A mathematical criterion for the validity of the Riemann hypothesis

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abstract

We already know in what situations there will be counterexamples for the Riemann hypothesis, but simply increasing Im (s) to find counterexamples for the Riemann hypothesis is still very slow. If there is only a counterexample when Im (s)=10 ^ 1000, or even 10 ^ 10000, then the performance requirements for the computer are very demanding. So, we must create a numerical order determinant to determine whether the Riemann hypothesis holds.

About ζ(s) =Re (ζ) + Im (ζ) i
We make s= ½+i * t, can be studied ζ(s)= ζ(½+i * t) Curve about t
Make an Im (ζ) - Re (ζ) curve, we conclude that
Im (ζ) > 0, Re (ζ) > 0 is the first quadrant
Im (ζ) > 0, Re (ζ) < 0 is the second quadrant
Im (ζ) < 0, Re (ζ) < 0 is the third quadrant
Im (ζ) < 0, Re (ζ) > 0 is the fourth quadrant
When the curve of Im (ζ) - Re (ζ) rotates clockwise, there are several possibilities
First Quadrant - Fourth Quadrant, Fourth Quadrant - Third Quadrant, Fourth Quadrant - Second Quadrant, Second Quadrant - First Quadrant - First Quadrant
When the curve of Im (ζ) - Re (ζ) rotates counterclockwise, there are several possibilities
Fourth Quadrant - First Quadrant, Third Quadrant - Fourth Quadrant, Second Quadrant - Fourth Quadrant, First Quadrant - Second Quadrant, First Quadrant - Third Quadrant
The remaining two cases, the third quadrant - second quadrant, and the second quadrant - third quadrant, only occur when the Riemann hypothesis has a counterexample
For the Riemann hypothesis, if there is no counterexample, then Im (ζ) - Re (ζ) It is a full curve.
If there is a counterexample, it will become a curve in the shape of Bagua, as shown in the following figure
We can make a judgment equation based on this characteristic

\[ \text{Im} (\zeta) \quad \text{Regarding } \text{Re} (\zeta) \quad \text{Take the derivative to obtain a function } g(t) \text{ with respect to the slope of } t \]

\[ g(t) = \frac{d \text{ Im} (\zeta)}{d \text{ Re} (\zeta)} \tag{1} \]

Then we let \( g(t) \) take the derivative of \( t \) and obtain the following equation

\[ g'(t) = \frac{\frac{d}{d t} \left( \frac{d \text{ Im} (\zeta)}{d \text{ Re} (\zeta)} \right)}{t} \tag{2} \]

One basis for determining whether the Riemann hypothesis is valid is

If there exists \( t \) such that \( g'(t) = 0 \), then the Riemann conjecture has a counterexample

\[ \text{Im} (\zeta) = \sum_{n = 1}^{+\infty} \frac{\sin (-t \ln n)}{\sqrt{n}} \tag{3} \]

\[ \text{Re} (\zeta) = \sum_{n = 1}^{+\infty} \frac{\cos (-t \ln n)}{\sqrt{n}} \tag{4} \]
Therefore, we can obtain

\[
g(t) = \frac{d \text{Im} (\zeta)}{d \text{Re} (\zeta)} = \frac{\sum_{n=1}^{\infty} -\ln n \cos (-t \ln n)}{\sum_{n=1}^{\infty} \ln n \sin (-t \ln n)}
\]
For taking the derivative of $g(t)$, we obtain

\[
g'(t) = \left\{ \begin{array}{c}
\sum_{n=1}^{+\infty} \frac{-\ln n \cos (-t \ln n)}{\sqrt{n}} \\
\sum_{n=1}^{+\infty} \frac{\ln n \sin (-t \ln n)}{\sqrt{n}}
\end{array} \right\} \cdot \frac{d}{dt} + \left\{ \begin{array}{c}
\sum_{n=1}^{+\infty} \frac{\ln n \cos (-t \ln n)}{\sqrt{n}} \\
\sum_{n=1}^{+\infty} \frac{-\ln n \sin (-t \ln n)}{\sqrt{n}}
\end{array} \right\} \cdot \frac{d}{dt}
\]

