0.

$t_{ik}$ refers to a matterless space, so, for real bodies, they are already locally separated, defined in a different metric. The two metrics differ from each other even if they are expressed in the same system of coordinates. The sum of the two cannot therefore be general covariant, and only integral conservation theorem can be formulated (i.e. not cordinate-independent).

The point mass of the Schwartzschild metric is idealization. Real bodies have finite volume. Their movement is characterized by the trajectory of their center of gravity. $T_{ik}$ divergence expresses local conservation in the case of a spherically symmetrical body, which is at rest in the frame of reference. In its centre no gravitational "force" is experienced. In the material-free space, $t_{ik}$ can disappear at every point, including at every point on the geodesic, where the centre of gravity of the test body is found. A test body shall not remain at rest in a gravitational field. Motion in geodesic orbit corresponds to inertial motion in special relativity. No energy is needed to maintain it. So the effect of gravitational field will not appear on the position of test body but on its shape.

If it were $t_{ik}$ covariant, $T_{ik}$ would have to contain it.
This is also not possible, because they have different metrics, and because in the gravitational field $T_{ik}=0$, so conservation theorems would also be damaged.
If $t_{ik}$ were generally covariant, it would mean absolute space, violate the equivalence principle, and there would be no relativity. $t_{ik}$ is determined by $g_{ik}$, i.e. a slight difference in test body's mass, does not changes the geodesic orbit. (If the mass of the test body is not significant compared to the gravitating mass in the centre of gravitational field.)

The conservation of matter and its gravitational field together cannot be subject to a general covariant conservation theorem. Acceleration, inertia are also relative, so gravitational energy to. So we need a local conservation theorem.

The energy of the gravitational field can be detected by its effect on matter. The test body moves on the geodesic, on the other hand, implements a local inertia system, according to the equivalence principle. This applies to any geodesic around any gravitational mass. If weight and inertial masses were not identical, stable planetary orbits would not exist. Relativity, equivalence principle and conservation laws are strictly connected.
We cannot talk about gravitational energy without its source. Consider the following example: In a centrally symmetrical, inhomogeneous gravitational field, two test bodies at the same height fall in free fall. Both are moving towards the center, giving the impression that the reason for their rapprochement is the attraction acting between them, which also does not depend on their mass. Let us now consider the case when, in the same gravitational field, two bodies move at a given distance from each other and fall freely, within the same radius. The distance between them is now growing. From this it may seem that a repulsive force arises between them.

Obviously, this cannot be caused by the energy of the gravitational field. In a falling cabin or on a space station, the thread pendulum does not swing. Known gravity must not be contributed by this additional force. If, in addition to gravity, force appears, the movement will not be geodesic. Such a force can not be covariant. The centrally symmetric force field is described by the Schwartzschild metric, the space-time of a falling cabin is Minkowski's.

The center of the orbit of a satellite orbiting the Earth is the center of the Earth. The center of mass of the Earth-Moon binary system is below the Earth's surface. Members of a binary star orbit around a point where there is no matter whatsoever, only their common gravitational field. This common gravitational field cannot be separated due to the nonlinearity of the field equations. This also speaks against localizability.

The space of a test body, moving on a geodesic is Minkowski-like. Assume a circular orbit. In a closed cabin, on the one hand, we cannot tell that we are in a gravity-free space, we fall vertically in free fall or orbit in a circular orbit. If all we know is that we are circulating, we cannot tell where, at what distance, in which direction is the center of gravity. We cannot even locate the source of the gravitational field. The quantities of a gravitational body and its gravitational field at a given point in space they cannot be localized.

This is also due to the fact that $t_{ik}$ cannot be localized. In an orbiting local system, we do not experience gravitational force. Its presence can only be determined by the tidal forces, which act not in one point, but in a finite volume, - causing deformation on a rigid body and shear in it –, which we can only get by integrating. A gyroscope can detect it, but this tool is not point like, and the detection is not possible in one point, but only moving on the geodesic.

About the „uniqueness” of the energy-moment pseudotensor: uniqueness means general covariance, what is not satisfied, and can not be.
From the field equations itself Einstein derived the motion equations, what are covariant only with linear transformations, same as the pseudotensors. Different type of gravitation field lead to different motion of bodies. In the same way, they represent different gravitation energies.
One can transform the Schwartzschild metric to the Kerr metric.
Between 2 different orbit, exists a transport. Exists transformation between arbitrary accelerating reference frames, it was the goal of the general relativity. Different pseudotensors can be transformed into each other to.

General relativity is consistent with the energy-moment pseudotensor of the gravitational field.

***** LISA can not work *****

If $R_{ik}$ is zero, the metric may be curved because the Weyl tensor, $C_{iklm}$ may differ from zero.

