Cosmological Structure Formation and Fractal Spacetime

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Abstract

Matter structures in cosmology include large-scale objects such as galaxies, galaxy clusters and Dark Matter halos. It is widely accepted that the formation of cosmic structures in the early Universe follows from the gravitational collapse of density perturbations. Here we argue that the genesis of cosmic structures is tied to the fractal topology of spacetime near the Big Bang singularity.

Key words: primordial cosmology, structure formation, Press Schecter formalism, dimensional condensation, fractal spacetime.

A key concept in cosmology is the *density contrast* defined as deviation of the local density from the mean density of the Universe [1–3],

$$\delta(\vec{x},t)=\frac{\Delta\rho}{\rho_m}=\frac{\rho(\vec{x},t)-\rho_m}{\rho_m} \tag{1}$$

The time evolution of (1) resembles the equation of a damped harmonic oscillator, that is,

$$\ddot{\delta}+2H\dot{\delta}-4\pi G\rho\delta=0 \tag{2}$$

where G and H are the Newton and Hubble constants, respectively. To account for the presence of *random fluctuations* in the primordial Universe, we add to (2) an internal perturbation assumed to take the form,

$$\eta(t)=\eta_0\delta-\eta(t) \tag{3}$$

The perturbation (3) consists of a constant amplitude term η_0 and a time-dependent noise encoded in $\eta(t)$. Equation (2) turns into

$$\ddot{\delta}+2H\dot{\delta}+\mu^2\delta=\eta(t) \tag{4}$$

in which the square of the dimensionless mass parameter is

$$\mu^2=\eta_0-4\pi G\rho>0 \tag{5}$$

A reasonable hypothesis is that (3) represents the sum of large periodic excitations, and, as a result, the contribution of the Hubble constant to the evolution of (1) may be considered negligible. Under these assumptions, the oscillator model (4) reflects the universal route to Hamiltonian chaos and fractal spacetime described in [4-5]. Stated differently, (4) is *on par* with the transition to chaos driven by *either* curvature fluctuations *or* the nonintegrable dynamics of particle interactions in the early Universe.

The model (4) belongs to the general class of *open systems* in which the oscillator under study undergoes both internal and external fluctuations, the latter being caused by coupling to an ever-changing environment. Along these lines of thought, refer to [6-7] and consider the overall Hamiltonian of the system presented as

$$H = H_0 + H_e + H_{\text{int}} \tag{6}$$

Following [7], the first term is the contribution of density perturbations δ, the second term comes from environmental fluctuations while the last term embodies the interaction between density perturbations and the environment as in,

$$H_0 = \frac{1}{2}(\dot{\delta}^2 + \mu^2 \delta^2) \tag{7a}$$

$$H_e = \frac{1}{2}(p^2 + \omega^2 q^2) \tag{7b}$$

$$H_{\text{int}} = -g\,(\delta \cdot q) + \frac{\delta^2 g^2}{2\omega^2} \tag{7c}$$

Here, p and q denote the environment momentum and field variables, g plays the role of a coupling parameter and ω is the circular frequency of environmental fluctuations. In general, the system (6) is characterized by *nonlocal (memory) effects*, with the memory kernel given by [7]

$$\mathrm{M}(t) = (\frac{g}{\omega})^2 \cos(\omega t) \tag{8}$$

It is known that many models in nonlinear physics and complex phenomena exhibit power-law density functions, such as (but not limited to) $1/f$ noise, Lévy flights, multifractal processes, fractional Brownian motion and self-organized criticality. In all these instances, the power spectrum obeys the generic scaling law,

$$P(\omega) \propto \omega^{\beta}, \quad 0 < \beta < 1 \tag{9}$$

and the corresponding memory kernel assumes the form,

$$\mathrm{M}(t) \propto t^{-\beta} \tag{10}$$

Note that, since (9) and (10) are *self-similar* functions, one can conveniently rescale the dimensionless time $t' = t^{\kappa}$ (with κ an arbitrary real number), and obtain a rescaled range for the continuous exponent β.

Relations (9) and (10) imply that, at some point in the cosmic evolution, the underlying four-dimensional spacetime of density perturbations acquires a *minimal fractal structure*. The most straightforward characterization of such structure can be done using the *continuous dimensional deviation,* which is defined as,

$$\varepsilon = 4 - D << 1 \tag{11}$$

With reference to the Press-Schecter (PS) theory of structure formation detailed in the Appendix and on account of (9) - (11), it makes sense to posit that the spectral index entering (A2) and (A3) becomes *dependent* on ε, that is,

$$n \Rightarrow n(\varepsilon) \tag{12}$$

The spectral index of (A3) can be accordingly parameterized as

$$\bar{n}(\varepsilon) = [\frac{3+n(\varepsilon)}{6}]^{-1} \tag{13}$$

and (A3) leads to

$$\boxed{\bar{n}(\varepsilon)\log(t_c) = \log(\frac{M}{M^*})} \tag{14}$$

This is our main result. It gives rise to several intriguing conclusions, as we now review below,

1) To comply with (14) within a given cosmic epoch, a shift in the continuous spacetime dimension ε *necessarily constrains* the shift in the magnitude of

M/M^*. This finding, along with the hierarchical clustering of cosmic structures, replicates the putative mass generation mechanism of particle physics via Dimensional Regularization [8-9]. On this basis, one can argue that an unforseen *duality* exists between density perturbations of mass M and the fractality of the spacetime background.

