Some Interesting Closed Form Expressions That Approximate Dimensionless Physical Constants

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TREASURE TROVE OF MATHEMATICS

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Abstract

The process of expressing the fundamental dimensionless physical constants in terms of nothing but integers, mathematical constants and arithmetical operations is known as enumeration. Many physicists believe that the derivation of the fundamental constants is impossible. However, I come from the school of thought that the physical universe is an incomplete system and, if we are ever to come to a true understanding of the physical universe, we must include the nonphysical in our theories.

I believe the nonphysical world has a mathematical structure, at least in part. As many physicists as possible should engage in enumeration as it could lead to a new theory that accounts for the values of these constants, in particular the fine structure constant, and we may learn some new physics along the way.

The expressions that approximate the fundamental constant should be as simple as possible, should only contain integers, mathematical constants and arithmetical operations. I will clue you in as to how to do these things for yourself; it’s simple in practice. All one has to do is go to the Wolfram Alpha website and type the value of the fundamental constant into the search box. In short order Alpha will return several possible closed form expressions. It’s that easy! So let’s get started.
Fine Structure Constant

We enter the experimental value of the inverse of the fine structure constant, $\frac{1}{\alpha} \sim 137.0359990(8451)$, into Wolfram Alpha and it returned several possible closed forms. The most interesting is:

$$58e! + \frac{340}{7} + \frac{144}{e} - \frac{545e}{7} \sim 137.035999084511.$$ 

Next we entered the experimental value of the fine structure constant, $\alpha \sim 0.0072973525693$, and Alpha returned two interesting closed forms:

$$\frac{7}{12} - \frac{9}{4e} + \frac{5e}{54} \sim 0.0072973525919$$

and

$$\frac{135}{8} - \frac{55 F_{FF}}{4} \sim 0.0072973525951$$

where $F_{FF}$ is the Fibonacci factorial constant,

$$F_{FF} \sim 1.226742010720353244417.$$ 

Proton-Electron Mass Ratio

This is my favorite one. It turns out that the proton-electron mass ratio is extremely well approximated by the fourth root of an integer! The experimental value of the proton-electron mass ratio is $m_p/m_e \sim 1836.15267343(11)$. We plug this value into Alpha and here’s what it found:

$$\sqrt[4]{11366719876399} \sim 1836.15267343109087.$$
I find that result absolutely amazing! What are the chances of that being a coincidence? In this case enumeration is hinting at some as yet unknown physics. It’s difficult to account for this occurrence in terms of any known physical theory. Now you can see the value of enumeration.

**Neutron-Electron Mass Ratio**

We plugged the experimental value of the neutron-electron mass ratio, \( m_n/m_e \sim 1838.68366173(89) \), into Alpha and it returned a simple closed form expression with surprising accuracy:

\[
\frac{33967}{570 \pi} + \frac{17377 \pi}{30} \sim 1838.683661738897.
\]

**Neutron-Proton Mass Ratio**

Lastly, we look at the neutron-proton mass ratio, which has the experimental value

\[
m_n/m_p \sim 1.00137841931(49),
\]

which we entered into Alpha. Alpha delivered a result that was so accurate that it predicts digits beyond our ability to measure with certainty. Here’s the concise result:

\[
\frac{e}{2} - \frac{160S_S}{799} \sim 1.0013784193149,
\]

where \( S_S \) is the Silverman constant given by

\[
S_S \sim 1.786576459365922.
\]
**Closing Thoughts**

Enumeration is a process that could shed light on aspects of reality that physics has been ignoring, i.e., the nonphysical. The physical universe is probably an incomplete system (in the Gödelian sense) and certain aspects of it will probably never be accounted for unless the physics community eventually entertains the nonphysical and its effects on the physical.

The closed form equations we choose as representative of a dimensionless physical constant should be as simple as possible, as we saw in the proton-electron mass ratio example. One should note that in principle an infinite number of equations could equate with any real number to arbitrary accuracy. So for that reason, we choose the simplest equations.