Tidal forces do not act on a single point, but in a finite volume, causing deformation. They do not change the test body's position. The extent of the local inertia system is determined by $t_{ik}$. Gravitational wave energy is also can be calculated from $t_{ik}$, and in empty space - where $T_{ik}=0$ - it is also not localizable. When encountered with a material body, they cause periodic deformation, thanks to which they can be detected. However, deformation cannot be determined independently of the cordinates. However, they do not have this effect on the empty space between bodies. Two bodies motionless relative to each other does not change its distance. Therefore, Feynmann's sticky bead argument is not valid. The wave acts on the rods, affecting their size. Since they do not form a rigid body, the beads react not by slipping, but also by deformation. Their position changes with the elongation and contraction of the rods only.

Consider a ball of dust falling freely in a central, inhomogeneous gravitational field. There is no interaction between its particles. With a greater radial distance comes a slight acceleration, and all particles move towards the center. Therefore, the sphere turns into an ellipsoid. Under the same conditions, the deformation of a rigid body becomes much smaller, and tension arises inside. When several extensive bodies fall together, they all behave in the same way, and their distances vary in the same way as particles in a powdery sphere.

Effect of gravitation waves is tension, not „gravitational push-pull”.
Between 2 separated bodies there can not be attractive – repulsive „force”.
LISA cannot operate because it is intended to be implemented with 3 unconnected satellites, so they do not form a rigid body.

## ***** THE COSMOLOGICAL CONSTANT *****

Lambda was first introduced into Riemann geometry by Helmholtz. [13] Based on the existence of rigid bodies, he believed that the geometry shall include that they can be freely shifted and rotated in space. The Riemann tensor then looks like this:

$$R_{ijkm} = \lambda(g_{ik}g_{jm} - g_{im}g_{jk})$$

This means a homogeneous space. Later, Clifford thought that matter could be thought of as the creasing or ripple of space on a flat plane. Einstein, after finding the field equations, was looking for a cosmological solution to them, whether they were suitable for describing the Universe as a whole. In 1916, the expansion of the Universe was not yet known. So he had to find a static solution. The very first solutions were dynamic ones. Another problem arises: general covariant boundary conditions cannot exist. Both problems were solved by Lambda, which he called cosmological constant. The form of the field equations extended specifically for this purpose is:

$$(R_{ik} - \tfrac{1}{2}g_{ik}R) + \Lambda g_{ik} + \kappa T_{ik} = 0$$

The metric we are looking for, which describes a homogeneous, isotropic, static space: [11]

$$ds^2 = \frac{\varrho^2}{\varrho^2 - r^2}\,dr^2 + r^2\,d\vartheta^2 + r^2\sin^2\vartheta\,d\varphi^2 - (dx^4)^2.$$

Eddington proved that this solution is unstable. If thickening occurs anywhere, the whole system begins to expand. In 1921, Friedmann, and independently Lemaitre, found a solution describing an expanding Universe. They determined the speed and acceleration of matter. These equations still include Lambda:

$$\left(\frac{\dot{a}}{a}\right)^2 + \frac{kc^2}{a^2} - \frac{\Lambda c^2}{3} = \frac{8\pi G}{3}\rho$$

$$2\frac{\ddot{a}}{a} + \left(\frac{\dot{a}}{a}\right)^2 + \frac{kc^2}{a^2} - \Lambda c^2 = -\frac{8\pi G}{c^2}p.$$

However, in 1930 it was rejected by both Einstein and Friedmann. The main reason for this is that Hubble published the results of his observations, the expansion of the Universe. Robertson and Walker developed their cosmological model without Lambda.

$$ds^2 = R(t)^2\left[dt^2 - \frac{dr^2}{1-kr^2} - r^2\left(d\theta^2 + \sin^2\theta\,d\varphi^2\right)\right]$$

Let us consider in more detail what exactly the role of Lambda was played and why it was removed: [9]

motion of the stars. One can arrange it so that the mean velocity of matter relative to this system shall vanish in all directions. There remain the (almost random) motions of the individual stars, similar to the motions of the molecules of a gas. It is essential that the velocities of the stars are known by experience to be very small as compared to the velocity of light. It is therefore feasible for the moment to neglect this relative motion completely, and to consider the stars replaced by material dust without (random) motion of the particles against each other.

The above conditions are by no means sufficient to make the problem a definite one. The simplest and most radical specialization would be the condition: The (naturally measured) density, $\rho$ of matter is the same everywhere in (four-dimensional) space, the metric is, for a suitable choice of coordinates, independent of $x_4$ and homogeneous and isotropic with respect to $x_1$, $x_2$, $x_3$.

The goal, then, is to describe a Universe whose matter is dusty, static, constant in time, spatially homogeneous and isotropic.

and isotropic with respect to $x_1$, $x_2$, $x_3$.

