Topological Theory of Hopf Bundle and Mass

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Abstract

Why a particle has the specific rest mass it does is an open question. The rudiments of an alternative theory to the Standard Model are put forward to try an answer this question. To perturb an object with Hopf fibration topology, a force in ordinary space jumps topologies. The size of the jump is the object's resistance to the force and the measure of its mass. Various mass splitting formulae carry the signature of a 3-sphere intersecting three dimensional space. The signature points to homotopic non-equivalence. Using this signature the masses of six lighter hyperons and electron are found to be functions of the proton and neutron.

Keywords: Hopf fibration, Higgs field, topology, mass splitting

The Higgs field imparts mass to fundamental particles. In the crowd analogy the field acts like a throng impeding a celebrity as they attempt to cross a room. [1] The stronger the interaction the slower the progress, the heavier the particle. If we dig a little bit deeper, particles that exhibit internal Lie group symmetry at higher energy states gain mass when spontaneous symmetry breaking couples with the Higgs field. [2, 3] The caveat is the field interacts with quarks, leptons and some bosons, but not photons; while the bulk of a Hadron's mass is due to quark confinement, not the Higgs field. Unable to predict why a particle has the precise mass that it does, the Standard Model leaves rest mass an open question. This is perhaps not surprising. Particle mass is a scalar constant. Its value is a real number not a complex number. Rest mass is not an evolving quantum state. Arguably the Standard Model is an unsuitable framework for understanding mass. In search of greater precision we rethink how a particle resists a force. The basic idea is that a particle's reluctance to interact with a field is due to homotopic non-equivalence between particle and field. The non-equivalence explains the entirety of a particle's mass. In order to act on the particle it is the field that breaks symmetry, not the particle.

In lieu of the Higgs scalar field, the theory considers a vector field. Ordinary space is Euclidean with connected topology and may be thought of as an $\mathbb{R}^3$ scalar field. Three dimensional physical space is assumed to be ordinary space. An ordinary force described by a three dimensional $\mathbb{R}^3$ vector shares the topology of ordinary space. We call ordinary three dimensional vector space '3-space'.

The theory also considers an S^3 Hopf fibration or Hopf bundle. The geometry is long understood.[4] A fibration maps a 3-sphere to a 2-sphere. The 3-sphere is the set of four dimensional points S^3. The 2-sphere is a two dimensional surface described by the set of three dimensional points S^2. A Hopf fibration continuously maps S^3 to S^2. This is done with Hopf maps. A Hopf map ($h : S^3 \to S^2$) is a surjective function that maps a subset of S^3 elements to a point in S^2. An individual Hopf map describes a circle (Hopf circle). Continuous mapping entails an infinite number of maps for each point in S^3; this requires an infinite bundle of circles connecting each S2 point to every point in S3. The total space is transitive. A 'Hopf-particle', as we shall call it, is a 3-sphere with S3 topology.

In geometry it is commonly understood that a 3-sphere intersecting ordinary space appears as a 2-sphere. We consider a Hop-particle / 3-space intersection. A 3-space force has the connected topology of a point. The particle's surface is a bundle of Hopf circles. The area where the external force makes contact with the Hopf surface raises the question of homotopic non-equivalence. The problem may be pictured as a cone mapping. If we imagine a cone, a point force at the apex of the cone is unable to pass to the base circle unless its connected topology is punctured. In reverse, only by *cutting* may the circle deform retract to a point. Alternatively, a particle with connected topology that deform retracts to a point offers no resistance to a point force. On this view, mass is relative to the different topologies of particle and force. For instance, a Hopf-particle is unable to resist a force with S3 topology as there is no need for the force to jump topologies. Resistance to change in location and speed (in 3-space) follows a point force breaking symmetry and jumping topologies. With a total space that is transitive, the size of the 3-sphere is the size of the jump, and it is the jump that is the measure of the resistance to a point force.

