Erosion protection of Tokamak and magnetic force on single charge
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Abstract: The walls of Tokamaks are heavily eroded by plasma which suggests that a mysterious force pushes the plasma to the wall. We have theoretically discovered this force and named it extra-force. This force appears only on single charges but not on current carrying wire, which is why the Lorentz force law does not contain it. This discovery not only brings new knowledge to electromagnetism, but also gives a solution against the erosion, which could improve the technology for controlled nuclear fusion. We have proposed an experiment to test this new force.

1. Introduction
A promising way to achieve controlled nuclear fusion is Tokamak which holds fusion fuel at around 100 million kelvins in a toroidal chamber. At this temperature the fusion fuel is in plasma state which is a mixture of positive ions and electrons. The plasma is confined with a powerful magnetic field which makes charged particles circling the confinement field lines. The coils of Tokamaks are so designed that the confinement field lines are parallel to the wall of the chamber and the Lorentz force is expected to retain the plasma not to hit the wall. However, the walls of running Tokamaks are heavily eroded by plasma as if charged particles do not completely obey the Lorentz force law. So, we suspect that on top of the Lorentz force a mysterious force is acted on the charged particles, which makes the particles escape the binding of the confinement field.

Previously, we have shown a flaw in the Lorentz force law, that it ignores the parallel-to-current force, see « From Coulomb’s force to magnetic force and experiments that show magnetic force parallel to current » [1]. This flaw suggests that the mysterious force could also be a force that the Lorentz force law ignores. In the following, we will derive the expression for the mysterious force by starting from the Coulomb’s law that expresses the electrostatic force that a charge \( q_a \) exerts on another charge \( q_b \):

\[
F_{ba} = \frac{q_a q_b}{4\pi\varepsilon_0} \frac{r}{|r|^3} \tag{1}
\]

When \( q_a \) and \( q_b \) move, relativistic effects come into play and modify the Coulomb’s force given in (1). There are two such effects: the dynamic effect and changing distance effect.

2. Relativistic effects
a. Changing distance effect
Changing distance effect means that when the distance between \( q_a \) and \( q_b \) changes, the force on \( q_a \) changes. Let \( r \) be the radial distance from \( q_b \) to \( q_a \), \( v_a \) the velocity of \( q_a \) and \( v_b \) that of \( q_b \), see Figure 1. The velocity of \( q_a \) relative to \( q_b \) is \( v = v_a - v_b \). Because the direction and magnitude of \( r \) vary with time, the force \( F_{ba} \) in (1) varies with time, see Figure 2.

![Figure 1](image1.png) ![Figure 2](image2.png)

For computing the variation of \( F_{ba} \), we transform the factor \( \frac{r}{|r|^3} \) in (1) with \( r = r_0 + \dot{r}t \), with \( t \) being the time and \( r_0 \) the radial distance when \( t = 0 \):

\[
\frac{r}{|r|^3} \rightarrow \frac{r_0 + \dot{r}t}{|r_0 + \dot{r}t|^3}.
\]

This formula allows to derive the Lorentz force law when \( q_b \) moves, which is a relativistic effect.
\[
\frac{r}{|r|^3} = \frac{r}{|r_0 + vt|^3} = r((r_0 + vt)^2)^{-\frac{3}{2}} = r(r_0^2 + 2r_0 \cdot vt + (vt)^2)^{-\frac{3}{2}} = \frac{r}{|r_0|^3} \left(1 + \frac{2r_0 \cdot vt + (vt)^2}{r_0^2}\right)^{-\frac{3}{2}}
\] (2)

For very small \(t\), the magnitude of \(vt\) is very small before that of \(r_0\), then we do the linear expansion to (2) and introduce \(r = r_0 + vt\), \(\frac{r}{|r|^3}\) becomes:

\[
\frac{r}{|r|^3} = \frac{r_0 + vt}{|r_0|^3} \left(1 - 3 \frac{r_0 \cdot vt}{r_0^2}\right)
\] (3)

Introducing (3) into (1) we obtain the instantaneous force on \(q_a\) at time \(t\):

\[
F_{ba} = \frac{q_b q_a}{4 \pi \varepsilon_0 |r_0|^3} \left((r_0 + vt) - 3(r_0 + vt) \frac{r_0 \cdot vt}{r_0^2}\right)
\] (4)

The charge \(q_a\) is moving, so we take the average force on \(q_a\) for the time interval \([-t_e, t_e]\) which equals the integral of (4) from \(-t_e\) to \(t_e\) divided by \(2t_e\):

\[
F'_{ba} = \frac{1}{2t_e} \int_{-t_e}^{t_e} F_{ba} dt = \frac{q_b q_a}{4 \pi \varepsilon_0 |r_0|^3} \left(\frac{1}{2t_e} \int_{-t_e}^{t_e} (r_0 + \left(v - 3r_0 \frac{r_0 \cdot v}{|r_0|^2}\right) t - 3v \frac{r_0 \cdot v}{|r_0|^2} t^2) dt\right)
\] (5)

In the integral in (5), we have the three functions \(l, t, t^2\) the integrals of which are:

\[
\int_{-t_e}^{t_e} dt = 2t_e \quad (6)
\]

\[
\int_{-t_e}^{t_e} t dt = t_e^2 - t_e^2 = 0 \quad (7)
\]

\[
\int_{-t_e}^{t_e} t^2 dt = \frac{1}{3}(t_e^3 + t_e^3) = \frac{2}{3}t_e^3 \quad (8)
\]

We introduce (6), (7) and (8) into the integral in (5) and obtain:

\[
\frac{1}{2t_e} \int_{-t_e}^{t_e} \left(r_0 + \left(v - 3r_0 \frac{r_0 \cdot v}{|r_0|^2}\right) t - 3v \frac{r_0 \cdot v}{|r_0|^2} t^2\right) dt = r_0 - t_e^2 v \frac{r_0 \cdot v}{|r_0|^2} \quad (9)
\]

