Some remarks on the generalization of atlases

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Abstract

We generalize atlases for flat stacks over smooth bundles by constructing
topic-global bijections between modules of differing order. We demonstrate
an adjunction between a special mixed module and a holonomy groupoid.

1 Notation and Conventions

Throughout, $Strat^{{\ast}}_M \subseteq Man$ will denote a point-for-point stratification on a
manifold $M$. We will let $Strat^\Delta_M$ denote the conical stratification, $Strat^u_M$ will
denote an open (portable) cover of a parameter space, which is dependent upon
the character $i$ for topological realization.

Call every $u_i$ a covering sieve, and let every

$$U_\alpha = \bigcup_i \alpha_i \in s_i \times s_i$$

Let, for every path $x \to -x$, there be a corresponding value $\Psi : X \to Y$.
In other words, we define a polar path (of polarity 1), by the map

$$\Pi_x : \pi \xrightarrow{\quad\pi^{-1}} -\pi$$

by inducing an isomorphism,

$$Id_x \simeq y \cdot i \in x_i$$

To explicitly define induction, as a first-principle operator, would be difficult,
though not entirely intractable. Recall from the adjunction

$$Hom(x, y) \xrightarrow{\sim} x \otimes y$$

that there is a path

$$\text{min} \to \text{max}(\exp(\pm x \times y))$$
where, for every nth order operation,
\[ x \otimes_n y \]
there is a rank n retract, consisting of 2n arrows \( \ker(x) \to \text{im}(x) \sim \ker(y) \to \text{im}(y) \). Let \( \mathcal{H}^2 \) denote the upper half-plane, and
\[ \mathcal{H}^1 \simeq \mathbb{A}^1 \]
hold by isometry between the cross product of diagonals
\[ \sqrt{\Delta^2} (x \times x) \cdot (y \times y) \simeq \text{Pull}_{\delta}(\text{Hom}(x, y)) \]

\textbf{Definition 1.} Let \( \mathcal{C}_\infty \) be an infinite ordered chain. Let every morphism be injective, and surjective, and therefore a bijection. We let, for every \( \varepsilon \in (x \in X) \times (y \in Y) \), there is a corresponding fraction, \( \frac{1}{n} \simeq \delta \).

Call every map \( A \xrightarrow{\delta} B \) of generalized spaces a \( \delta \)-pushout, and call its inverse \( \delta^{-1} \) a \( \delta \)-pullback.

The \( \leftrightarrow \) will denote a monomorphism, and \( \rightarrow \) will denote an epimorphism.

\section{Lucid sets and inner homs}

Let \( \mathcal{C} : \mathcal{C} \longrightarrow \mathcal{C}_{\text{SET}} \) be a perfect immersion. Call the image of a distinguished character \( i \in \mathcal{C} \) a lucid map, if its pullback is an etale object in the \( \mathcal{C} \)-category, which we will later enrich.

Let there be a bijection between an index \( \mathfrak{A} \), and a category \( \mathcal{C} \), and another between \( \mathcal{C}_{\text{SET}} \) and \( \mathcal{C} \). Let the index generate a class of open submersions

\[ U_{\alpha_i} : x \leftrightarrow y \cap * \]

\textbf{Proposition 1.} There is a \( \delta \)-pushout for every element \( x_i \in \mathcal{C} \).

\textbf{Example 1.} The \( \mathcal{C}^1 \) space has a mirror with Holder continuity \( \mathcal{C}^\infty \rightarrow |\mathcal{C}|_\pm \). This allows the bi-crossed module of inner homs

\[ \Pi; x \star y \]

to have a one-to-one bijection with orbital elements, which, as have been previously shown, act as local isotropy groupoid realizations.

\textbf{Example 2.} The counting operad \( x \star_+ y \) induces a transitive relationship,

\[ \mathcal{R}_{xy} : \sum_{i=0}^{\infty} x_i \to y_i \]
on the set of open objects in the collective isotropy group module,

\[ \bigcup_i \mathcal{G}_{x_i} \]
**Example 3.** As the fiber $x \Rightarrow y$ splits, we obtain morphisms $pr_0$, $pr_1$, and $pr_2$, inducing a conical foliation on an abstract space. The critical point may be written $z_k$, where the harmonic function

$$\sum_{n=\aleph_0}^{\aleph_\omega} \frac{1}{n} \alpha_n + \frac{1}{n+1} \alpha_{n+1} - \frac{1}{n+\infty} \alpha_{n+\infty}$$

gives us the value of $k$.