2) In light of (14), *gravitational formation of bound structures* at the critical density perturbations δ_c and critical time t_c acts as dual to *topological condensation* of fractal dimensions.

3) Gravitational phenomena near the Big Bang singularity and the horizon of massive Black Holes appear to be tied to spacetime fractality well above the Fermi scale of electroweak interactions. In turn, this result implies that, as large-scale condensate of spacetime dimensions, Dark Matter shares properties common to gravitation and particle physics.

4) A glance at Figs 1 and 2 below reveals a striking similarity between (A4) and the probability distribution of avalanche sizes in *self-organized criticality*, as applied to the sandpile paradigm [11-12].

APPENDIX

The Press-Schecter (PS) theory

The PS theory studies the evolution of cosmic structures as a function of the cosmic time [1-3, 10]. Primordial density perturbations (1) are assumed to be Gaussian and the probability distribution of perturbations with mass M takes the form,

$$p(\delta)=\frac{1}{2\sqrt{\pi}\sigma(M)}\exp[-\frac{\delta^2}{2\sigma^2(M)}] \tag{A1}$$

Here, $\sigma^2(M)=\langle\delta^2\rangle$ represents the variance of the distribution, equal to the mean of the square of density fluctuations. The PS theory is founded on the key hypothesis that density perturbations gravitationally collapse into bound objects upon reaching a critical amplitude δ_c at a critical time t_c. In this context, a reasonable approximation is to consider both δ_c and t_c constant parameters *within a given cosmic epoch* of the early Universe.

The power spectrum of density fluctuations $P(k)$ and the critical time in the PS theory are, respectively,

$$P(k) \propto k^n \tag{A2}$$

$$t_c = \frac{\delta_c}{\sqrt{2\langle\delta^2\rangle}} = (\frac{M}{M^*})^{(3+n)/6} \tag{A3}$$

in which $M^* \propto (\delta_c^{-2})^{(3+n)/6}$ denotes an epoch-dependent reference mass and n is the spectral index. The number of structures of a given mass M assumes the form [3]

$$N(M) = \frac{\overline{\rho}}{\sqrt{\pi}} \frac{\gamma}{M^2} (\frac{M}{M^*})^{\gamma/2} \exp[-(\frac{M}{M^*})^{\gamma}] \tag{A4}$$

where $\overline{\rho} = M/V$ is the mean density of the background, V the volume and

$$\gamma = 1 + \frac{n}{3} \tag{A5}$$

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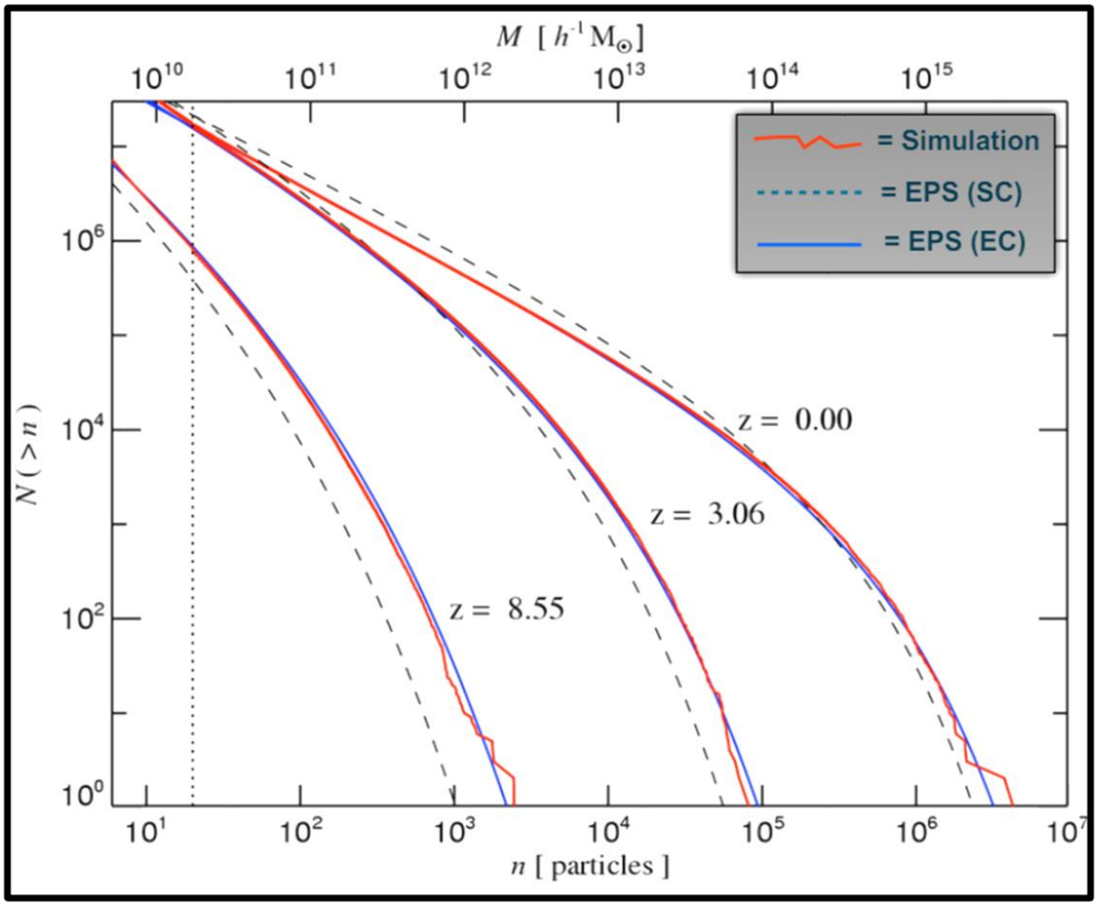