The value of \(t_e\) is determined as the time that light takes to travel from \(q_b\) to \(q_a\), which makes the changing distance effect a relativistic effect, see the equation (45) of From Coulomb’s force to magnetic force and experiments that show magnetic force parallel to currents. With \(c\) being the speed of light, \(t_e\) equals:

\[
t_e = \frac{|r_0|}{c} \quad (10)
\]

By combining (5), (9) and (10), we obtain the average force on \(q_a\):

---

2
\[ F'_{ba} = \frac{q_b q_a}{4 \pi \varepsilon_0 |r_0|^3} \left( r_0 - \frac{|r_0|^2}{c^2} \frac{r_0 \cdot v}{|r_0|^2} \right) \] (11)

b. Dynamic effect

Dynamic effect means that the intensity of the force is directly modified by the relative velocity. When the charge \( q_a \) moves relatively to \( q_b \), the reference frame of \( q_a \) moves relatively to that of \( q_b \) and the time of \( q_a \) dilates by the factor \( 1/\sqrt{1 - \frac{v^2}{c^2}} \) with respect to that of \( q_b \). The force on \( q_a \) increases by the same factor due to dynamic effect, see the equation (8) in «From Coulomb’s force to magnetic force and experiments that show magnetic force parallel to current»\(^{[1]}\). Let \( F' \) be a static force and \( F'' \) the force modified by the dynamic effect, the relativistic transformation for force is:

\[ F'' = \frac{F'}{\sqrt{1 - \frac{v^2}{c^2}}} \] (12)

By applying (12) to (11), the modified force on \( q_a \) equals:

\[ F''_{ba} = \frac{q_b q_a}{4 \pi \varepsilon_0 |r_0|^3} \left( r_0 - \frac{|r_0|^2}{c^2} \frac{r_0 \cdot v}{|r_0|^2} \right) \left( 1 - \frac{v^2}{c^2} \right)^{-\frac{1}{2}} \] (13)

c. Force modified by the two effects

Because \( v^2/c^2 \) is very small before 1, we do the linear expansion to the term \( \left( 1 - \frac{v^2}{c^2} \right)^{-\frac{1}{2}} \):

\[ \left( 1 - \frac{v^2}{c^2} \right)^{-\frac{1}{2}} \approx 1 + \frac{1}{2} \frac{v^2}{c^2} \] (14)

We introduce (14) into (13) which then becomes:

\[ F''_{ba} = \frac{q_b q_a}{4 \pi \varepsilon_0 |r_0|^3} \left( r_0 + \frac{1}{2} \frac{v^2}{c^2} \frac{r_0 \cdot v}{|r_0|^2} \left( 1 + \frac{1}{2} \frac{v^2}{c^2} \right) \right) \] (15)

We neglect the terms of order \( 1/c^4 \) and replace \( v \) with \( v_a - v_b \). \( F''_{ba} \) is expressed as:

\[ F''_{ba} = \frac{q_b q_a}{4 \pi \varepsilon_0 |r_0|^3} \left( r_0 + \frac{1}{2} \frac{v_a - v_b}{c^2} \frac{(v_a - v_b)}{|r_0|^2} \right) \] (16)

\( F''_{ba} \) is the force that the charge \( q_b \) acts on \( q_a \) and modified by the relativistic dynamic effect and changing distance effect.

3. Couple of charges

a. Force a couple of charges acts on a single charge

The force \( F''_{ba} \) is a force that a single charge exerts on a single charge but not yet a magnetic force which is exerted by a neutral material carrying current. An elementary magnetic force is exerted by an elementary unit of current carrying wire which is a couple of charges formed by a fixed positive charge and a free electron. The couple of charges is neutral, the positive charge is denoted by \( \Theta_b \), the electron by \( \Theta_b \) and the couple of charges by \( \Theta_b \Theta_b \). The force that \( \Theta_b \Theta_b \) exerts on \( q_a \) is the elementary magnetic force exerted by a current carrying wire, see Figure 3.
When the free electron is static the force that $\Theta_b \Theta_b$ exerts on $q_a$ is zero because the electrostatic forces that $\Theta_b$ and $\Theta_b$ act on $q_a$ cancel. When the free electron is moving the force from $\Theta_b$ is modified by the relativistic effects while that from $\Theta_b$ is not. In this case, the forces that $\Theta_b$ and $\Theta_b$ act on $q_a$ cannot sum to zero, so $\Theta_b \Theta_b$ exerts a net force on $q_a$. This is how elementary magnetic force emerges from a neutral wire when the free electrons start to move.

Let us compute the elementary magnetic force by first computing the force $\Theta_b$ acts. The velocity of $q_a$ is $v_a$, the charge of $\Theta_b$ is $q_b = e$, it is static and $v_b = 0$. We introduce these $v_b$, $q_b$ and $v_a$ into (16) and get:

\[
F_+^m = \frac{e q_a}{4 \pi \varepsilon_0 |r_0|^2} \left( r_0 + \frac{r_0 \cdot v_a^2}{c^2} \, \frac{v_a}{|r_0|^2} - \frac{|r_0|^2}{c^2} v_a \cdot r_0 \right) \quad (17)
\]

For the free electron $\Theta_b$, its velocity is $v_b$ and its charge is $q_b = -e$. The velocity of $q_a$ is still $v_a$. We introduce these $v_b$, $q_b$ and $v_a$ into (16) and obtain the force that $\Theta_b$ acts on $q_a$:

\[
F_-^m = -\frac{e q_a}{4 \pi \varepsilon_0 |r_0|^2} \left( r_0 + \frac{r_0 \cdot (v_a - v_b)^2}{c^2} \, \frac{v_a - v_b}{|r_0|^2} \right) \quad (18)
\]