![Diagram](https://example.com/diagram.png)

Recall that, for a totally lossless projection of a perfect map, there exists a perfect inverse. In formal terms:

**Axiom 1.** $(\text{Perf}(x) \Rightarrow \text{Perf}(y)) \Rightarrow \text{Perf}((x \cdot y)^{-1})$

In fact, by this we mean strictly

$$\text{Perf}(x, y) \Rightarrow \text{Perf}(x, y \cdot, -)$$

So, for $C$ a small category, whose objects are etale, the topological realization

$$c \in C \Rightarrow \{\ast\}$$

, we are given (for free), a logical implication

$$c \Rightarrow C_{\text{SET}}$$

be “remembering” the inner hom constructed in example 1.

Some topics of interest for this implication may include portability, which further implies holonomy if the underlying stratification element is a manifold. Thus, as a result, it may be worth considering orbifolds as well, leading to a more nuanced theory of orbispaces.

I am inclined to state that, at the macroscopic level, everything that is observed is portable; i.e., it exhibits actions which are derivatives with respect to time. This is to say, everything observed in the practical world, is a submodule of the enriched vector space overlying the stack $\mathcal{A}$ from which the topology is derived.

**Proposition 2.** Let

$$\text{Hol}_n \simeq (\mathcal{A} \rightarrow \text{Strat}^\bullet)$$

Then, there is an arithmetic mapping

$$n \rightarrow \bullet$$

which is locally contractible if it is simply connected.
Remark 1. \( \text{Hol}_n \), of course, denotes the holonomy groupoid of Ehresmann, with rank \( n \) isomorphisms. Recall that a rank \( n \) isomorphism is a cutting of a fiber \( q \) into \( n \) isotropic (of equal arity) segments.

**Proposition 3.** \( \mathcal{A} \rightarrow \text{Strat}^\omega \) may be extended to a logical implication

\[ \mathcal{A} \iff \text{Strat}^\omega \]

**Proof.** The inverse, \( \text{Id}_n^{-1} \) may be composed with \( \text{Id}_n \) to yield the null groupoid.\(^2\) \( \square \)

### 3 Classifying spaces, atlases, and diffeomorphisms

Let \( \mathcal{C} \rightarrow \mathcal{C}_+ \) be an additive, polar mapping. Let there be a manifold \( \mathcal{M} \rightarrow \mathcal{C}_+ \) generated by taking the first derivative of a tangent fiber at any arbitrary point, with respect to time.

Let there be a Haefligger classifying space,

\[ \Gamma^q \cdot BG \]

such that \( \pi_q(e) \Rightarrow \partial U_{q, e} \).

Suppose that the implication splits as \( \Pi_R = x_y, y^x \).

Then,

**Proposition 4.** There is a canonical bijection

\[ ((x_y \times y^x) \cdot (y^x \times x_y)) \leftrightarrow \pi_n(\Psi_\theta) \]

Recall from [4] the van Est theorem:

**Theorem 1.** If \( G \) is (topologically) \( p_0 \)-connected, the map induced by \( VE \) in cohomology is an isomorphism for \( p \leq p_0 \) and injective for \( p = p_0 + 1 \), in the map:

\[ VE_{k-hom} : C^p_k-hom(\mathfrak{v}) \rightarrow C^p_k-hom(\mathfrak{v}) \]

That is to say, for two \( p \)-cochains obeying the (generalized) cocycle condition, there is a totally lossless projection\(^3\)

\[ \mathfrak{v}_p \leftrightarrow v_p \]

such that

\[ \mathfrak{v}_p^n \cdot v_p^n = \sum_{i=0}^{n} fib_{pro}(v_i) \]

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\(^2\) Consult [1] for the relevant literature. It is a masterful work of art. [2] is highly recommended as well, but not, of course, necessary.

