A NEW NUMERICAL INTERPRETATION OF THE CONCEPT OF EXPONENTORY (Θ NOTATION)

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0- Abstract

In this paper I show a possible change in the theory of series beyond product. Instead of a resolution Bottom-to-Top we will see a necessary application of the method for exponents that is a process Top-to-Bottom. That implies a change in the numerical results in a same proposition of a series.

1- Introduction

The idea developed by me [1] about serial operator of exponents has shown recently some problems in the numerical interpretation if we approach the series of functions to hyper-operations algebras. It is not in fact possible an interpretation of resolution of the exponential serial operation in a Bottom-to-Top resolution of the resultant exponential tower. It was firstly my idea to get a simplification of the computations and obtain with simple series not too big numbers. But it is a contrary interpretation of the classic literature in mathematics since Euler [3] (fifth Fermat number $2^{(2^{16})}+1 = 4,294,967,297$ is not prime) obviously using Top-to-Bottom resolution, and maybe before him.

2- The interpretation change

In my first approach of the operator exponentory I defined it as

$$\begin{align*}
\frac{b}{\Theta} \, f(n) &= f(a) \uparrow f(a+1) \uparrow \cdots \uparrow f(b-1) \uparrow f(b) \\
\frac{n=a}{\rightarrow}
\end{align*}
$$

(1)

The arrow indicates the order of resolution. But adapting it to the theories like tetration [2] (itered exponentiation) that is why the order of resolution have to change to:
\[
\begin{align*}
\begin{array}{c}
\Theta \\
\end{array}
\begin{array}{c}
f(n) = f(a) \uparrow f(a+1) \uparrow \ldots \uparrow f(b-1) \uparrow f(b) \\
\end{array} \\
\begin{array}{c}
n=a \\
\end{array}
\end{align*}
\] (2)

3- Numerical examples

First we are going to view my own order of resolution (which was wrong in a classic point of view).

\[
\begin{align*}
\begin{array}{c}
5 \\
\Theta \\
n=3 \\
\end{array}
\begin{array}{c}
f(n) = 3 \uparrow 4 \uparrow 5 = 81 \uparrow 5 = 3486784401 \\
n=3 \\
\end{array}
\end{align*}
\] (3)

Now, in the other hand, the correct operation order following tradition of exponential towers:

\[
\begin{align*}
\begin{array}{c}
5 \\
\Theta \\
n=3 \\
\end{array}
\begin{array}{c}
f(n) = 3 \uparrow 4 \uparrow 5 = 5 \uparrow 1024 = 3,73 \cdot 10^{488} \\
n=3 \\
\end{array}
\end{align*}
\] (4)

4- First property of exponentory operator

I want to express here something related to the topic as an extra. Exponentory has neutral element in single lineal variable:

\[
\begin{align*}
\begin{array}{c}
b \\
\Theta \\
n=1 \\
\end{array}
\begin{array}{c}
f(n) = 1 \\
n=1 \\
\end{array}
\end{align*}
\] (5)

The proof is very simple, any power with 1 in the basis has a result of 1. ( \( 1^n = 1 \forall n \in \mathbb{C} \) )
Which implies that every finite or infinite significant series should start in a number \( n > 2 \) on single lineal variable.

5- Conclusions

In my way to obtain more reasonable results in numeric applications of exponentory I misunderstood tradition in process of resolution of exponential towers, but if the mathematician can assume that very large numbers will be obtained in the use of the \( \Theta \) notation, the resolution Top-to-Bottom is more accurate.
6- References