Is it Possible to Arbitrarily Slow Down Time in a Limited Volume With an Energy-Impulse Tensor Whose Components Can be Reduced Arbitrarily?

Part II:Summary of Introduction and Computations of Density and Energy in Warped Region

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Abstract

A solution is presented that describes a region of space, box or warp bubble, where time gets slowed down by an arbitrary factor, while reducing the components of the energy-impulse tensor by any chosen amount.

1 Introduction:

This papers investigates if it is possible, in the framework of general relativity, to slow down time by an arbitrary factor with respect to an observer in an inertial reference frame within a warp bubble, if the energy-impulse tensor has negative or positive components and if they can be reduced by an arbitrary value and a way to test this in a laboratory.

Note: all notations here are those used by Landau and Lifshitz in the second book ("The Classical Theory of Fields") of their well known Course of Theoretical Physics [2].

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We start with the metric

$$ds^2 = a(x,y,z)\,dt^2 - b(x,y,z)^2 dx^2 - b(x,y,z)^2 dy^2 - b(x,y,z)^2 dz^2 \qquad (1)$$

The differential proper time is:

$$d\tau = a(a,y,z)\,dt \qquad (2)$$

$a(x,y,z)$ is chosen in a simplified way (for a sphere with radius R and thickness $\Delta \ll 1$) as:

- 1)- $a(x,y,z)=1$ for every $r > R + \frac{\Delta}{2}$
- 2)- $a(x,y,z)=1$ for every r such that $R - \frac{\Delta}{2} < r < R + \frac{\Delta}{2}$ (cavity)
- 3)- $a(x,y,z) = A = constant \ll 1$ for $0 < r < R - \frac{\Delta}{2}$

then we get:

- 1)- $\tau = At$ within the ball $0 < r < R - \frac{\Delta}{2}$,(zero initial conditions)
- 2)-Where $r = (x^2 + y^2 + z^2)^{\frac{1}{2}}$

t is the coordinate time in the exterior of the warp bubble (that is, proper time for observers whose reference frame is inertial, i.e., moving in a very weak gravitational field at a speed which is far smaller than the speed of light). τ tau is the proper time within the warp bubble (in our case).

When A<<1 the proper time becomes very small and it is like an hibernation of time within the warp bubble, we could call a chamber in this situation a “stasis chamber”.

2 Appendix I:

We choose a spherically symmetric warp bubble with a shell having radius R and thickness

$$\Delta \ll 1$$

with the following convergence values , $r = (x^2 + y^2 + z^2)^{\frac{1}{2}}$.

for $b(x, y, z)$:

- 1)- $b(x, y, z) = 1$ for every r such that $r > R + \frac{\Delta}{2}$
- 2)- $b(x, y, z) \gg 1$ for every r such that $R - \frac{\Delta}{2} < r < R + \frac{\Delta}{2}$ (cavity wall)
- 3)- $b(x, y, z) = 1$ for every r such that $0 < r < R - \frac{\Delta}{2}$

3 The Einstein Tensor in contravariant form is:

$$G^{tt} = -\frac{1}{a(x,y,z)^2\, b(x,y,z)^4}\left(2\left(\frac{\partial^2}{\partial y^2} b(x,y,z)\right) b(x,y,z) - \left(\frac{\partial}{\partial x} b(x,y,z)\right)^2 + 2\left(\frac{\partial^2}{\partial x^2} b(x,y,z)\right) b(x,y,z) + 2\left(\frac{\partial^2}{\partial z^2} b(x,y,z)\right) b(x,y,z) - \left(\frac{\partial}{\partial y} b(x,y,z)\right)^2 - \left(\frac{\partial}{\partial z} b(x,y,z)\right)^2\right)$$

$$G^{xx} = \frac{1}{b(x,y,z)^6\, a(x,y,z)}\left(2\left(\frac{\partial}{\partial x} a(x,y,z)\right)\left(\frac{\partial}{\partial x} b(x,y,z)\right) b(x,y,z)\right.$$