\(^3\) See [5]
Let \( G_0 = \text{Id}_x \in X \) for some \( x \in \mathcal{G} \cap X \). This identity extends to a classifying space, \( BG \), by way of a categorical stratification, \( \text{Strat}_q^G \), via the map
\[
\text{Strat}_q^G \cdot \Gamma^q
\]
for \( q \neq n \in \mathbb{N} \). Here, \( x^n_i \) denotes the \( q \)th orbital of a point-like object in a topological stack. We may extend this to a map of displays:
\[
\phi_{\mathcal{X} \rightarrow \mathcal{Y}} : (x^n_i)^q \rightarrow \tilde{x}_j
\]
where \( \tilde{x}_j \) denotes the \( j \)th representative generator of a jet bundle \( J_x(\phi) \).

Denote by \( \tilde{\mathcal{X}}^{\text{MixT}_xJ_n} \) the mixed module obtained by flattening a section of a topological space \( X \) to a discrete foliation \( \tilde{\mathcal{X}} \simeq \mathcal{F}_x \) over the tangent space of a jet bundle of order \( n \).

**Proposition 5.** The diagram
\[
\begin{array}{ccc}
\tilde{\mathcal{X}}^{\text{MixT}_xJ_n} & \xrightarrow{|\sim|} & \mathcal{X} \\
\downarrow \text{fiber}(x_i) & & \downarrow \text{pr}_0 \\
S^n & \xrightarrow{\text{pr}_1} & \mathcal{Y} \times (x_i \circ \text{Id}_\delta)
\end{array}
\]

is commutative, and the projections are totally lossless.

**Proof.** Assume \( \mathcal{X} \) is perfect. Then, \( \mathcal{Y} \times (x_i \circ \text{Id}_\delta) \) must be perfect also. Since we have the quotient uniformity
\[
\tilde{\mathcal{X}}^{\text{MixT}_xJ_n} / \sim = [x_i] \in \mathcal{X}
\]
serving as the geometric realization for each tangent fiber
\[
T_{x_i} \in \text{fiber}(\mathcal{X})
\]
where \( \mathcal{X} \) is a topological space, we conclude our proof by noting that, for every section of \( T_{x_i} \), there is a \( \delta \)-pullback \( \phi^{-1}T_{x_i} \). \( \square \)

At this point, we may construct a gerbe, \( \mathcal{G}^{\text{Hol}}_\mathcal{X} \), by inducing a levelwise, piecewise differentiable structure on the tangent bundle. This is given by the formula
\[
\mathcal{G}^{\text{Hol}}_\mathcal{X} = \{ \mathcal{X} \times_x, \mathcal{X}|x_i \in \sum_{i=0}^{n} X_i \{ \partial^0 x + \partial x + \partial^2 x + \ldots + \partial^n x \} \}
\]
Further yet, there is a diffeomorphism
\[
\mathcal{G}^{\text{Hol}}_\mathcal{X} \simeq \mathcal{J}^n(T_x x_i (\text{fiber}(x)))
\]

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on connections parameterized by the nth order jet bundle over a typical fiber of $x$. This is because of the famous link between the structure sheaf, $\mathcal{O}_X$, of a stack of which $x$ is a germ, and the orbit group, $\mathcal{O}_x$ of a topological stack including $x$ as a point-like object.

**Definition 2.** Call $A$ an atlas if it is obtained by a composition of transition maps

$$A = \int_0^n (\phi(\partial^n(x_i)) \circ \phi^{-1}(\partial^n(x_i)) \circ \ldots \circ \phi(\partial^0(x_i)) \circ \phi^{-1}(\partial^0(x_i)))$$

**Example 4.** Consider an atlas $A$ where every natural transformation $\phi^{-1} \Rightarrow \phi$ is totally lossless. This is called the perfect atlas.

**Example 5.** Let $A$ be an atlas, and let every $x_i$ belong to the category $\text{Strat}_M$ of stratified manifolds, with an unspecified stratification. Then, a corner, $\partial(\mu(x + y))$, is the orthogonal pseudo-orthogonal stratification of an atlas $A$, written $A(\text{Strat}_M)$. 

**Proposition 6.** There is an adjunction

$$\mathcal{G}^{\text{Hol}}_X \leftrightarrow \mathcal{M}^{\text{Mix}}_T_x J^n$$

**Proof.** This follows from the famous “tensor-hom” adjunction. 

4 References