The sum of (17) and (18) equals:

\[
F_1 = F_+^m + F_-^m = \frac{e q_a}{4 \pi \varepsilon_0 |r_0|^2} \left( r_0 + \frac{r_0 \cdot v_a^2}{c^2} \, \frac{v_a}{|r_0|^2} - \frac{|r_0|^2}{c^2} v_a \cdot r_0 \right) - \frac{e q_a}{4 \pi \varepsilon_0 |r_0|^2} \left( r_0 + \frac{r_0 \cdot (v_a - v_b)^2}{c^2} \, \frac{v_a - v_b}{|r_0|^2} \right) \quad (19)
\]

We develop the terms $v_a^2 - (v_a - v_b)^2$ and $v_a(r_0 \cdot v_a) - (v_a - v_b)(r_0 \cdot (v_a - v_b))$ in (20) and (21):

\[
v_a^2 - (v_a - v_b)^2 = 2v_a \cdot v_b - v_b^2 \quad (20)
\]

\[
v_a(r_0 \cdot v_a) - (v_a - v_b)(r_0 \cdot (v_a - v_b)) = v_b(r_0 \cdot v_a) + (v_a - v_b)(r_0 \cdot v_b) \quad (21)
\]

By introducing (20) and (21) into (19), we get:

\[
F_1 = \frac{e q_a}{4 \pi \varepsilon_0 c^2 |r_0|^2} \left( r_0(v_a \cdot v_b) - v_b(r_0 \cdot v_a) - (v_a - v_b)(r_0 \cdot v_b) - \frac{r_0 v_b^2}{2} \right) \quad (22)
\]

$F_1$ is the elementary magnetic force that the couple of charges $\Theta_b \Theta_b$ acts on the single charge $q_a$.

b. Force a couple of charges acts on another couple of charges

Suppose that the object at the point $a$ is a couple of charges, $\Theta_b \Theta_b$. The force that $\Theta_b \Theta_b$ acts on $\Theta_b \Theta_b$ equals the force that $\Theta_b \Theta_b$ acts on $\Theta_b$ plus that on $\Theta_b$. Let us first compute the force on $\Theta_b$, which is a fixed positive charge, so $q_a = e$ and $v_a = 0$. We introduce these $q_a$ and $v_a$ into (22) and get:
\[ F_{1+} = \frac{e^2}{4\pi \varepsilon_0 c^2 |r_0|^3} \left( v_b(r_0 \cdot v_b) - \frac{r_0 v_b^2}{2} \right) \]  

\( \Theta_s \) is a moving electron, its velocity is \( v_s \) and its charge is \( q_s = -e \). We introduce these \( q_s \) and \( v_s \) into (22) and get:

\[ F_{1-} = -\frac{e^2}{4\pi \varepsilon_0 c^2 |r_0|^3} \left( v_b(r_0 \cdot v_b) - v_b(r_0 \cdot v_a) - (v_a - v_b)(r_0 \cdot v_b) - \frac{r_0 v_b^2}{2} \right) \]  

The sum of these two forces equals:

\[ F_2 = F_{1+} + F_{1-} \]

\[ = \frac{e^2}{4\pi \varepsilon_0 c^2 |r_0|^3} \left( v_b(r_0 \cdot v_b) - \frac{r_0 v_b^2}{2} \right) \]

\[ - \frac{e^2}{4\pi \varepsilon_0 c^2 |r_0|^3} \left( r_0(v_a \cdot v_b) - r_0(v_a \cdot v_a) - (v_a - v_b)(r_0 \cdot v_b) - \frac{r_0 v_b^2}{2} \right) \]

\[ F_2 = \frac{e^2}{4\pi \varepsilon_0 c^2 |r_0|^3} \left( -r_0(v_a \cdot v_b) + v_b(r_0 \cdot v_a) + v_a(r_0 \cdot v_b) \right) \]

\( F_2 \) is the elementary magnetic force that a couple of charges in a current exerts on that of another current. Notice that \( F_2 \) is the magnetic force on a couple of charges while \( F_1 \) in (22) is that on a single charge. For the ions in a Tokamak we must use \( F_1 \), not \( F_2 \).

In fact, we have already derived \( F_2 \) in « From Coulomb’s force to magnetic force and experiments that show magnetic force parallel to currents »[1], see the equation (54) in this article. But, in the demonstration I have separately computed the dynamic effect for \( \Theta_s \Theta_b \) and \( \Theta_b \Theta_s \) and the changing distance effect for \( \Theta_s \Theta_b \) and \( \Theta_b \Theta_s \); then, I have bluntly added the two effects together, which is not rigorous. Here, the derivation of \( F_2 \) is mathematically rigorous.

### 4. Force from current-element

#### a. \( b \) is a current element

In a Tokamak the magnetic force on an ion is acted by a coil, so we will compute the force that a current element \( dI_b \) acts on an ion \( q_a \). Suppose \( dI_b \) contains \( n \) free electrons which move at the mean velocity \( v_a \). Then, \( dI_b \)

\[ dI_b = I_b dl = -n \cdot e v_b \]  

We express \( v_b \) with \( dI_b \):

\[ v_b = -\frac{dI_b}{n \cdot e} \]  

The number of couple of charges equals that of free electrons, so we introduce (28) into (22) and get:

\[ F_1 = \frac{eq_a}{4\pi \varepsilon_0 c^2 |r_0|^3} \left( r_0(v_a \cdot \left( -\frac{dI_b}{n \cdot e} \right) - \frac{dI_b}{n \cdot e} (r_0 \cdot v_a) - (v_a - v_b)(r_0 \cdot \frac{dI_b}{n \cdot e}) - \frac{r_0}{2} v_b \left( -\frac{dI_b}{n \cdot e} \right) \right) \]