\[
= \left[ \begin{array}{c}
\sum_{n=1}^{+\infty} \frac{\ln n \sin (-t \ln n)}{\sqrt{n}} \\
\sum_{n=1}^{+\infty} \frac{-\ln n \cos (-t \ln n)}{\sqrt{n}}
\end{array} \right] \cdot \frac{d}{dt} + \left[ \begin{array}{c}
\sum_{n=1}^{+\infty} \frac{\ln n \sin (-t \ln n)}{\sqrt{n}} \\
\sum_{n=1}^{+\infty} \frac{-\ln n \cos (-t \ln n)}{\sqrt{n}}
\end{array} \right] \cdot \frac{d}{dt}
\]

\[
= \left[ \begin{array}{c}
\sum_{n=1}^{+\infty} \frac{\ln n \sin (-t \ln n)}{\sqrt{n}} \\
\sum_{n=1}^{+\infty} \frac{-\ln n \cos (-t \ln n)}{\sqrt{n}}
\end{array} \right]^2 \cdot \frac{d}{dt}
\]
\[
\sum_{n=1}^{+\infty} \frac{\ln n \cos (-t \ln n)}{\sqrt{n}} = \sum_{n=1}^{+\infty} \frac{\ln n \sin (-t \ln n)}{\sqrt{n}} + \sum_{n=1}^{+\infty} \frac{2 \ln n \cos (-t \ln n)}{\sqrt{n}} - \frac{2 \ln n \sin (-t \ln n)}{\sqrt{n}} \]

\[
\sum_{n=1}^{+\infty} \sum_{m=1}^{+\infty} \frac{\ln n \sin (-t \ln n)}{\sqrt{n}} \cdot \frac{\ln m \sin (-t \ln m)}{\sqrt{m}} = \sum_{n=1}^{+\infty} \sum_{m=1}^{+\infty} \frac{\ln n \cos (-t \ln n)}{\sqrt{n}} \cdot \frac{\ln m \cos (-t \ln m)}{\sqrt{m}} + \sum_{n=1}^{+\infty} \sum_{m=1}^{+\infty} \frac{2 \ln n \sin (-t \ln n)}{\sqrt{n}} \cdot \frac{2 \ln m \cos (-t \ln m)}{\sqrt{m}} - \sum_{n=1}^{+\infty} \sum_{m=1}^{+\infty} \frac{\ln n \sin (-t \ln n)}{\sqrt{n}} \cdot \frac{\ln m \cos (-t \ln m)}{\sqrt{m}}\]

\[
\sum_{n=1}^{+\infty} \frac{\ln n \sin (-t \ln n)}{\sqrt{n}} = \sum_{n=1}^{+\infty} \frac{\ln n \sin (-t \ln n)}{\sqrt{n}} \cdot \frac{\ln n \cos (-t \ln n)}{\sqrt{n}} + \frac{\ln n \sin (-t \ln n)}{\sqrt{n}} \cdot \frac{\ln n \sin (-t \ln n)}{\sqrt{n}} \cdot \frac{\ln n \cos (-t \ln n)}{\sqrt{n}} \cdot \frac{\ln n \cos (-t \ln n)}{\sqrt{n}}
\]
If we set $g'(t)=0$, then we have

\[
\sum_{n=1}^{+\infty} \sum_{m=1}^{+\infty} \frac{\ln n \ln m \cos (-t \ln n + t \ln m)}{\sqrt{n m}} = 0 \tag{9}
\]

Similarly, we can set $u'(t)=0$ and obtain

\[
\sum_{n=1}^{+\infty} \sum_{m=1}^{+\infty} \frac{(-1)^n \ln n \ln m \cos (-t \ln n + t \ln m)}{\sqrt{n m}} = 0 \tag{10}
\]

If we set $u'(t)=0$, then we have

\[
\sum_{n=1}^{+\infty} \sum_{m=1}^{+\infty} \frac{(-1)^n \ln n \ln m \cos (-t \ln n + t \ln m)}{\sqrt{n m}} = 0 \tag{11}
\]
For (11), in cases where accuracy requirements are not high, $t > 14.13412514$, there are $s=0.5+ \sigma i * t \ (\sigma \neq 0)$ is a counterexample of the Riemann hypothesis.

References

1. viXra:2005.0284 The Riemann Hypothesis Proof  Authors: Isaac Mor