It is this case which I at first considered the most natural idealized description of physical space in the large; it is treated on pages 103–108 of this book. The objection to this solution is that one has to introduce a negative pressure, for which there exists no physical justification. In order to make that solution possible I originally introduced a new member into the equation instead of the above mentioned pressure, which is permissible from the point of view of relativity. The equations of gravitation thus enlarged were:

$$(1) \qquad (R_{ik} - \tfrac{1}{2}g_{ik}R) + \Lambda g_{ik} + \kappa T_{ik} = 0$$

where $\Lambda$ is a universal constant ("cosmologic constant"). The introduction of this second member constitutes a complication of the theory, which seriously reduces its logical simplicity. Its introduction can only be justified

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Einstein, on the other hand, did not consider the introduction of negative pressure justifiable. Instead, it changed the geometry for that material distribution. Lambda, on the other hand, can only be constant, partly because of static and partly because by the Bianchian identities this is required. In Friedmann's second cosmological model, space coordinates are not independent of time. Space is isotropic, but it is no longer static and density is not constant.

by the difficulty produced by the almost unavoidable introduction of a finite average density of matter. We may remark, by the way, that in Newton's theory there exists the same difficulty.

The mathematician Friedman found a way out of this dilemma.* His result then found a surprising confirmation by Hubble's discovery of the expansion of the stellar system (a red shift of the spectral lines which increases uniformly with distance). The following is essentially nothing but an exposition of Friedman's idea:

### FOUR-DIMENSIONAL SPACE WHICH IS ISOTROPIC WITH RESPECT TO THREE DIMENSIONS

We observe that the systems of stars, as seen by us, are spaced with approximately the same density in all directions. Thereby we are moved to the assumption that the *spatial* isotropy of the system would hold for all observers, for every place and every time of an observer who is at rest as compared with surrounding matter. On the other hand we no longer make the assumption that the average density of matter, for an observer who is at rest relative to neighboring matter, is constant with respect to time. With this we drop the assumption that the expression of the metric field is independent of time.

to neighboring matter, is constant with respect to time. With this we drop the assumption that the expression of the metric field is independent of time.

We now have to find a mathematical form for the condition that the universe, *spatially speaking*, is isotropic everywhere. Through every point $P$ of (four-dimensional) space there is the path of a particle (which in the following will be called "geodesic" for short). Let $P$ and $Q$ be two

* He showed that it is possible, according to the field equations, to have a finite density in the whole (three-dimensional) space, without enlarging these field equations *ad hoc*. Zeitschr. f. Phys. 10 (1922).

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In this model, Lambda is no longer included:

We can choose the coordinate system $(x_1, x_2, x_3)$ so that the line element becomes conformally Euclidean:

$$\text{(2d)} \qquad d\underset{0}{\sigma}^2 = A^2(dx_1^2 + dx_2^2 + dx_3^2) \text{ i.e. } \underset{0}{\gamma_{ik}} = A^2\delta_{ik}$$

where $A$ shall be a positive function of $r(r = x_1^2 + x_2^2 + x_3^2)$ alone. By substitution into the equations, we get for $A$ the two equations:

$$\text{(3)} \qquad \begin{cases} -\frac{1}{r}\left(\frac{A'}{Ar}\right)' + \left(\frac{A'}{Ar}\right)^2 = 0 \\ -\frac{2A'}{Ar} - \left(\frac{A'}{A}\right)^2 - BA^2 = 0 \end{cases}$$

The first equation is satisfied by:

$$\text{(3a)} \qquad A = \frac{c_1}{c_2 + c_3 r^2}$$

(3a) $$A = \frac{c_1}{c_2 + c_3 r^2}$$

where the constants are arbitrary for the time being. The second equation then yields:

(3b) $$B = 4\,\frac{c_2 c_3}{c_1^{\,2}}$$

About the constants $c$ we get the following: If for $r = 0$, $A$ shall be positive, then $c_1$ and $c_2$ must have the same sign. Since a change of sign of all three constants does not change $A$, we can make $c_1$ and $c_2$ both positive. We can also make $c_2$ equal to 1. Furthermore, since a positive factor



can always be incorporated into the $G^2$, we can also make $c_1$ equal to 1 without loss of generality. Hence we can set:

(3c) $$A = \frac{1}{1 + cr^2};\ B = 4c$$

There are now three cases:

$c > 0$ (spherical space)
$c < 0$ (pseudospherical space)
$c = 0$ (Euclidean space)

By a similarity transformation of coordinates ($x_i' = ax_i$, where $a$ is constant), we can further get in the first case $c = \frac{1}{4}$, in the second case $c = -\frac{1}{4}$.

*Summary.* The hypothesis of *spatial* isotropy of our idealized universe leads to the metric:



APPENDIX FOR THE SECOND EDITION

(2) $$ds^2 = dx_4^2 - G^2A^2(dx_1^2 + dx_2^2 + dx_3^2)$$

where $G$ depends on $x_4$ alone, $A$ on $r$ ($= x_1^2 + x_2^2 + x_3^2$) alone, where:

(3) $$A = \frac{1}{1 + \frac{z}{4} r^2}$$

We must now further satisfy the field equations of gravitation, that is to say the field equations without the "cosmologic member" which had been introduced previously *ad hoc:*

(4) $$(R_{ik} - \tfrac{1}{2}g_{ik}R) + \kappa T_{ik} = 0$$

By substitution of the expression for the metric, which was based on the assumption of spatial isotropy, we get after calculation:

$$R_{ik} - \frac{1}{2} g_{ik}R = \left(\frac{z}{G^2} + \frac{G'^2}{G^2} + 2\frac{G''}{G}\right) GA\delta_{ik} \qquad (i, k = 1, 2, 3)$$

Eventually:

SOLUTION OF THE EQUATIONS IN THE CASE OF NON-VANISHING SPATIAL CURVATURE

If one considers a spatial curvature of the spatial section ($x_4$ = const), one has the equations:

(5) $$zG^{-2} + \left(2\frac{G''}{G} + \left(\frac{G'}{G}\right)^2\right) = 0$$
$$zG^{-2} + \left(\frac{G'}{G}\right)^2 - \frac{1}{3}\kappa\rho = 0$$

The curvature is positive for $z = +1$, negative for $z = -1$.

