Abstract: A source S moving with a constant velocity \( v \) emits a signal \( s \) moving with the velocity of light \( c \) relative to source S position with time \( t \). The velocity \( v \) of signal \( s \) is the sum of \( v \) and \( c \). The time of flight \( t \) for the signal \( s \) when emitted from S and observed at O is calculated using purely Galilean transformation of velocities in Euclidean Space Geometry. O must reside in the \( s \) light cone to observe \( s \) and avoid the artificially introduced infinities that plague classical relativity models. The geometrical interpretation of the physics is valid for velocities greater than \( c \).

Introduction
Assume a source S moves with constant velocity on the x axis in Euclidean Space Geometry. When S crosses the origin it emits a spherical signal \( s_1 \) which moves with velocity \( c \). The center of \( s_1 \) remains coincident with S. An observer O at coordinate \( x, y \), receives \( s_1 \) at \( t_1 \). S at time \( t_0 \) emits \( s_2 \). O receives signal \( s_2 \) at time \( t_2 \). O measures the time \( \tau' \), \( t_2 - t_1 \), a transverse relative time shift. The derivation of equations to calculate \( \tau' \) with S velocity at 2 times \( c \), \( 3 \times 10^8 \) (m/s), are presented. The scale used in the figures is 1 unit of time in seconds (s) that light travels with velocity \( c \) in meters (m) when source S has velocity 0 (m/s).

Geometry \( v \) near \( c \)
The extinction shift principle\(^1\) shows when a stationary Observer O is perpendicular to Source S moving with velocity \( v \) when S emits signal \( s_2 \), O will measure \( \tau' \) (1) a transverse relative time shift, not a time dilation. Refer to Figure 1 for geometric relationships.

\[
\tau' = \frac{\tau_0}{\sqrt{1 - \frac{v^2}{c^2}}} \tag{1}
\]

From geometry we get (2).

\[
(\tau' c)^2 = (\tau_0 c)^2 + (\tau' v)^2 \tag{2}
\]

Now the Observer is limited on capability to receive \( s_2 \) as \( v \) approaches \( c \) (\( 3 \times 10^8 \) m/s). This limitation is removed when O is positioned inside \( s_2 \) light cone. The next section will detail the geometric layout for O to receive \( s_1 \) and \( s_2 \) and get \( \tau' \).

Transverse relative time shift - Geometry when $v = 2c$

Let $S$ travel with velocity $2c$ (m/s) emit a signal $s_1$ at time equal to 0 (s) as it crosses the origin going in the $x$ direction. Let there also be an observer $O$ at coordinate $x$, $y$ or $O(x,y)$, where $x = 22c$ (m) and $y = 5c$ (m). The Observer coordinates are in meters (m). At 1 (s) $S$ will have traveled the distance $6 \times 10^8$ (m). The signal $s_1$ would have traveled $3 \times 10^8$ (m) in the $-y$ direction and $9 \times 10^8$ (m) on the $x$-axis, $[2c \text{ (m/s)} + 1c \text{ (m/s)}][1 \text{ (s)}] = 3c \text{ (m)}$, see geometry in Figure 2.1.

In 2 seconds $S$ will have traveled to $4c$ (m). Signal $s_1$ would have traveled $2c$ in the $-y$ direction and $8c$ (m) along the $x$ axis, see Figure 2.2.

In 3 seconds $S$ will have traveled to $6c$ (m). Signal $s_1$ would have traveled $3c$ in the $-y$ direction and $9c$ (m) along the $x$ axis, see Figure 2.3.

Sometime after 7 seconds but before 8 seconds the $s_1$ would arrive at $O$ in $t_1$ seconds, see Figure 3.

**Figure 2.1:** $S$ with velocity $2c$ (m/s), time = 1 (s).

**Figure 2.2:** $S$ with velocity $2c$ (m/s), time = 2 (s).

**Figure 2.3:** $S$ with velocity $2c$ (m/s), time = 3 (s).
Determining $t_1$

The transient time $t_1$ for the light pulse to reach $O(x_1, y_1)$ can be determined from geometry, see Figure 4. The velocity of the Source $S$ is $v$ (m/s). The distance traveled by $S$ in $t_1$ (s) equals $vt_1$ (m). The distance $s_1$ travels radially about $S$ is $ct_1$ (m).