$$+\left(\frac{\partial^2}{\partial y^2} b(x,y,z)\right) a(x,y,z)\, b(x,y,z) + \left(\frac{\partial}{\partial x} b(x,y,z)\right)^2 a(x,y,z)$$

$$+\left(\frac{\partial^2}{\partial z^2} b(x,y,z)\right) a(x,y,z)\, b(x,y,z) + \left(\frac{\partial^2}{\partial y^2} a(x,y,z)\right) b(x,y,z)^2 - \left(\frac{\partial}{\partial y} b(x,y,z)\right)^2 a(x,y,z)$$

$$+\left(\frac{\partial^2}{\partial z^2} a(x,y,z)\right) b(x,y,z)^2 - \left(\frac{\partial}{\partial z} b(x,y,z)\right)^2 a(x,y,z)\Bigg)$$

$$G^{xy} = -\frac{1}{b(x,y,z)^6\, a(x,y,z)}\Bigg(\left(\frac{\partial^2}{\partial y\,\partial x} a(x,y,z)\right) b(x,y,z)^2 - \left(\frac{\partial}{\partial y} b(x,y,z)\right)\left(\frac{\partial}{\partial x} a(x,y,z)\right) b(x,y,z)$$

$$-\left(\frac{\partial}{\partial x} b(x,y,z)\right)\left(\frac{\partial}{\partial y} a(x,y,z)\right) b(x,y,z) - 2\left(\frac{\partial}{\partial y} b(x,y,z)\right)\left(\frac{\partial}{\partial x} b(x,y,z)\right) a(x,y,z)$$

$$+\left(\frac{\partial^2}{\partial y\,\partial x} b(x,y,z)\right) a(x,y,z)\, b(x,y,z)\Bigg)$$

$$G^{xz} = -\frac{1}{b(x,y,z)^6\, a(x,y,z)}\Bigg(\left(\frac{\partial^2}{\partial z\,\partial x} a(x,y,z)\right) b(x,y,z)^2 - \left(\frac{\partial}{\partial z} b(x,y,z)\right)\left(\frac{\partial}{\partial x} a(x,y,z)\right) b(x,y,z)$$

$$-\left(\frac{\partial}{\partial x} b(x,y,z)\right)\left(\frac{\partial}{\partial z} a(x,y,z)\right) b(x,y,z) - 2\left(\frac{\partial}{\partial z} b(x,y,z)\right)\left(\frac{\partial}{\partial x} b(x,y,z)\right) a(x,y,z)$$

$$+\left(\frac{\partial^2}{\partial z\,\partial x} b(x,y,z)\right) a(x,y,z)\, b(x,y,z)\Bigg)$$

$$G^{yy} = \frac{1}{b(x,y,z)^6\, a(x,y,z)} \left(2 \left(\frac{\partial}{\partial y} a(x,y,z) \right) \left(\frac{\partial}{\partial y} b(x,y,z) \right) b(x,y,z) \right.$$

$$+ \left(\frac{\partial^2}{\partial x^2} b(x,y,z) \right) a(x,y,z)\, b(x,y,z) + \left(\frac{\partial}{\partial y} b(x,y,z) \right)^2 a(x,y,z)$$

$$+ \left(\frac{\partial^2}{\partial z^2} b(x,y,z) \right) a(x,y,z)\, b(x,y,z) + \left(\frac{\partial^2}{\partial x^2} a(x,y,z) \right) b(x,y,z)^2 - \left(\frac{\partial}{\partial x} b(x,y,z) \right)^2 a(x,y,z)$$

$$\left. + \left(\frac{\partial^2}{\partial z^2} a(x,y,z) \right) b(x,y,z)^2 - \left(\frac{\partial}{\partial z} b(x,y,z) \right)^2 a(x,y,z) \right)$$