The equation without Lambda allows both positive and negative curvature. (Citations are from Einstein’s book, The Meaning of Relativity, [9] )

The following figure shows the speed at which galaxies move away from each other over time.

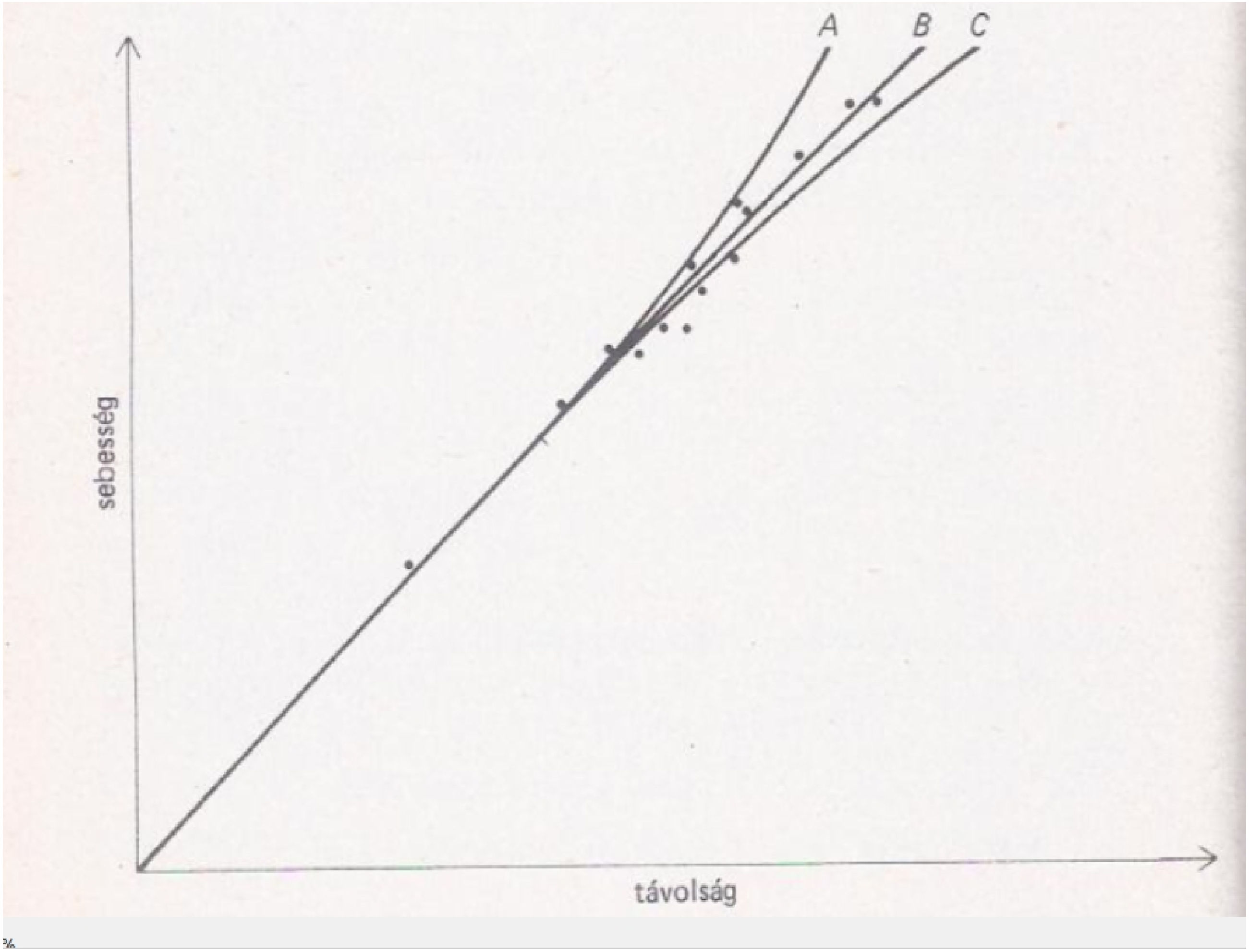
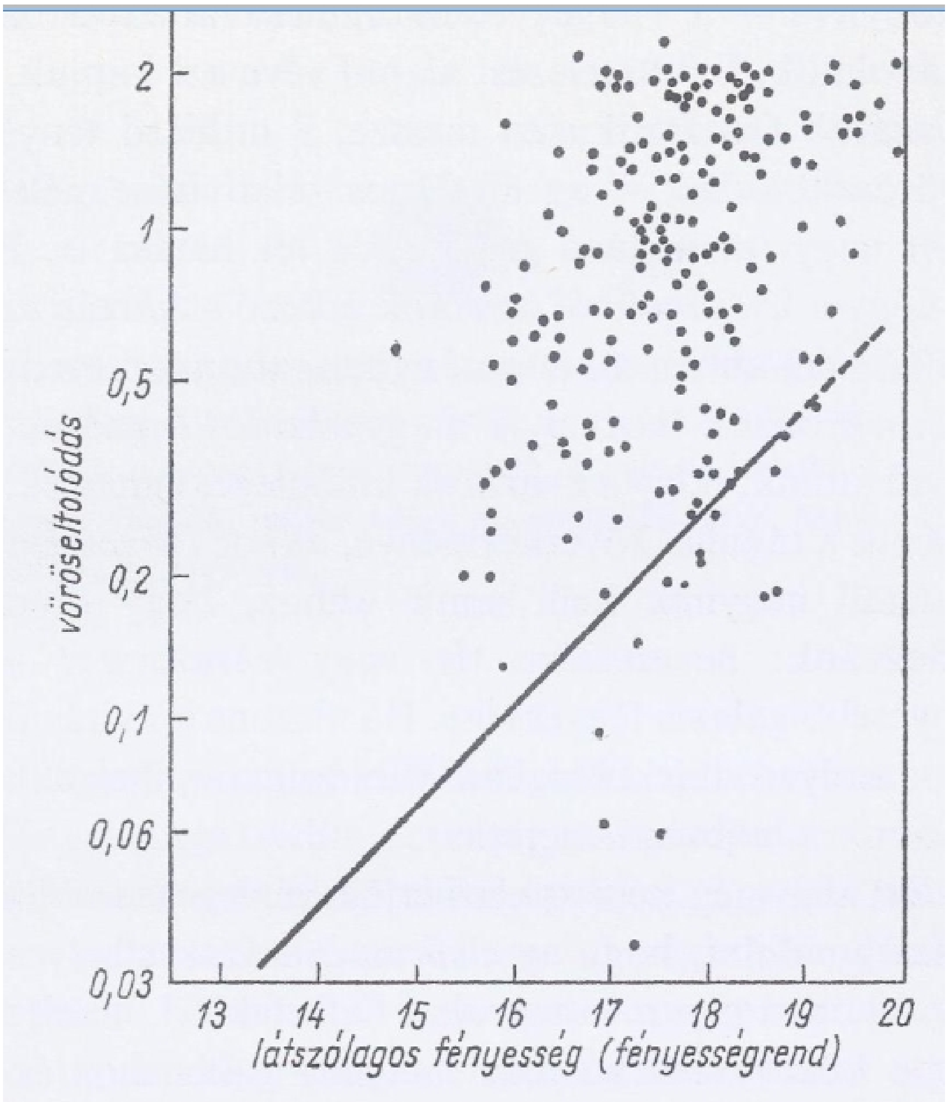



