Comment on Alan Macdonald's articles about the "World's Fastest Derivation of the Lorentz Transformation"

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Abstract

In his 1981 till today papers about the "World's Fastest Derivation of the Lorentz Transformation", Alan Macdonald has published ingenious thoughts on the Lorentz transformation. However, contrary to his explicit statement in footnote 3, he implicitly does assume that the transformation sought for is linear. For this, we propose a small addition to his text.

The Lorentz transformation is often considered the mathematical heart of the theory of special relativity. For this, it enters virtually all other theories compatible with the latter one but Dirac's "instant form of relativistic dynamics' [1] (for reviews and further developments of the latter one, see, e.g. [2]). Accordingly, there is quite a huge variety of derivations of it. The motivations for searching novel derivations are quite different, too; they reach from making a minimum of assumptions (what leads to rather large a number of steps) till being fastest (requiring more or stronger assumptions).

In his 1981 till today papers about his "World's Fastest Derivation of the Lorentz Transformation" [3][4][5], Alan Macdonald has invented quite an original approach to reach that goal. He relaxes the implicit and additional, respectively, assumption of classical mechanics that the reading of a clock is independent of its motion. The fastness of his derivation is partially due to his use of constancy of the speed of light w.r.t. all inertial frames (Einstein's second postulate, his assumption A) and his assumption "(B) Inertial frames are homogeneous and spatially isotropic (which is not necessary [6]). And, contrary to his explicit statement in footnote 3, he implicitly assumes the linearity of the transformation, see below.

From his crucial figure, Macdonald [5] rightly derives the relations (c = 1)

$$X' = 0\,, \quad X = vT\,, \quad T = \gamma T'.$$

"Thus on O,

$$T + X = \gamma(1+v)(T' + X') \tag{1}$$

$$T - X = \gamma(1-v)(T' - X')\,." \tag{2}$$

However, since $X' = 0$, that equations should have been written more generally as

$$T + X = \gamma(1+v)(T' + \alpha X') \tag{3}$$

$$T - X = \gamma(1-v)(T' - \beta X')\,. \tag{4}$$

The coefficients α and β are not necessarily equal unit as in eqs. (1) f. Because the transformation sought for is not assumed to be linear ([5] fn. 3), α and β may even depend on T' and X'.

Now, within kinematics, the linearity of the transformation can be shown without additional assumptions [6]. As a consequence, not only the transformation between the vectors (T, X) and (T', X') is linear but also the transformation between any vectors ($\tilde{T} := aT + bX$, $\tilde{X} := cT + dX$) and ($\tilde{T}' := aT' + bX'$, $\tilde{X}' := cT' + dX'$), where the coefficients $a, \ldots, d$ may depend on v, and $ad \neq bc$. In particular, the case $a = b = c = 1, d = -1$ is allowed. This justifies eqs. (1) f.

Thus, to justify his eqs. (1) f. above [5], we propose Alan Macdonald to replace footnote 3 by a reference to the linearity of the transformation. It would not touch his claim "World's Fastest Derivation of the Lorentz Transformation".

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[1] P. A. M. Dirac, "Forms of Relativistic Dynamics," Rev. Mod. Phys. 21 (1949) 3, 392–399, https://journals.aps.org/rmp/pdf/10.1103/RevModPhys.21.392.

[2] https://www.researchgate.net/profile/Eugene-Stefanovich.

[3] Alan Macdonald, "Derivation of the Lorentz transformation," Am. J. Phys. **49** (5), 493 (1981).

[4] Alan Macdonald, "Still the 'World's Fastest Derivation of the Lorentz Transformation'," 2006, https://arxiv.org/abs/physics/0606046.

[5] Alan Macdonald, "World's fastest derivation of the Lorentz transformation," http://www.faculty.luther.edu/~macdonal/LorentzT.pdf.

[6] P. Enders & R. Rupp, "Illustrative axiomatic derivation of the special Lorentz transformation from merely the properties of empty space and inertial systems," 2023, http://viXra.org/abs/2306.0035.