Fig. 1: Scaling behavior of the Press-Schecter theory [10]

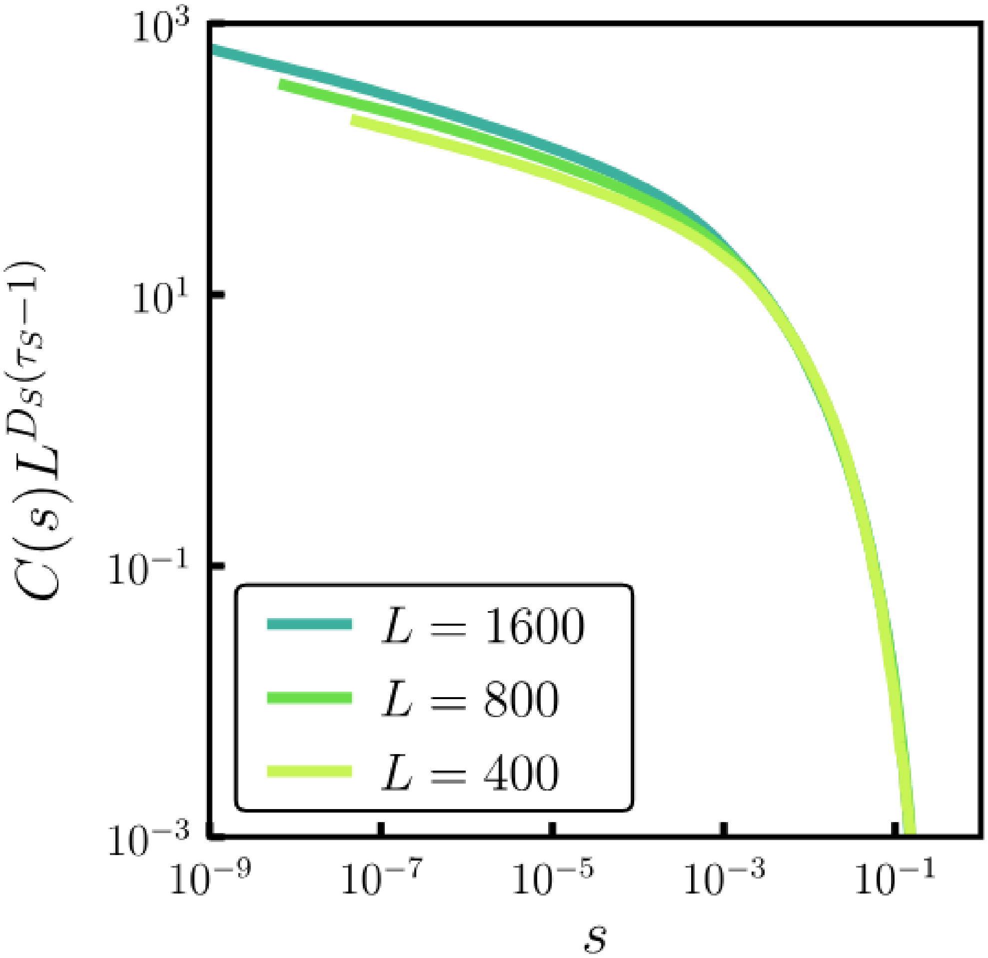

Fig. 2: Scaling behavior of the 2D sandpile model [11-12]