Image by Kauffmann [14] from his book, 1977. At this time, no one used Lambda. Since 1930, every physicist and astronomer has known, that the expansion of the Universe can be uniform, slowing down, or accelerating. Previous observations have already shown that that most likely a slight acceleration.

This figure (Omnes) [15] shows possible ways in which the Universe expands, as astronomers already knew in 1973.
Here we can see the redshift of quasars, depending on the apparent brightness:




The Robertson-Walker metric in 1931 also did not include Lambda. The assumption of dark energy accelerating the movement of galaxies is therefore not justified.

Therefore, the two equations below are not really equations. The two sides cannot be equal.

$$G_{\mu\nu} = 8\pi G(T_{\mu\nu} + \rho_\Lambda g_{\mu\nu})$$

$$G_{\mu\nu} = 8\pi G(T_{\mu\nu} - \bar{\rho}_{DE} g_{\mu\nu})$$

The first error that stands out is the line over density. Averaging is not a covariant operation. It is probably included to eliminate the inhomogeneous distribution of the substance. It is no coincidence that Friedmann rejected Lambda along with homogeneity. Both versions have the other error, since

$$G_{\alpha\beta} = R_{\alpha\beta} - \frac{1}{2}Rg_{\alpha\beta}$$

So the left side of this equation,

$$R_{ik} - \frac{1}{2}g_{ik}R = \frac{8\pi k}{c^4}T_{ik}$$

and the right side of this was used,

$$R_{ik} = \frac{8\pi k}{c^4}\left(T_{ik} - \frac{1}{2}g_{ik}T\right)$$

slightly transformed, so, like this:

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = -\kappa\left(T_{\mu\nu} + T_{\mu\nu}^{\text{vac}}\right)$$

It is obviously wrong.