The velocity of the Source $S$ is $v$ (m/s). The distance traveled by the $S$ in $t_1$ (s) is $vt_1$ (m). The distance the signal $s_1$ travels radially about $S$ is $ct_1$. 

Angles A, B, D have corresponding sides a, b, d.

$$a = ct_1 \quad (3)$$

$$b^2 = x_1^2 + y_1^2 \quad (4)$$

$$d = vt_1 \quad (5)$$

$$\cos(A) = \frac{x_1}{b} = \frac{x_1}{\sqrt{x_1^2 + y_1^2}} \quad (6)$$
From the law of cosines we get (7). Combine (3)(4)(5)(6) into (7) to get (8) where the t, x, y subscripts are dropped. Solve for $t_1$ (9).

\[ a^2 = b^2 + d^2 - 2bd \cos(A) \quad (7) \]

\[ 0 = -(ct_1)^2 + (x^2 + y^2) + (vt_1)^2 - 2\sqrt{x^2 + y^2}vt_1 \frac{x}{\sqrt{x^2 + y^2}} \quad (8) \]

\[ t_1 = -vx \pm \sqrt{c^2x^2 + c^2y^2 - v^2y^2} \frac{c^2 - v^2}{c^2 - v^2} \quad (9) \]

\[ ^2 \text{Standard Mathematical Tables, 27th Edition, CRC PRESS, Copyright 1984, p. 144} \]

\[ ^3 \text{Mathematica 12.3.1 Kernel for Microsoft Windows (64-bit), Copyright 1988-2021 Wolfram Research, Inc.} \]
The input and output to Mathematica is in Figure 5. Now $t_1$ is the time at which $O$ observes $s_1$. We use the $t_1(+) \text{ solution to calculate } t_1$. Let $S$ emit a signal $s_2$ at time $t$ equal to 3 (s). $s_2$ time of flight to $O$ is $t_2$, some time after 6 (s). See Figure 6 for geometry of $s_1$ and $s_2$.

The ratio $\tau'/\tau_0$ is the transverse relative time shift factor observed by $O$ at $O(x,y)$.

$$\tau_0 + t_2 = t_1 + \tau' \quad (10) \quad \tau' = \tau_0 + t_2 - t_1 \quad (11)$$
Transverse relative distance

At time $t_1$ S location is $vt_1$ and O receives $s_1$. At time $(\tau_0 + t_2)$ S location is $v(\tau_0 + t_2)$ when O receives $s_2$. Now $\tau'$ (s) times the Source S velocity is equal to the actual distance traveled by S when O receives the $t_1$ and $t_2$ signal. The actual signal $t_1$ was emitted at $t_0$ where time is 0 seconds, the coordinate origin. S travels to $v\tau_0$ (m) in $t_2$ (s). What is interesting is that O perceives that S has traveled $v\tau'$ (m), a relative distance shift $x'(12)$. Where $x_0$ (m) is the actual distance traveled by S with velocity v in $\tau_0$ (s) (13).

$$x' = v\tau' \quad x_0 = v\tau_0 \quad (12) \quad (13)$$

Transverse relative length

Let there be two Sources $S_1$ and $S_2$ moving with velocity v and separated by distance $v\tau_0$ (m), see Figure 7. $S_1$ and $S_2$ move to the right as if there were a virtual rod of length $\Delta x$ between them. Let X be the distance from the origin to the rods center at time $t=0$ (s). $S_1$ and $S_2$ emit signals $s_1$ and $s_2$ at time $t_1 = t_2 = 0$ (s), $v = 2c$ (m/s).

In 1 (s) the rod would have moved to position $2c$ (m) and $s_1$ and $s_2$ would have expanded by $1c$ in radius as shown in Figure 7.1. In 2 (s) the rod would be in position shown in Figure 7.2, in 3 (s), Figure 7.3, in 4 (s) Figure 7.4 and in 7 seconds Figure 7.7.