We multiply both sides of (29) with \( n \) and cancel the \( e \)’s, we obtain:

\[ nF_1 = \frac{qa}{4\pi \varepsilon_0 c^2 |r_0|^3} \left( -r_0(v_a \cdot dI_b) + dI_b(r_0 \cdot v_a) + (v_a - v_b)(r_0 \cdot dI_b) + \frac{r_0}{2} (v_b \cdot dI_b) \right) \]
b. Expression with double vector product

Using the double vector product identity, \( A \times (B \times C) = -C(A \cdot B) + B(A \cdot C) \), we transform the term \(-r_o(v_a \cdot dI_b) + dI_b(r_0 \cdot v_a)\) in (30) into:

\[
-r_o(v_a \cdot dI_b) + dI_b(r_0 \cdot v_a) = v_a \times (dI_b \times r_o) \quad (31)
\]

We denote \( n F_i \) with \( F_c \), introduce (31) into (30) and obtain:

\[
F_c = nF_1 = \frac{q_a}{4 \pi \varepsilon_0 c^2 |r_0|^3} \left( v_a \times (dI_b \times r_0) + (v_a - v_b)(r_0 \cdot dI_b) + \frac{r_0(v_b \cdot dI_b)}{2} \right) \quad (32)
\]

\( F_c \) is the magnetic force that the current element \( dI_b \) acts on the single charge \( q_a \).

c. Lorentz force

Today, magnetic force is computed with the Lorentz force law. Let \( q_e \) be a single charge moving at the velocity \( v_0 \) in the magnetic field \( B \). According to the Biot–Savart law the magnetic field that \( dB \) creates on the charge \( q_e \) equals:

\[
dB = \frac{dI_b \times r_o}{4 \pi \varepsilon_0 c^2 |r_0|^3} \quad (33)
\]

Then the Lorentz force on \( q_e \) equals:

\[
F_{\text{Lorentz}} = \frac{q_a v_a \times dB}{q_a} = \frac{q_a}{4 \pi \varepsilon_0 c^2 |r_0|^3} v_a \times (dI_b \times r_0) \quad (34)
\]

d. Difference between \( F_c \) and the Lorentz force

Let us subtract (32) with (34), the difference between \( F_c \) and the Lorentz force equals:

\[
F_c - F_{\text{Lorentz}} = \frac{q_a}{4 \pi \varepsilon_0 c^2 |r_0|^3} \left( v_a \times (dI_b \times r_0) + (v_a - v_b)(r_0 \cdot dI_b) + \frac{r_0(v_b \cdot dI_b)}{2} \right)
- \frac{q_a}{4 \pi \varepsilon_0 c^2 |r_0|^3} v_a \times (dI_b \times r_0) \quad (35)
\]

This force is the part of magnetic force that the Lorentz force law ignores. We denote this force as \( F_e \) and call it “extra-force”. We rearrange (35) and obtain:

\[
F_e = F_c - F_{\text{Lorentz}} = \frac{q_a}{4 \pi \varepsilon_0 c^2 |r_0|^3} \left( (v_a - v_b)(r_0 \cdot dI_b) + \frac{r_0(v_b \cdot dI_b)}{2} \right) \quad (36)
\]

Because \( F_e \) is missing in the Lorentz force law, it could be the mysterious force that pushes the plasma to the wall.

e. Value of the extra-force

The value of the extra-force is evaluated with respect to the Lorentz force. Let us take a current element \( dI_b \) and a free electron in another current element \( dI_e \). Suppose that \( dI_b = dB \) and the velocities of free electrons are equal, \( v_b = v_e \). The radial distance \( r_0 \) is perpendicular to \( dB \) and \( dI_e \). The reference frame is defined by the unit vectors \( e_x \) and \( e_y \), then we have:

\[
\begin{align*}
r_0 &= r_0 e_x \\
dI_b &= dI_b e_y \\
v_a &= -v_e e_y \\
v_b &= -v_e e_y
\end{align*}
\]
The Lorentz force that \( dI_b \) acts on one electron of \( dI_a \) is computed with (34) and (37) for \( q_a = -e \) :

\[
F_{\text{Lorentz}} = \frac{-e(-v_a dI_b r_0)}{4\pi \varepsilon_0 c^2 |r_0|^3} e_y \times (e_y \times e_x) \\
= \frac{e v_a dI_b r_0}{4\pi \varepsilon_0 c^2 |r_0|^3} e_x
\]  

(38)

The extra-force that \( dI_b \) exerts on the same electron is computed with (36) and (37):

\[
F_e = \frac{-e}{4\pi \varepsilon_0 c^2 |r_0|^3} \left( 0 \times (r_0 \cdot dI_b) - \frac{v_b dI_b}{2} r_0 e_x \right) \\
= \frac{1}{2} \frac{ev_a dI_b r_0}{4\pi \varepsilon_0 c^2 |r_0|^3} e_x \\
= -\frac{F_{\text{Lorentz}}}{2}
\]  

(39)

So, in this case the extra-force equals minus half the Lorentz force.

**f. Force that a current element exerts on another current element**

Let us compute the force that a current element \( dI_b \) exerts on another current element \( dI_a \). Suppose that \( dI_a \) contains \( m \) free electrons and \( dI_b \) contains \( n \), then \( dI_a \) contains \( m \Theta_a \) and \( dI_b \) contains \( n \Theta_b \). Because \( F_z \) is the force that one \( \Theta_b \Theta_a \) acts on one \( \Theta_a \Theta_a \), the force that \( dI_b \) exerts on \( dI_a \) equals \( m \cdot n \) times \( F_z \). So, we multiply both sides of (26) with \( m \cdot n \) and obtain :

\[
F_{iba} = mnF_z = \frac{1}{4\pi \varepsilon_0 c^2 |r|^3} \left( -r \cdot (dI_a \cdot dI_b) + dI_b (r \cdot dI_a) + dI_a (r \cdot dI_b) \right)
\]  

(40)

\[
F_{iba} = \frac{1}{4\pi \varepsilon_0 c^2 |r|^3} \left( dI_a \times (dI_b \times r) + dI_b (r \cdot dI_a) \right)
\]  

(41)

\( F_{iba} \) is the magnetic force that the current element \( dI_b \) acts on the current element \( dI_a \).