Einstein's static solution was born from:

$$(R_{ik} - \tfrac{1}{2}g_{ik}R) + \Lambda g_{ik} + \kappa T_{ik} = 0$$

At her

$$T_r^r = \mu^0 u_r u^r = -\mu_0 c^2. \qquad \lambda = -\frac{1}{2}\varkappa\mu_0 c^2.$$

Thus, if

$$\rho_\Lambda g_{\mu\nu}$$

equals to

$$^1/_2\, g_{\mu\nu} T$$

apart from a multiplier of 1/2, We could get here:

$$R_{ik} = \frac{8\pi k}{c^4}\left(T_{ik} - \frac{1}{2} g_{ik} T\right)$$

However, since

$$R_{\mu\nu} - \Lambda g_{\mu\nu} = \kappa\left(T_{\mu\nu} - \frac{1}{2} T\, g_{\mu\nu}\right)$$

come true,

$$G_{\mu\nu} = 8\pi G(T_{\mu\nu} - \bar{\rho}_{DE}\, g_{\mu\nu}),$$

what is equal to

$$R_{\mu\nu} - \tfrac{1}{2} g_{\mu\nu} R = -\kappa\left(T_{\mu\nu} + T_{\mu\nu}^{\mathrm{vac}}\right),$$

cannot be fulfilled.

It can stand only on the left and be only constant.

$$\Lambda g_{ik}$$

In Einstein's cosmological solution the energy-moment tensor is equal to $T_{00}$, i.e. the energy-moment tensor only 1 of its elements differs from zero, the pressure is zero. This modification is suitable only for describing a homogeneous Universe, what has no structure.

$$(R_{ik} - \tfrac{1}{2}g_{ik}R) + \Lambda g_{ik} + \kappa T_{ik} = 0$$

This one

$$G_{\mu\nu} = 8\pi G(T_{\mu\nu} - \bar{\rho}_{DE}g_{\mu\nu})$$

is not suitable for describing anything.

To the right, the "energy density of vacuum" for the same reason can not be added to $T_{ik}$, like the pseudotensor. It would also play a role in space outside of material bodies, but the idea is that it would be repulsive. Such a force has no place in field equations. It cannot be covariant and does it violate conservation laws.

The cosmological principle: 1. no privileged place, 2. no privileged direction 3. distribution of the substance is homogeneous, refers to 3-dimensional space, not 4-dimensional spacetime. Thus, to the right of the field equations there cannot be a quantity associated with the acceleration of expansion next to $T_{ik}$ from the outset, and which cannot therefore be covariant. Moreover, the Lambda means a strong restriction for the possible metric. (In a homogen isotropic space can not be electromagnetic radiation neither gravitational vawes.)

Thus, the acceleration of the expansion of the Universe did not first arise in 1999, but it was detected for the first time that the rate of expansion is slowing down first, at a distance of about 5-7 billion light years. The acceleration occurs at a distance of 8-10 billion light years. The expansion that slows down first and accelerates later cannot be explained by the cosmological constant since its value must be strictly constant. Moreover, the 1930s Friedmann and Robertson and Walker solutions made the matter of the Universe powdery, it was considered pressureless, just like Einstein. So this idea is obviously wrong:

$$T = \begin{pmatrix} \Lambda & 0 & 0 & 0 \\ 0 & -\Lambda & 0 & 0 \\ 0 & 0 & -\Lambda & 0 \\ 0 & 0 & 0 & -\Lambda \end{pmatrix}$$

## ***** Inflationary Expansion of the Early Universe *****

Since the Lambda’s value must be constant, nor can it induce the supposed early, exponentially accelerating expansion of the Universe. The argument that it is not the motion of matter that exceeds the speed of light, but that empty space expands, cannot be accepted. As already mentioned, Lambda can only be constant. Its role was originally to compensate for the expansion of the Universe.

Obviously, it cannot work in the opposite sense. On the other hand, the gravitational field cannot be separated from gravitational matter. Thus, this argument cannot be correct. Finally, gravitational field has inertia in the same way as matter, so empty space cannot expand faster than the speed of light.

## ***** SCHWARTZSCHILD SINGULARITY *****

„On The Gravitational Field Of A Mass Point According To Einstein’s Theory”. This is the title of Karl Schwartzschild's paper, which was the first exact solution of field equations. In vacuum $T_{ik}=0$, so the equation is

$$R_{ik} = 0$$

for the entire space – except for the mass point. So we are looking for the gravitational field of a naked singularity… The gravitation field is spherically symmetrical. For empty space, the metric really gives a good description.
The parameter "m", which appears in the metric, represents mass, it is not predetermined. This means:  the metric is mass-independent, whith maintaining symmetry. On the other hand, density is infinite, regardless of mass.
Since the metric does not depend on mass, it remains valid up to infinitesimal size. General relativity is scale-independent.
If in the centre $T_{ik}=T_{00}$=infinite, the boundary condition, Minkowski-like metric in the infinite can not be fulfilled. In a realistic case the volume and density is finite.
Looking at the Earth's interior at any volume, we do not find infinite density, and in zero volume there is zero mass.
After gravitational collapse of a large star, the size of the black hole is determined by the Schwartzschild radius. Here it takes an extreme value of $g_{00}$ and $g_{11}$. According to the popular view, this means only the singularity of coordinates. In the same way as at the North and South Poles, where meridians of longitude meet. Such a singularity can be eliminated by coordinate transformation, they say.
We can shift the North Pole to Nairobi or Puerto Vallarta, but the Earth's axis of rotation still meets the surface in the same place, the equator will be on the same place. The position of the poles is not only geometric arbitrariness, but has a physical cause. According to the same interpretation, a true singularity is in the center with infinite density. So nothing happened, it was before the collapse, and all stars and planets are actually naked singularities. In a singularity the Ricci scalar and the Kretschman scalar are equal to infinite.  But by the Schwartzschild metric, it's equally infinite in the middle of a ping-pong ball. In real we know, by the Birkhoff theoreme, inside of an empty sphere there is not gravitational force.