**Figure 7.1**: $s_1$, $s_2$ and rod position at time $t = 1$ (s).

**Figure 7.2**: $s_1$, $s_2$ and rod position at time $t = 2$ (s).
From geometry we see that observer O receives both signals \( s_1 \) and \( s_2 \) at the same \( t \) when located on \( X \) (14) and \( Y \) (15).

\[
X = vt + \frac{1}{2} \Delta x \quad (14) \quad Y = \sqrt{(ct)^2 - \frac{1}{4} \Delta x^2} \quad (15)
\]

**Figure 7.3**: \( s_1, s_2 \) and rod position at time \( t = 3 \) (s).

**Figure 7.4**: \( s_1, s_2 \) and rod position at time \( t = 4 \) (s).

**Figure 7.7**: \( s_1, s_2 \) and rod position at time \( t = 7 \) (s).
Now let a rod of length \( \Delta x \) moving with velocity \( v \) emit signals \( s_1 \) and \( s_2 \) at time \( t = 0 \) (s) when \( s_1 \) is at the origin. Let observer \( O \) be at coordinates \( X, Y \) inside the \( s_2 \) light cone, see Figure 8. From geometry in Figure 9 substitute \( x \cdot \Delta x \) for \( x \) in (8) to get (16). Solve (16) for \( t_1 \) to get (17) using Mathematica, Figure 10.

\[
0 = -(ct_1)^2 + ((x - \Delta x)^2 + y^2) + (vt_1)^2 - 2\sqrt{(x - \Delta x)^2 + y^2}vt_1 \frac{x_1}{\sqrt{(x - \Delta x)^2 + y^2}}
\]

(16)

\[
t_2 = \frac{v\Delta x - vx \pm \sqrt{c^2\Delta x^2 - 2c^2\Delta xx + c^2x^2 + c^2y^2 - v^2y^2}}{c^2 - v^2}
\]

(17)

Figure 9: Rod of length rod with length \( \Delta x \) moving with velocity \( v \) emit signals \( s_1 \) and \( s_2 \) at time \( t = 0 \) (s) when \( s_1 \) is at the origin.

Figure 10: Mathematica solution when \( O \) observes \( s_2 \) at \( t_2 \) as a function of \( v, \Delta x, x, y, c \).

Now for the moving rod \( O \) receives \( s_1 \) at \( t_1 \) after \( s_2 \) at \( t_2 \) therefore \( \tau' \) is (18), \( \tau_0 \) is 0 (s).

\[
\tau' = \tau_0 + t_1 - t_2 = t_1 - t_2
\]

(18)
Let a rod of length $\Delta x = 3c$ (m), moving with velocity $v = 2c$ (m/s) and Observer O at $x = 16.5c$ (m), $y = 3c$ (m), from (17) $t_2 = 3.96$ (s), (9) $t_1 = 5.78$ (s), and (18) $\tau' = 1.82$ (s) (19).

$$\tau' = t_1 - t_2 = t_1 - t_2 = 5.78 - 3.96 = 1.82 \text{ (s)}$$ (19)

From O calculates the rod length $\Delta x'$ to be $1.09\times10^9$ (m) (20). $\Delta x'$ is the “transverse relative length” observed and measured by 0.

$$\Delta x' = v \tau' = 2(3\times10^8)(1.82) = 1.09\times10^9 \text{ (m)}$$ (20)

O calculates the per unit transverse relative length of the moving rod to be 0.6071 (21)

$$\frac{\text{Observer calculated rod length}}{\text{Actual rod length}} = \frac{v \tau'}{\Delta x}$$ (21)

Observation

The ratio $\tau'/\tau_0$ is the transverse relative time shift factor observed by O at O(x,y). The Observer must be downstream and inside of both the $s_1$ and $s_2$ light cones to observe both signals when the signal source S has any velocity $v$. If $v$ is equal to 0 (m/s) then (8) reduces to (13) (14) the expected physics in Euclidean space with Galilean geometry.

$$0 = -(ct_1)^2 + (x^2 + y^2) + (vt_1)^2 - 2\sqrt{x^2 + y^2}vt_1 \frac{x}{\sqrt{x^2 + y^2}}$$ (8)

$$0 = -(ct_1)^2 + (x^2 + y^2)$$ (13)

$$(ct_1)^2 = (x^2 + y^2)$$ (14)

The calculations for the provided example are in Table 1. The procedure outlined in this work can be extended to estimate the incoming velocity of cosmic rays that initiate double pulses in detectors like Super-Kamiokande\(^4\). The velocity of light $c$ must be adjusted to account for the properties of the light conducting medium.

\(^4\)http://www-sk.icrr.u-tokyo.ac.jp/sk/index-e.html
Table 1: Calculations for time and distance quantities when \( v=2c \).