Five equations characterise Hopf-particle rest mass. Eq. (1) tells us mass is determined by the size of the 3-sphere. I.E.

$$M = 2\pi^2 r^3. \tag{1}$$

For example, if the mass of the proton is 938.272 then $r \approx 3.622$. The 3-space occupied by a Hopf-particle is the interior of a 2-sphere. The volume is as Eq. (2). In the case of the proton, $V_p \approx 199.108$.

$$V = \frac{4\pi}{3} r^3. \tag{2}$$

The density of the interior of the 2-sphere is as Eq. (3).

$$\rho = \frac{M}{V} = \frac{3\pi}{2}. \tag{3}$$

When $\rho > 1$, hypermass (H) is the difference between mass and volume, as Eq. (4).

$$H = M - V. \tag{4}$$

Hopf-particle mass has the Hopf/hypermass signature (h-signature), as Eq. (5).

$$M = H\left(\frac{\rho}{\rho - 1}\right). \tag{5}$$

Ignoring the standard deviations, the following set of equations use the 2022 CODATA recommended rest mass energy values in MeV [5] for proton, neutron and electron, as (6).

$$\begin{aligned} M_p &= 938.272\ 089\ 43\ (29), \\ M_n &= 939.565\ 421\ 94\ (48), \\ M_e &= 0.510\ 998\ 950\ 69\ (16). \end{aligned} \tag{6}$$

The h-signatures found in the rest mass data suggests lighter hyperons are Hopf-particles. For instance, Σ rest mass h-signatures are functions of the proton and neutron masses, as Eqs. (7, 8, 9).

$$M_{\Sigma^+} = (2M_p - M_n)\Big(\frac{\rho}{\rho - 1}\Big) \approx 1189.3712. \tag{7}$$

$$M_{\Sigma^0} = M_n\Big(\frac{\rho}{\rho - 1}\Big) \approx 1192.6546. \tag{8}$$

$$M_{\Sigma^-} = (4M_n - 3M_p)\Big(\frac{\rho}{\rho - 1}\Big) \approx 1197.5797. \tag{9}$$

Eq. (7) matches the Particle Data Group (PDG) current fit for M_{Σ^+} (1189.37 ±0.07).[6] The PDG fit for M_{Σ^0} is 1192.642 ±0.024, Eq. (8) is particularly close to Wang 1192.65 ±0.020.[7] However, Eq. (9) is over four standard deviations over the PDG fit (1197.449 ±0.030). The PDG value for M_{Σ^-} draws on three results. Schmidt (1197.43) and Gurev (1197.417) are too low to be the value derived here.[8, 9] Schmidt is an old paper from 1965, and Gurev is a little cited proof of concept. Eq. (9) is within one standard deviation of Gall (1197.532 ±0.057) [10]. Improved experimental accuracy affirming Gall is needed to support this thesis.

Eqs. (10, 11) suggest the Ξ masses are functions of the neutral and negative Σ masses, less the proton's 3-space volume.

$$M_{\Xi^0} = M_{\Sigma^0}\big(\frac{\rho}{\rho - 1}\big) - V_p \approx 1314.8104. \tag{10}$$

$$M_{\Xi^-} = M_{\Sigma^-}\big(\frac{\rho}{\rho-1}\big) - V_p \approx 1321.0622. \tag{11}$$

Eq. (10) is within one standard deviation of the PDG fit (1314.86 ± 0.20) and is close to Fanti (1314.82 $\pm$0.06)[11].

The h-signatures so far presented appear ad hoc. However, Eqs, (7, 8, 9, 10, 11) are a non-arbitrary solution to Eqs. (12, 13) in MeV. (A scaling factor is needed for other unit systems; a point we elaborate on later).

$$M_{\Sigma^0}\left(\frac{M_{\Xi^-} - M_{\Xi^0}}{M_{\Sigma^-} - M_{\Sigma^0}}\right) - M_{\Xi^0} - V_p = \ 0. \tag{12}$$

$$M_{\Sigma^-}\left(\frac{M_{\Xi^-} - M_{\Xi^0}}{M_{\Sigma^0} - M_{\Sigma^-}}\right) - M_{\Xi^-} - V_p = \ 0. \tag{13}$$