$$G^{yz} = - \frac{1}{b(x,y,z)^6\, a(x,y,z)} \left(\left(\frac{\partial^2}{\partial z\, \partial y} a(x,y,z) \right) b(x,y,z)^2 - \left(\frac{\partial}{\partial z} b(x,y,z) \right) \left(\frac{\partial}{\partial y} a(x,y,z) \right) b(x,y,z) \right.$$

$$- \left(\frac{\partial}{\partial y} b(x,y,z) \right) \left(\frac{\partial}{\partial z} a(x,y,z) \right) b(x,y,z) - 2 \left(\frac{\partial}{\partial y} b(x,y,z) \right) \left(\frac{\partial}{\partial z} b(x,y,z) \right) a(x,y,z)$$

$$\left. + \left(\frac{\partial^2}{\partial z\, \partial y} b(x,y,z) \right) a(x,y,z)\, b(x,y,z) \right)$$

$$G^{zz} = \frac{1}{b(x,y,z)^6\, a(x,y,z)} \left(2 \left(\frac{\partial}{\partial z} a(x,y,z) \right) \left(\frac{\partial}{\partial z} b(x,y,z) \right) b(x,y,z) \right.$$

$$+ \left(\frac{\partial^2}{\partial x^2} b(x,y,z) \right) a(x,y,z)\, b(x,y,z) + \left(\frac{\partial}{\partial z} b(x,y,z) \right)^2 a(x,y,z)$$

$$+\left(\frac{\partial^2}{\partial y^2}b(x,y,z)\right)a(x,y,z)\,b(x,y,z)+\left(\frac{\partial^2}{\partial x^2}a(x,y,z)\right)b(x,y,z)^2-\left(\frac{\partial}{\partial x}b(x,y,z)\right)^2a(x,y,z)$$

$$\left.+\left(\frac{\partial^2}{\partial y^2}a(x,y,z)\right)b(x,y,z)^2-\left(\frac{\partial}{\partial y}b(x,y,z)\right)^2a(x,y,z)\right)$$

Einstein Equation

$$G^{ik}=\frac{8\pi G}{c^4}T^{ik} \qquad [2]$$

If $b=b(x,y,z)\gg 1$ and $\partial b/\partial x^i\leq b$, $\partial^2 b/\partial x^i\partial x^k\leq b$, $x^i, x^k=x,y,z$, $i,k=1,2,3$ in the cavity wall of the warp bubble or box the components of the impulse-energy tensor can be reduced by an arbitrary amount.

In the spherical coordinates, the metric (1) is, for a spherical chamber :

$$ds^2=a(r)^2dt^2-b(r)^2dr^2-b(r)^2r^2d\theta^2-b(r)^2r^2\sin(\theta)^2d\phi^2$$

the value for a(r) and b(r) are given on pag 2 and 3

The components of the Einstein tensor are:

$$G^{tt}=-\frac{2\frac{d^2}{dr^2}b(r)b(r)r-\left(\frac{d}{dr}b(r)\right)^2r+4b(r)\frac{d}{dr}b(r)}{a(r)^2b(r)^4r}$$

$$G^{rr}=\frac{2b(r)\left(\frac{d}{dr}b(r)\right)\left(\frac{d}{dr}a(r)\right)r+\left(\frac{d}{dr}b(r)\right)^2a(r)r+2b(r)^2\left(\frac{d}{dr}a(r)\right)+2b(r)\left(\frac{d}{dr}b(r)\right)a(r)}{b(r)^6ra(r)}$$

$$G^{\vartheta\vartheta}=\frac{(\frac{d^2}{dr^2}b(r))b(r)a(r)r+(\frac{d^2}{dr^2}a(r))b(r)^2r-(\frac{d}{dr}b(r))^2a(r)r+b(r)^2(\frac{d}{dr}a(r))+b(r)(\frac{d}{dr}b(r))a(r)}{b(r)^6r^3a(r)}$$

$$G^{\phi\phi}=\frac{(\frac{d^2}{dr^2}b(r))b(r)a(r)r+(\frac{d^2}{dr^2}a(r))b(r)^2r-(\frac{d}{dr}b(r))^2a(r)r+b(r)^2(\frac{d}{dr}a(r))+b(r)(\frac{d}{dr}b(r))a(r)}{b(r)^6r^3\sin(\vartheta)^2a(r)}$$

The components are now only four, which simplifie~~d~~s the problem. Howerer, the energy density could be negative and also the three pressures, in the warped region of the warp bubble.