<table>
<thead>
<tr>
<th>Eq.</th>
<th>Signal</th>
<th>( S_1 )</th>
<th>( S_2 )</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( c ) (m/s)</td>
<td>3.000E+08</td>
<td>3.000E+08</td>
<td>Velocity of light ( c ) when Source is at rest.</td>
</tr>
<tr>
<td>2</td>
<td>( v/c ) (m/s)</td>
<td>2.000E+00</td>
<td>2.000E+00</td>
<td>Ratio of Source velocity to that of ( c ).</td>
</tr>
<tr>
<td>3</td>
<td>( v ) (m/s)</td>
<td>6.000E+08</td>
<td>6.000E+08</td>
<td>Velocity of Source.</td>
</tr>
<tr>
<td>4</td>
<td>( \tau_0 ) (s)</td>
<td>3.000E+00</td>
<td>Time duration between ( s_1 ) and ( s_2 ) start events by Source traveling at velocity ( v ).</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>( \Delta x ) (m) = 3( c )</td>
<td>1.800E+09</td>
<td>Rod length</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>( X ) (m) = 16.5( c )</td>
<td>4.950E+09</td>
<td>4.950E+09</td>
<td>Observer x-axis coordinate</td>
</tr>
<tr>
<td>7</td>
<td>( Y ) (m) = 3( c )</td>
<td>9.000E+08</td>
<td>9.000E+08</td>
<td>Observer y-axis coordinate</td>
</tr>
<tr>
<td>8</td>
<td>(17) ( t_2 ) for rod</td>
<td>-</td>
<td>3.959E+00</td>
<td>Time (s) of receipt of ( s_2 ) by Observer in moving rod example</td>
</tr>
<tr>
<td>9</td>
<td>(9) ( t_1 ) for rod</td>
<td>5.780E+00</td>
<td>-</td>
<td>Time (s) of receipt of ( s_1 ) by Observer in moving rod example</td>
</tr>
<tr>
<td>10</td>
<td>(19) ( \tau' = t_1 - t_2 ) (s)</td>
<td>1.821E+00</td>
<td>-</td>
<td>( \tau_0 = 0 ) since ( s_1 ) and ( s_2 ) are emitted at time ( t_2 = 0 ) seconds.</td>
</tr>
<tr>
<td>11</td>
<td>(20) ( \Delta x' = v \tau' ) (m)</td>
<td>1.093E+09</td>
<td>-</td>
<td>Observer calculated rod length knowing rod velocity and ( \tau' ), a transverse relative length.</td>
</tr>
<tr>
<td>12</td>
<td>(21) ( v \tau'/\Delta x )</td>
<td>6.071E-01</td>
<td>-</td>
<td>Observer calculated per unit &quot;transverse relative length&quot; when velocity of rod is ( 2C ) and the observers position is ( x, y ).</td>
</tr>
</tbody>
</table>

List of acronyms

- \( c \) Speed of light in meters per second, assumed \( 3\times10^8 \) (m/s)
- \( m \) Distance in meters (m)
- \( O \) Observer location \( X \) (m), \( Y \) (m)
- \( S \) Source for signal \( s_n \)
- \( s \) Time in seconds (s)
- \( s_n \) Signal \( s_n \) location at time \( t_n \) where \( n = 1, 2, \ldots \)
- \( t \) Reference time in seconds for all space, typically time \( t = 0 \) (s) when source \( S \) passes the origin where \( X=Y=0 \) (m)
- \( t_n \) Time in seconds elapsed from initiation of signal \( s_n \)
- \( \tau_0 \) Time difference in seconds (s) when source \( S \) sends signals \( s_1 \) and \( s_2 \)
- \( \tau' \) Time difference in seconds (s) when observer \( O \) receives signals \( s_1 \) and \( s_2 \)
- \( v \) Velocity of signal source in meters per second (m/s)
- \( X \) Observer \( O \) x-axis coordinate
- \( x_n \) Distance in meters (m) on x-axis location of signal \( s_n \) initiation to Observer \( O \)
- \( \Delta x \) Rod length in meters (m)
- \( \Delta x' \) Rod length in meters (m)
- \( Y \) Observer \( O \) y-axis coordinate in meters (m)
- \( y_n \) Observer \( O \) y-axis coordinate in meters (m)