Unfortunately, Eq. (11) is over nine standard deviations adrift of the PDG fit for M_{Ξ^-}. The present PDG recommended value (1321.71 ± 0.07) draws on a 2006 study of a large 1992-1995 data sample [12]. Faced with an unlikely nine standard deviation downward adjustment, Eq. (14) introduces the electron mass as a fudge factor ≈ 0.511.

$$M^*_{\Xi^-} = (M_{\Sigma^-} + M_e)\left(\frac{\rho}{\rho-1}\right) - V_p \approx 1321.7109. \tag{14}$$

The Ω^- mass is derived using the adjusted $M^*_{\Xi^-}$, as Eq. (15).

$$M_{\Omega^-} = \left(\frac{3M_{\Xi^0} + 2M^*_{\Xi^-}}{5}\right)\left(\frac{\rho}{\rho-1}\right) \approx 1672.4824. \tag{15}$$

The most recent PDG fit for the Ω^- mass is 1672.45 ± 0.29 MeV. From Eq (15) and Eqs. (1, 2) we get $V_\Omega = 354.9118$. The adjusted formulae with fudge factor also appear ad hoc, but Eqs (7, 8, 9, 10, 12, 14) are the solution to Eqs. (16, 17), again in MeV only, with additional scaling factors needed for other units.

$$M_{\Sigma^0}\left(\frac{M^*_{\Xi^-}-M_{\Xi^0}}{M_{\Sigma^-}-M_{\Sigma^0}}\right)-M_{\Xi^0}-V_{\Omega^-}-\frac{\rho}{\rho-1}=\ 0. \tag{16}$$

$$M_{\Sigma^-}\left(\frac{M^*_{\Xi^-}-M_{\Xi^0}}{M_{\Sigma^-}-M_{\Sigma^0}}\right)-M_{\Xi^-}-V_{\Omega^-}-\frac{\rho}{\rho-1}=\ 0. \tag{17}$$

Eq. (18) is trivially true regardless of the system of units.

$$M_p+\frac{M_e}{3\left(\frac{M^*_{\Xi^-}-M_{\Xi^0}}{M_{\Sigma^-}-M_{\Sigma^0}}-\frac{\rho}{\rho-1}\right)}=M_n. \tag{18}$$

The triviality is due to the equivalence at Eq. (19).

$$\frac{M^*_{\Xi^-}-M_{\Xi^0}}{M_{\Sigma^-}-M_{\Sigma^0}}-\frac{\rho}{\rho-1}=\frac{M_e}{3(M_n-M_p)} \tag{19}$$

If M_{Ξ^-} is not adjusted, Eq. (18) insists M_n is an infinite mass. The adjustment $M^*_{\Xi^-}$ means Eq. (18) is trivially true whilst also leaving M_n and $M^*_{\Xi^-}$ a close match to observation.

Eqs. (20, 21) are each an h-signature proportional to the electron mass in electron volts.

$$\left(\frac{M_e}{M_{\Sigma^0}\left(\frac{M^*_{\Xi^-}-M_{\Xi^0}}{M_{\Sigma^-}-M_{\Sigma^0}}\right)-M_{\Xi^0}-V_{\Omega^-}}\right)\left(\frac{\rho}{\rho-1}\right)\approx 0.511. \tag{20}$$

$$\left(\frac{M_e}{M_{\Sigma^-}\left(\frac{M^*_{\Xi^-}-M_{\Xi^0}}{M_{\Sigma^-}-M_{\Sigma^0}}\right)-M_{\Xi^-}-V_{\Omega^-}}\right)\left(\frac{\rho}{\rho-1}\right)\approx 0.511. \tag{21}$$

As the number ≈ 0.511 holds true for or any system of units, we know it is not unwittingly introduced at an earlier stage. For instance, the CODATA 2022 values for proton and electron are more accurately known in u, as (22).