In « From Coulomb’s force to magnetic force and experiments that show magnetic force parallel to current »\(^{[1]} \), the Lorentz force law and the Biot–Savart law have been correctly derived from (41). So, equation (41) is correct. Since (41) is derived from equation (22), equation (22) must be correct too and \( F_i \) can be safely used for the ions in a Tokamak.

**5. Force in Tokamak**

**a. Extra-force in a Tokamak**

If the extra-force is the mysterious force, it must point to the wall. The total extra-force on an ion is computed by integrating the extra-force over the entire confinement coil. Let us take a current element \( dI_b \), the mean velocity of the free electrons in \( dI_b \) is \( v_b \), the velocity of the ion \( q_a \) is \( v_a \) and the radial distance from \( dI_b \) to \( q_a \) is \( r = r_a \). For computing the extra-force on \( q_a \) we introduce these \( v_b, q_b, v_a \) and \( r \) into (36) and get :

\[
F_E = \oint F_e = \oint \frac{q_a}{4\pi \varepsilon_0 c^2} \left( \oint \frac{v_a (r \cdot dI_b)}{|r|^3} - \oint \frac{v_b (r \cdot dI_b)}{|r|^3} + \oint \frac{r (v_a \cdot dI_b)}{2|r|^3} \right)
\]  

(42)

This integral is for a given time, so \( v_a \) the velocity of the charge \( q_a \) is constant over the integration and is extracted from the first integral :

\[
\oint \frac{v_a (r \cdot dI_b)}{|r|^3} = v_a \oint \frac{r \cdot dI_b}{|r|^3}
\]  

(43)

The integral in (43) \( \oint \frac{r \cdot dI_b}{|r|^3} = 0 \) because the confinement coil is a closed loop. Then, equation (42) becomes :
\[ F_E = \frac{q_a}{4\pi \varepsilon_0 c^2} \left( -\int \frac{\mathbf{v}_b \cdot (d\mathbf{I}_b)}{|r|^3} + \int \frac{r(\mathbf{v}_b \cdot d\mathbf{I}_b)}{2|r|^3} \right) \] (44)

This equation shows that the direction of \( \mathbf{F}_E \) is determined by the sign of the charge \( q_a \). However, \( \mathbf{F}_E \) cannot easily be computed for the entire coil. So, we will compute the extra-force that a cross section of the toroidal coil exerts on one ion. Near the wall, the extra-force from the other part of the toroidal coil becomes negligible before that of the taken cross section and the direction of the total extra-forces is the same as that from the taken cross section.

Suppose that the taken cross section is a circular coil which we plot in Figure 4. The reference frame is determined by the unit vectors \( \mathbf{e}_r \) and \( \mathbf{e}_\theta \), the local unit vectors at the point of \( d\mathbf{I}_b \) are \( \mathbf{e}_r \) and \( \mathbf{e}_\theta \). The ion has the charge \( q_a \) and is at the position \( X_0 \mathbf{e}_x \). \( d\mathbf{I}_b \) is at the position \( Re \). The local unit vectors \( \mathbf{e}_r \) and \( \mathbf{e}_\theta \) equal:

\[
\mathbf{e}_r = \cos \theta \mathbf{e}_x + \sin \theta \mathbf{e}_y \quad (45)
\]

\[
\mathbf{e}_\theta = -\sin \theta \mathbf{e}_x + \cos \theta \mathbf{e}_y \quad (46)
\]

![Figure 4](image)

The radial distance from \( d\mathbf{I}_b \) to \( q_a \) equals the position of \( q_a \) minus the position of \( d\mathbf{I}_b \):

\[
\mathbf{r} = X_0 \mathbf{e}_x - Re \quad (47)
\]

\( d\mathbf{I}_b \) is expressed as:

\[
d\mathbf{I}_b = l_b d\mathbf{e}_\theta \quad (48)
\]

The mean velocity of the free electrons \( \mathbf{v}_b \) is opposite to \( d\mathbf{I}_b \) and is expressed as:

\[
\mathbf{v}_b = -\mathbf{v}_b \mathbf{e}_\theta \quad (49)
\]

For the integral \( \oint \frac{\mathbf{v}_b \cdot r \cdot d\mathbf{I}_b}{|r|^3} \) in (44) the term \( \mathbf{r} \cdot d\mathbf{I}_b \) is computed with (47) and (48):

\[
\mathbf{r} \cdot d\mathbf{I}_b = (X_0 \mathbf{e}_x - Re_x) \cdot (l_b d\mathbf{e}_\theta)
= X_0 l_b d\mathbf{e}_x \cdot \mathbf{e}_\theta - Rl_b d\mathbf{e}_x \cdot \mathbf{e}_\theta
= X_0 l_b d\mathbf{e}_x \cdot \mathbf{e}_\theta
\]

The dot product \( \mathbf{e}_x \cdot \mathbf{e}_\theta \) is computed with (46):

\[
\mathbf{e}_x \cdot \mathbf{e}_\theta = -\sin \theta \quad (51)
\]

By combining (49), (50) and (51) we get:

\[
\mathbf{v}_b (\mathbf{r} \cdot d\mathbf{I}_b) = (-\mathbf{v}_b \mathbf{e}_\theta) X_0 l_b d(\sin \theta)
= X_0 \mathbf{v}_b l_b \sin \theta d\mathbf{e}_\theta
\]