The problem if we count the value of the scalar

$$K = \frac{48G^2M^2}{c^4r^6}$$

through the Schartzschild metric. Under the surface of the black hole $T_{ik}$ is not equal to zero. This metric is not valid under the surface of a real body.
The limit is the Schwatzschild radius. There must be matter. We can not extend the Schwartzschild metric under the surface of the Earth by any kind of transformation.
At the event horizon $g_{11}$ is infinite. Moreover, at the event horizon the speed of light is 0. Einstein understood at the age of 16, there is not a comoving reference frame with the light. So, this surface is not part of the Schwartzschild metric. The Kruskal-Szekeres extension is not applicable in a vacuum and below the surface of the Earth, in two completely different metrics. We can use a spherical system of coordinates in Euclidean space. Then at the poles there is singularity of coordinates. On the surface of the Earth the „singularity" does not mean infinte density, but has physical meaning. Coordinate values have not meaning, but the singularity appears in the value of metric tensor in the Schwartzschild metric. Zero and infinite values can not be transformed into finite quantities. Singularity of metric does not mean infinite density. If $T_{ik}=0$, the metric can not relate to density of matter. Infinite density could appear only if $T_{ii}$ equal to infinite, what can not be valid.
If we extend the Schwartzschild metric to the centre, under the gravitational radius or under the surface of a real body, the energy will be described by $t_{ik.}$ But in case of a real star or planet, under the surface the energy is described by the $T_{ik}$. Obviously, on the same place can not be both.
We cannot assume that during the gravitational collapse of a star, its matter moves as if it were in Schwartzschild space. The pressure cannot be ignored, it has weight as well. The energy-moment tensor we can not consider as $T_{00}$. The rest energy is equal to trace of $T_{ik,}$ the Laue scalar. In the centre of a gravitational mass described by nonzero $T_{ik,}$ and the metric is not singular there. [16] Arbitrary choice of coordinates does not mean we can ignore the presence of matter, can be valid only if the initial and boundary conditions are regarded.

The Schwartzschild metric is an exact solution, and exact descrition of the spherical symmetric gravitational field, because in its centre there is NOT singularity. The limit of contraction can therefore be $r_g$, without contradiction.

## ***** MACH PRINCIPLE *****

In the General Theory of Relativity the Lambda expresses the Mach principle. [11] With the introduction of Lambda, no additional boundary condition is required in the static homogen cosmological solution. As it was mentioned before, this model is unstable, and is not a good description of the Universe. So Friedmann and Einstein rejected the Lambda.

Einstein was influenced by Mach and by his principle, what is a response to Newton's bucket experiment. Mach criticized Newton's idea of absolute space as an absolute frame of reference against which bodies move. According to Newton, if you rotate a bucket of water in empty space, the surface of the water will become concave. But if there is only a bucket of water, what do we compare rotation to? The frame of reference would be a rigid, infinite, absolute coordinate system. He himself knew that his theory was flawed on this point: if the Universe is finite, gravity would cause matter to compress. And if it were infinite, we would experience infinite potential everywhere. Mach's solution was to relate all motion, including rotation, to the system of stars, which would also explain inertial mass. Mach himself said, the effect is the same when the stars rotate around the stationary bucket.

It can not be good. This would mean absolute rotation, and that the bucket is the center of the Universe.
A rotating system cannot be extended to arbitraty distance, the peripheral speed would exceed the speed of light. On the other hand, there is no equivalent distribution of matter to a rotating system. Stars can be landmarks in the short term, but inertial mass cannot be explained by the physical effects of stars. They would also be suitable for designating coordinate systems only if the Universe were homogeneous and static, but it would mean an absolute reference frame again. Obviously, neither Mach nor Einstein knew about the expansion of the Universe in 1907. If we imagine 2 rotating bodies with parallel axes of rotation but rotating in opposite directions, or if the axes of rotation are perpendicular to each other, it is obvious that Mach's idea cannot work.

Newton's thought experiment, by the way, was wrong. In a stationary bucket, the surface of the water is really flat, and in a rotating bucket it is concave - on the surface of the Earth, in its gravitational field.
In a space free of gravity, water takes on a spherical shape, not following the shape of the bucket. Even if you turn the bucket, the water is not obliged to follow it.