As can be seen, time flows more slowly inside the warp bubble compared to outside of it. This means that the structure could be calibrated so that, for example, 5 minutes pass inside the warp bubble while a month passes outside in an inertial system or on Earth (which is nearly inertial, with low speeds compared to the speed of light and very weak gravitational fields). It would be as if an observer inside the bubble had traveled into the future. This could be interesting for potential experimental verification (new mode for verify general relativity other the classics). Essentially, this is a machine for time travel into the future.

4 Computations of energy density and energy for a spheric chamber in spheric polar coordinates in warped region (energy of warp bubble) approssimated solution:

We choose the funtions given by these equations:

$b(r)=\frac{2^p}{(1+(\tanh(\sigma(r-R)))^2)^p}$ $P\gg1$ inside and outside warped region $b(r)$ about 1

and $b(r)\gg1$ in warped region, $b(r)$ is analytical and $\sigma\gg1$

$a(r)=constant\,\frac{(1-(\tanh(\sigma(r-R)))^2)}{2}$ with $constant\ll1$ $\sigma\gg1$ analytical

Taking a small neighborhood of sphere with $R=10\,m$ radius in warped region.

$$G^{tt}\cong\frac{(\sigma P)}{(2^{P})^{2}}>0 \quad \text{is small} \qquad \rho=k\,G^{tt}>0$$

$$G^{rr}\cong\frac{1}{(2^{P})^{4}}>0 \quad \text{is small} \qquad p_{r}=k\,G^{rr}>0$$

$$G^{\theta\theta}\cong-\frac{(\sigma P)}{(2^{P})^{4}}<0 \quad \text{is small} \qquad p_{\theta}=k\,G^{\theta\theta}<0$$

$$G^{\varphi\varphi}=\frac{G^{\theta\theta}}{(\sin(\theta))^{2}}<0 \quad \text{is small} \quad (\vartheta\neq0) \qquad p_{\varphi}=k\,G^{\varphi\varphi}<0$$

$k=\frac{c^{4}}{8\pi G}$ the components of Einstein tensor inside and outside warped region is about zero (order less of 10^{-200}),for exemple $P=140,\sigma=5000,R=10\,m\,,\Delta\cong10^{-4}\,m$

$$M\cong\frac{\rho}{c^{2}}\pi R^{2}\Delta\cong10^{-12}\,kg$$ mass source in warped region.

$$\frac{\rho}{c^{2}}=10^{-10}\frac{kg}{m^{3}}$$ mass density in warped region

5 Conclusion:

This paper demonstrates the possibility of slowing down time by an arbitrary factor, potentially approaching the "freezing" of time (hibernation of time) within a limited volume, such as a warp bubble or box, relative to an inertial observer. It also explores the capability to reduce the components of the energy-impulse tensor by an arbitrary value within the cavity wall of the warp bubble or box. Importantly, it suggests that using ordinary matter, rather than exotic matter, may achieve this reduction, even with partially negative components, but with $\rho>0$. However, the significant slowing down of the speed of light within the volume containing matter, though compensable, could pose challenges depending on the desired reduction of the energy-impulse tensor components.

The Einstein tensor exhibits singularity for $r=0,\vartheta=0$ in a spherical chamber. Nonetheless, a Planck length cutoff exists below which one cannot descend, thereby eliminating physical singularity and leaving only a mathematical one. Consequently, this implies that general relativity must be replaced by a theory of quantum gravity. This solution allows for verifying general relativity beyond other classical methods. Indeed, the Jacobian is zero for these values, meaning that the coordinate map in this case does not make sense and therefore is no longer a good coordinate system.

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