For the integral \( \oint \frac{\mathbf{v}_b \cdot r \cdot d\mathbf{I}_b}{2|r|^3} \) in (44), we compute the dot product \( \mathbf{v}_b \cdot d\mathbf{I}_b \) in (53) with (48) and (49). Then \( \mathbf{r}(\mathbf{v}_b \cdot d\mathbf{I}_b) \) is expressed in (54):
\[ \mathbf{v}_b \cdot d\mathbf{I}_b = -v_b I_b dl \quad (53) \]
\[ r(\mathbf{v}_b \cdot d\mathbf{I}_b) = -v_b I_b r dl \quad (54) \]

Introducing (52) and (54) into (44) we get:

\[ F_E = \frac{q_a}{4\pi \varepsilon_0 c^2} \left( -\oint X_a v_b l_b \sin \theta \, d\mathbf{e}_\theta - \oint v_b l_b \, r dl \right) \]
\[ = -\frac{q_a v_b l_b}{4\pi \varepsilon_0 c^2} \oint \left( X_0 \sin \theta \, \mathbf{e}_\theta + \frac{r}{2} \right) \left| \mathbf{r} \right|^3 dl \quad (55) \]

We take advantage of the symmetry of the circle and add the value of \( X_a \sin \theta \, \mathbf{e}_\theta + \frac{r}{2} \) for the angle \( \theta \) with that for the symmetrical angle \( -\theta \):

\[ \left( X_0 \sin \theta \, \mathbf{e}_\theta + \frac{r}{2} \right) + \left( X_0 \sin \theta \, \mathbf{e}_\theta + \frac{r}{2} \right)_{-\theta} \]
\[ = (X_0 \sin \theta \, \mathbf{e}_\theta + X_0 \sin (-\theta) \, \mathbf{e}_\theta + \frac{1}{2} (X_0 \mathbf{e}_x - R \mathbf{e}_r) + X_0 \mathbf{e}_x - R (\mathbf{e}_r)_{-\theta}) \]
\[ = X_0 \sin \theta (\mathbf{e}_\theta - \mathbf{e}_{-\theta}) + \frac{1}{2} (2X_0 \mathbf{e}_x - R((\mathbf{e}_r)_{\theta} + (\mathbf{e}_r)_{-\theta})) \quad (56) \]

With (45) and (46) we get the symmetrical unit vectors \((\mathbf{e}_r)_{-\theta}\) and \(\mathbf{e}_\theta\) and the values of \((\mathbf{e}_r)_{\theta} + (\mathbf{e}_r)_{-\theta}\) and \(\mathbf{e}_\theta - \mathbf{e}_{-\theta}\):

\[ (\mathbf{e}_r)_{-\theta} = \cos \theta \, \mathbf{e}_x - \sin \theta \, \mathbf{e}_y \]
\[ \mathbf{e}_{-\theta} = \sin \theta \, \mathbf{e}_x + \cos \theta \, \mathbf{e}_y \]
\[ (\mathbf{e}_r)_{\theta} + (\mathbf{e}_r)_{-\theta} = \cos \theta \, \mathbf{e}_x + \sin \theta \, \mathbf{e}_y + \cos \theta \, \mathbf{e}_x - \sin \theta \, \mathbf{e}_y \]
\[ = 2 \cos \theta \, \mathbf{e}_x \]
\[ \mathbf{e}_\theta - \mathbf{e}_{-\theta} = -2 \sin \theta \, \mathbf{e}_x \]

We introduce (59) and (60) into (56) and get:

\[ \left( X_0 \sin \theta \, \mathbf{e}_\theta + \frac{r}{2} \right)_{\theta} + \left( X_0 \sin \theta \, \mathbf{e}_\theta + \frac{r}{2} \right)_{-\theta} \]
\[ = -2X_0 \sin \theta \sin \theta \, \mathbf{e}_x + \frac{1}{2} (2X_0 \mathbf{e}_x - R \cos \theta \, \mathbf{e}_x) \]
\[ = -2X_0 \sin^2 \theta \, \mathbf{e}_x + (X_0 - R \cos \theta) \mathbf{e}_x \quad (61) \]

The integral in (55) is over a circle. Because of the symmetry we integrate over half a circle, that is, from \( \theta = 0 \) to \( \theta = \pi \). The elementary length of a circle is \( dl = Rd\theta \). We introduce (61) and \( dl = Rd\theta \) into (55) and reset the boundary of the integral, we get:

\[ F_E = -\frac{q_a v_b l_b}{4\pi \varepsilon_0 c^2 R} \int_0^\pi \left( -2X_0 \sin^2 \theta + (X_0 - R \cos \theta) \right) \frac{R d\theta}{|\mathbf{r}|^3} \]
\[ = \frac{q_a v_b l_b}{4\pi \varepsilon_0 c^2 R} \left( \frac{X_0}{R} \right) \int_0^\pi \sin^2 \theta \, d\theta - \frac{X_0}{R} \cos \theta d\theta \quad (62) \]

Let us simplify the form of \( F_E \) with the function \( K \) in (63), then \( F_E \) is written in (64):

\[ K = 2 \frac{X_0}{R} \int_0^\pi \sin^2 \theta \frac{d\theta}{|\mathbf{r}|^3} - \int_0^\pi \frac{X_0}{|\mathbf{r}|^3} \cos \theta d\theta \quad (63) \]
\[ F_E = \frac{q_a v_b l_b}{4\pi \varepsilon_0 c^2 R} K \mathbf{e}_x \quad (64) \]
For determining $K$ we have numerically computed its second part function $f(X_0)$ which cannot be determined analytically:

$$f(X_0) = -\int_0^\pi \frac{X_0/R - \cos \theta}{|r/R|^2} d\theta$$

(65)

And plotted its values in Figure 5:

Figure 5

![Figure 5](image)

Figure 5 shows that the function $f(X_0)$ is positive for $0 < X_0 < R$. On the other hand, the value of $\int_0^\pi \frac{\sin^2 \theta}{|r/R|^2} d\theta$ is positive. So, the function $K$ is positive, see (63). For positive ions, $q_a > 0$, then the extra-force $F_{E\oplus}$ is in the same direction as the unit vector $e_x$, see (64):

$$F_{E\oplus} = \left| \frac{ev_B I_b}{4\pi e_0 c^2 R} K \right| e_x$$

(66)

For electrons, $q_a < 0$ and the extra-force $F_{E\oplus}$ is in the opposite direction as $e_x$:

$$F_{E\oplus} = -\left| \frac{ev_B I_b}{4\pi e_0 c^2 R} K \right| e_x$$

(67)

As $e_x$ is directed to the nearest current element on the wall, the positive ions are attracted by the wall while the electrons are pushed away from the wall, see Figure 6. In consequence, the wall is eroded by positive ions, not by electrons.