The Mach principle here means that for the bucket to rotate, it is necessary to have at least one distant body, which 1: or motionless relative to it, 2: orbits around it. The two cases cannot be distinguished. Furthermore, the rotation of the bucket, the inertial mass, is not affected by the distribution and distance of the stars or if those approache or move away from it. Also, the speed of rotation does not depend on the possible own rotation of the distant object.

Einstein had another thought experiment: There are 2 liquid spheres, one spherical, the other ellipsoid. Presumably, the ellipsoid rotates. Motion, on the other hand, is relative, so the other fluid sphere could also be an ellipsoid. Now let's not take into account that the rotation must have been created by some effect. We relate the movements of bodies to each other, but this is not a point-like, idealized case. The spherical shape is center-symmetrical, the ellipsoid is axisymmetrical. This is already shown by the form.

The fact that the weight and inert masses are identical also means that the two have the same source. The Sun and Moon do not circle the Earth daily. The Earth's rotation around its axis can be detected without an external reference point by the Coriolis force. It is enough to pull the plug out of the bathtub. (Water should be in it.)
In case of rotation, a prominent direction, the axis of rotation, and the plane of rotation perpendicular to it appears. It selects the rotation body. Points only in the plane of rotation are rotating around the centre of mass, so pressure and shear must be regarded.
If 2 satellit is orbiting around the Earth, at an altitude of 500 and 36.000 km, the weighty and inertial mass can differ significantly - in the frame of reference of the Earth. In the local systems of satellites, on the other hand, we experience weightlessness. From this it can be seen that inertia is relative, and cannot be related to the masses of stars or their effects.

The $T_{ik}$ contains the rest energy, moment, pressure and shear. In the gravitational field of the rotating body, the Thirring-Lense effect occurs. Thus, distant stars can designate a temporal coordinate system. In this case the distance of the stars is irrelevant, because the coordinates have no physical meaning or effect, so we cannot look for the source of inertial mass in this, only in the matter itself.
When the Moon is orbiting the Earth, between them the attracting force depends on product of these masses. If the inert mass of Moon would depend on far stars, equivalence principle would not be satisfied, would not be geodesic orbit. $T_{ik}$ is localized and includes the weight and inertial mass,

these are not separated. Equality of $T_{0i}$ and $T_{i0}$ express the equivalence principle. [13] On the other hand, conservation laws of weight and inert mass are valid locally, both are relative.

Abraham proved in 1914 by simple calculation that if we attribute the inertial mass of the Sun to the influence of external mass, there should be 1 million solar masses near it. [17]

Einstein eventually rejected the Mach principle, even before Lambda.

## ***** Mathematical problems in the development of General Relativity *****

The equations of the empty space

$$R_{ik} = 0$$

are satisfied by

$$\gamma_{ik} = \varphi_{i,\,k} + \varphi_{k,\,i}$$

however we choose the value of $\varphi_i$. ( gamma here is the metric tensor.) [13]

Thats why Einstein thought, equations of the gravitational space can not be generally covariant, because in such a theory, the principle of causality would not apply either. With 4 independent functios, the metric would not be determined by the energy-moment tensor. However, the new theory had to be generally relativistic, consistent with the equivalecia principle, and ensure the validity of conservation theorems. Abandoning covariance was a 2-year misstep. In March 1914, in a letter to Besso, the "laid-down" (angepasste) covariance first appeared.

$$\sum_{\alpha\beta\mu} \frac{\partial}{\partial x_\alpha}\left(\sqrt{-g}\;\gamma_{\alpha\beta} g_{\sigma\mu} \frac{\partial \gamma_{\mu\nu}}{\partial x_\beta}\right) = k(T_{\sigma\nu} + t_{\sigma\nu})$$

$$\sum_{\alpha\beta\mu\nu} \frac{\partial}{\partial x_\nu}\,\frac{\partial}{\partial x_\alpha}\left(\sqrt{-g}\;\gamma_{\alpha\beta} g_{\sigma\mu} \frac{\partial \gamma_{\mu\nu}}{\partial x_\beta}\right) = 0.$$

These "fitted" transformations can be reduced to linear, so they do not include accelerating transformations and contradict the equivalence principle. Thus, on November 4, in his lecture at the Prussian Academy, he already returned to general covariance. The next problem was the unimodularity. According to this, if the determinant of $g_{ik}$ is equal to -1, the equations can only be covariant if the trace of $T_{ik}$ is equal to 0. This would mean that the structure of the material is electromagnetic in nature. Einstein renounced this assumption in a lecture on 18 November, and on 25 November he presented the form of field equations we know today.

$$R_{\mu\nu} = \frac{8\pi G}{c^4}\left(T_{\mu\nu} - \frac{1}{2} g_{\mu\nu} T\right)$$

These equations do not require $T_{ik}$ to be traceless, only that the frame of reference can be chosen so that the determinant of the metric tensor is -1.

The freely selectable function therefore means only that we can determine the gravitational field independently of the coordinates. The fact that coordinates have no physical meaning was unusual for Einstein, but it was essentially the original objective.

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