$$
\begin{aligned}
M_n &= 1.008\ 664\ 916\ 06\ (40),\\
M_p &= 1.007\ 276\ 466\ 57\ 89\ (83),\\
M_e &= 0.000\ 548\ 579\ 90\ 90\ 441\ (97).
\end{aligned} \tag{22}
$$

Using (22) for the input values (ignoring the standard deviation), Eqs. (20, 21) give the answer 0.511 007 366 716. With the CODATA 2022 values for mass in kg, we get 0.511 007 405 833. The mass energy equivalent in MeV from (6) givews 0.511 007 615 223. The cluster is a tad too high for the electron mass. However, the function is sensitive to its inputs. As the neutron mass is the more uncertain it makes sense to use the proton and electron masses to dial-in the neutron mass. Using M_p and M_e from (22) and M_n^* from (23), Eqs. (20, 21) gives the CODATA 2022 MeV numerical value 0.510 998 950 69, as (6).

$$
M_n^* = 1.008\ 664\ 915\ 876\ 394\ 0717. \tag{23}
$$

Obviously (23) attempts a degree of accuracy six decimals beyond M_p u to get an accurate dial-in. However, M_n^* represents a downward adjustment of less than one standard deviation, which equates to less than one fifth of an electron volt. Given the uncertainties, a mass energy for the neutron that resolves Eq. (20, 21) is found in the range 939.565 421 76 $\pm$0.000 000 06 MeV.

At first blush, the number $\approx$ 0.511 signals mass is an electromagnetic phenomena. Eqs. (20, 21) also establish electron mass is a function of baryon mass. However, the pressing question is how (20, 21) arrive at a dimensionless number proportional to electron mass in the human system of electron volts? The obvious answer is to regard the $1/10^6$ ratio as an SI/cgs scaling problem. To explain the difference we look to the commensurable field density values found in the cgs system where 1 unit of electrical field density corresponds to 1 unit of magnetic field density. This is the 1:1 correspondence of 1 statV/cm for 1 Mx/cm^2. Let us call this a 'balanced' system. The 1:1 balance is lost in SI units where 1 V/m : 3.34×10^{-9} Wb/m^2. Whilst a balanced system reflects a natural principle, the cgs unit for charge, the esu, is chosen so that Coulomb's constant = 1. This is mathematically tidy but no more than a convention. To establish a balanced system, where the charge value is not artificially adjusted, we convert statV to SI units.

$$\frac{1\ statV}{c} = \frac{299792458}{10^6}\ V \times \frac{1}{c} = \frac{1}{10^6\ N}. \tag{24}$$

At Eq. (24) the cgs/SI scaling factor $\frac{299792458}{1{,}000{,}000}$ adjusts for the speed of light (cm/m) and mass (g/kg). Dividing by c gives a force in newtons. For a balanced natural system where $c = 1$ and force F has the generic dimensions $l \cdot M \cdot t^{-2}$, and q is a generic elementary charge, Eq. (24) is proportional to the generic formula, as Eq. (25).

$$\frac{5.11 \times 10^5\ e}{10^6\ N} \propto \frac{0.511\ q}{F}. \tag{25}$$

Replacing the number 0.511 in Eq. (25) with Eqs. (20, 21), and allowing $\frac{M \cdot q}{F} = n$, then by $F = Ma$ and $q = a$, we get Eq. (26).

$$n \cdot \frac{F}{q} \approx 0.511. \tag{26}$$

Further allowing $n = 1$, the length and time dimensions cancel in Eq. (26) leaving $M_e \approx 0.511$; which is in agreement with interpreting the h-signatures of Eqs. (20, 21) as the mass of the electron.

In conclusion, a more rigorous mathematical foundation is needed for the mass spitting formulae presented in this paper. However, these formulae clearly show the lighter hyperons and electron each have an h-signature. Reducing nine free parameters to two provides a hint mass is due to homotopic non-equivalence. Deriving the electron mass as a pure number is also not insignificant.

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