![Figure 6](image)
b. Keeping the plasma away from the wall

We could protect the wall if we could repel the positive ions. But, if an electromagnetic force can repel positive ions, the same force will forcefully attract electrons and the electrons would erode the wall instead of positive ions. So, it seems impossible to repel the plasma from the wall.

But I have figured out a way to do so. Let us imagine a cloud of positive ions, in the middle of which is a negative charge. Because the negative charge attracts the positive ions, the cloud would not expand. Can we apply this principle to limit the expansion of the plasma in a Tokamak?

A cloud of plasma would expand under its proper pression. But if it contains an excess of electrons it becomes negatively charged and would interact with the extra-force like a negative cloud. More precisely, the electrons in excess would drift outward due to the electrostatic force among them. At equilibrium, the electrons in excess would concentrate on the surface of the cloud and rearrange within the plasma such that the electric potential is constant. In a Tokamak’s chamber, the surface of the cloud is a thin layer near the wall which we call the skin of the plasma.

How would the skin interact with the extra-force? Let us take a bunch of particles from the skin which contains m positive ions and n electrons. The extra-force on one positive ion is given by (66) and that on m positive ions equals:

$$mF_{\Theta\Theta} = m \frac{ev_bI_b}{4\pi \varepsilon_0c^2R} K e_x$$  \hspace{1cm} (68)

The extra-force on one electron is given by (67) and that on n electrons equals:

$$nF_{\Theta\Theta} = -n \frac{ev_bI_b}{4\pi \varepsilon_0c^2R} K e_x$$  \hspace{1cm} (69)

Then, the extra-force on the bunch of particles equals:

$$F_{\Theta\Theta} = mF_{\Theta\Theta} + nF_{\Theta\Theta} = (m - n) \frac{ev_bI_b}{4\pi \varepsilon_0c^2R} K e_x$$  \hspace{1cm} (70)

Because the skin is richer in electron, n is bigger than m, so m - n < 0 and:

$$F_{\Theta\Theta} = - (m - n) \frac{ev_bI_b}{4\pi \varepsilon_0c^2R} K e_x$$  \hspace{1cm} (71)

$F_{\Theta\Theta}$ is the extra-force on the bunch and is opposite to $e_x$, which means that the bunch would be repelled by the wall, creating a void space between the wall and the skin, see Figure 7.

![Figure 7](image_url)

However, there can be many objections to this point of view. For example, why don’t the positive ions move to the wall as the extra-force pushes them in this direction? In fact, the positive ions and the electrons of the bunch...
being very close and the electrons more numerous, the electrostatic force of the electrons is much stronger than the extra-force and the positive ions would stay near the electrons in spite of the extra-force.

Figure 7 shows a bunch in the skin that contains one positive ion and two electrons. Because \( F_{\text{eo}} \), the net extra-force on the bunch, points away from the wall and the positive ions is locked with the two electrons, the positive ions will follow the two electrons and move away from the wall. In consequence, the three particles will move as a whole under the force \( F_{\text{eo}} \). As all bunches of the skin would move as a whole, the skin itself would move away as a whole leaving a void space near the wall.

Another objection would be: is the extra-force strong enough to hold the skin away? I think that, because the plasma is already strongly confined by the confinement field, the charged particles that could escape the confinement field plus the extra-force would be rare. Only experiment could determine how much particles could make to the wall. We reasonably expect that the negatively charged skin would give a good protection to the wall.

Moreover, we have a way to still improve the protection by negatively charging the wall itself. This way, on top of the extra-force, the wall would act a negative electrostatic force on the skin which would be pushed further away making the void space larger. This idea shows that, in the case where you do not agree with the extra-force, you can still create a void space near the wall by negatively charging the wall.

For fine-tuning the protection, we now have two adjustable parameters: the quantity of electrons in excess in the plasma and the quantity of negative charge in the wall. The real key of this technique of protection is adding electrons to the plasma.

6. How to test the extra-force

Because the extra-force is not predicted by the Maxwell’s theory and has eluded experimentation for more than one and half century, the physical community would not accept this new force except it is shown in experiment. In order to test the action of the extra-force I have designed an experiment which will show the influence of the extra-force on the electron beam of a cathode ray tube.

The cathode ray tube is placed outside and in the middle of a very long solenoid. According to the Maxwell’s theory, the magnetic fields inside and outside a long solenoid are parallel to the solenoid. Moreover, outside and in the middle region the magnetic field is zero. Let \( B \) be the magnetic field and \( v_e \) the velocity of the electrons of the beam. The cathode ray tube is parallel to the solenoid so that \( v_e \) is parallel to the solenoid. Because \( B \) is also parallel to the solenoid, \( B \) and \( v_e \) are parallel vectors and the Lorentz force on the electrons which is \( q_e v_e \times B \) equals zero. In consequence, the electron beam should not be deflected and the spot on the screen of the cathode ray tube made by the electron beam should stay still in the center, see Figure 8.

![Figure 8](Image)

The total extra-force that the solenoid exerts on one electron is computed with (44). The charge of an electron is \( q_e = -e \), a current element of the solenoid is \( dI_b \). We introduce these \( v_e \), \( q_e \) and \( dI_b \) into (44) and get:

\[
F_E = -\frac{e}{4\pi \varepsilon_0 c^2} \left( -\oint \frac{v_e \cdot dI_b}{|r|^3} + \oint \frac{r (v_e \cdot dI_b)}{2|r|^3} \right)
\]  

(72)
As \( \mathbf{F}_e \) is not zero, the electron beam should be deflected and the spot on the screen should move accordingly. For testing the action of the extra-force, we will vary the current in the solenoid to make the extra-force vary with the current. If the extra-force is real, the light spot would move in synchronicity with the current, see Figure 8. So, the result of this experiment would be:

- If the spot moves on the screen, the extra-force exists and the Maxwell’s theory is not fully correct.
- If the spot does not move, the extra-force does not exist.

The interior of a solenoid is similar to the interior of the toroidal chamber of a Tokamak. So, it is interesting to test the extra-force in the interior of a large solenoid. The result of this experiment would be the same as that in the exterior.

### 7. Discussion

In this article we have derived four forces:

1. The force that one single charge exerts on another single charge, see (16). We call it “charge-to-charge force”.
2. The force that one current element exerts on a single charge, see (32). We call it “current-to-charge force”.
3. The difference force between the “current-to-charge force” and the Lorentz force on a single charge, see (36). We call this force “extra-force”.
4. The force that one current element exerts on another current element, see (41). Because this force is different from the Lorentz force we call it “current-to-current force”.

The current-to-charge force is derived from the Coulomb’s law and thus, respects the Newton's third law. The current-to-current force and current-to-charge force are both sums of charge-to-charge forces and thus, respect the Newton's third law too. In the contrary, the Lorentz force law does not respect the Newton's third law and is flawed.

Tokamaks are designed with the most accurate electromagnetic theory today: the Maxwell’s theory. In spite of the highest care taken in the design, Tokamaks still suffer from erosion. But we are unable to find a solution with the Maxwell’s theory. So, the Maxwell’s theory is not sufficiently correct for current to plasma interaction.

We have derived the new extra-force in (36) which, according to our computation, pushes the plasma to the wall. So, we have understood how erosion occurs, which enables us to propose a technical solution to prevent erosion. This solution consists of adding more electrons into the plasma which would become negatively charged and be repelled by the extra-force. So, we have not only improved the understanding of electromagnetism, but also brought real technical progress.

For showing the existence of the extra-force we have proposed an experiment using solenoid and cathode ray tube. Previously, I have done a similar experiment which is shown in this video [https://www.youtube.com/watch?v=DXu88treBj4](https://www.youtube.com/watch?v=DXu88treBj4). The explanation of this experiment is in «Aharonov–Bohm effect in CRT experiment» [2]. But in this experiment the electron beam is perpendicular to the solenoid rather than parallel.

The experiments with solenoid are important because they would validate the extra-force which is a magnetic force in a region where the Maxwell’s theory predicts no magnetic force. The reason that the Lorentz force law does not contain the extra-force is that this law was established with the data of former experiments which were all done with current carrying wires. A conductor wire is formed by free electrons and positive ion. Although the extra-forces on the positive and negative charges exist within the wire, they are opposite and cancel out. This is why the extra-force does not appear on a neutral wire. In consequence, the Lorentz force law cannot contain the extra-force because the latter has never been reported in any experimental data and was known to no one.

Because of its founding experiments, the subject of the Lorentz force law is current carrying wires in closed loop. Single charge has never been tested for Lorentz force and thus is not the subject of the Lorentz force law. A physical law is valid only for its subject but not for things that are not its subject. So, the Lorentz force law is valid only for current carrying wires in closed loop, but not for single charges.
A more radical way to make swift progress is to directly add electrons in a functioning Tokamak and see if the plasma separates from the wall in someplace. Because there are many working Tokamaks all over the world, many teams can do this experiment with the existing equipment without having to create new one and then, these experiments can be done fast. The validation of this concept would accelerate the achievement of controlled nuclear fusion. So, I highly recommend interested teams to perform the experiments. The first successful team would get the most credit for this important step to the future.

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Letter to readers

Dear readers,

I have derived the extra-force because I thought that the magnetic force on a single charge could be different from the Lorentz force. Why did I have that intuition? We know that currents are composed of positive charges and moving electrons. So, the Lorentz force is surely the difference between the force on the positive charges and that on the moving electrons. Because the moving electrons have a velocity while the positive charges do not, it is surely the velocity that modifies the electrostatic force. The only way physical quantities could be modified in moving frame is through relativity. So, I have tried to modify electrostatic force with relativity and fortunately, was successful in deriving the 4 magnetic forces.

I have proposed to inject additional electrons into functioning Tokamak to protect its wall. This technique would not only make Tokamak work longer, but also allow higher temperature of the plasma because the latter would no longer touch the wall. This technique would increase the density of the plasma because the extra-force is a compressing force so that more plasma can be loaded. In consequence, the confinement time, temperature and density would all be improved at once, making the Lawson criterion and controlled nuclear fusion closer to reach.

The Maxwell’s theory is very successful for explaining the majority of electromagnetic phenomena, which makes us believe that it can correctly predict all electromagnetic phenomena. However, there are things it does not predict but really exist, for example, the extra-force. When these things happen, for example, the erosion of the wall of Tokamaks, we do not understand why they occur, let alone to fix their consequence if disastrous. So, it is important to correct the Maxwell’s theory when a flaw is found.

The missing of the extra-force is a flaw in the Maxwell’s theory and its discovery a breakthrough for electromagnetism. So, this article topples the Maxwell’s theory again and will be no doubt rejected by all respectable journals. In consequence, I will not submit it for publication but only post online. I’m sure that you, my online readers, would have the wise to recognize its value and would promote it either by sharing it with your colleagues or by performing the proposed experiments. If you succeed in doing the experiments, you will get honorable credit for your contribution.

Kuan Peng, 13 March 2024