Remarkable phenomenon in quantum mechanics

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Abstract

It happens that Nagata and Nakamura discuss a novel inconsistency within quantum mechanics when accepting we use the property of the Kronecker delta notation without extra assumptions about the reality of observables. Here, we discuss a further strong novel inconsistency within quantum mechanics without the property of the Kronecker delta notation. Based on the argumentations, we propose an experimental accessible inconsistency in terms of imperfect sources and detectors. In more detail, we encounter an imperfect quantum state, the dark count, and the quantum efficiency, which cannot be avoidable from a real experimental situation. Such an error of the number of particles becomes less and less important as we increase trials more and more by using the strong law of large numbers. Our discussion gives in some sense the limitation of von Neumann's model for quantum measurement theories.

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I. INTRODUCTION

Quantum mechanics (cf. [1–7]) gives explanations for the microscopic behaviors of the nature. We see researches concerning the mathematical formulations of quantum mechanics. For example, the mathematical foundations of quantum mechanics is discussed by Mackey [8]. On the quantum logic approach to quantum mechanics is also discussed by Gudder [9]. Conditional probability and the axiomatic structure of quantum mechanics are also reported by Guz [10].

von Neumann's mathematical model for quantum mechanics is logically successful [4]. The axiomatic system for the mathematical model is a consistent one. Thus, we cannot say that von Neumann's mathematical model has an inconsistency. What is the inconsistency to be discussed in this paper? We cannot expand von Neumann's mathematical model more in handling real experimental data [11, 12]. Mathematically, von Neumann's model is logically consistent, which fact is true. However, von Neumann's theory is questionable in the sense that the mathematical model does not always expand to real experimental data. And there is the inconsistency if we apply von Neumann's model to expanding even a simple physical situation. In short, von Neumann's mathematical model might not be useful in that case.

The inconsistency to be discussed in this paper is significant. von Neumann's mathematical model has the qualification to be true axiomatic system for quantum mechanics. Therefore, we cannot modify the axioms based on the nature of Matrix theory. Nevertheless, we encounter an inconsistency, probably due to the nature of Matrix theory, within von Neumann's theory, that is, his mathematical model is perfect but his physical thought might be a little bit questionable today.

Of course our analyses rely on von Neumann's model and the possible positions of the inconsistencies lie outside von Neumann's model. Thus we cannot sometimes expand von Neumann's model to experimental situations. In short, we might have to limit in some sense his model for quantum measurement theories.

On the other hand, the incompleteness argument to quantum mechanics itself is discussed by Einstein, Podolsky, and Rosen [13]. A hidden-variable interpretation of quantum mechanics is a topic of research [2, 3] and the no-hidden-variable theorem is discussed by Bell, Kochen, and Specker [14, 15]. The Kochen-Specker theorem based on the Kronecker delta notation is also discussed by Nagata, Patro, and Nakamura [16]. The Kronecker delta notation seems to be necessary for quantum mechanics when using Matrices and Vectors.

Recently, Nagata and Nakamura discuss a novel inconsistency within quantum mechanics when accepting we use the property of the Kronecker delta notation without extra assumptions about the reality of observables [17]. Here, we discuss a further strong novel inconsistency within quantum mechanics without the property of the Kronecker delta notation. Based on the argumentations, we propose an experimental accessible inconsistency in terms of imperfect sources and detectors.

In more detail, we encounter an imperfect quantum state, the dark count, and the quantum efficiency, which cannot be avoidable from a real experimental situation. If we use the quantum predictions by $2N$ trials, then the inconsistency increases by an amount that grows linearly with $4N^2$. In fact, such an error of the number of particles becomes less and less important as we increase trials more and more by using the strong law of large numbers. Our discussion gives in some sense the limitation of von Neumann's model for quantum measurement theories.

II. REMARKABLE PHENOMENON IN QUANTUM MECHANICS

Let σ_z be z-component Pauli observable. It could be defined as follows:

$$\sigma_z \equiv \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \tag{1}$$

Let $|\uparrow\rangle$ and $|\downarrow\rangle$ be eigenstates of σ_z such that $\sigma_z|\uparrow\rangle = +1|\uparrow\rangle$ and $\sigma_z|\downarrow\rangle = -1|\downarrow\rangle$. The measured results of trials are either $+1$ or -1 in the ideal case.

When we consider a quantum optical experiment, we have the following relation with the photon polarization states:

$$\begin{aligned} |\uparrow\rangle &\leftrightarrow |H\rangle, \\ |\downarrow\rangle &\leftrightarrow |V\rangle, \end{aligned} \tag{2}$$

where $|H\rangle$ is a quantum state interpreted by a horizontally polarized photon and $|V\rangle$ is a quantum state interpreted by a vertically polarized photon.

Let us introduce the random noise admixture $\rho_{\rm noise}(=\frac{1}{2}I)$ into the quantum states, where I is the two-dimensional identity operator. We consider the noisy quantum states emerged from an imperfect source as follows:

$$\begin{aligned} \rho_1 &= (1-\epsilon)|\uparrow\rangle\langle\uparrow| + \epsilon \times \rho_{\rm noise}, \\ \rho_2 &= (1-\epsilon)|\downarrow\rangle\langle\downarrow| + \epsilon \times \rho_{\rm noise}. \end{aligned} \tag{3}$$

The value of $\epsilon(<1)$ is interpreted as the reduction factor of the contrast observed in the single-particle experiment. Then we have ${\rm tr}[\rho_1\sigma_z] = +1-\epsilon$ and ${\rm tr}[\rho_2\sigma_z] = -1+\epsilon$.

We introduce a value V which is the sum of two data in an experiment. The measured results of trials are either $+1-\epsilon$ or $-1+\epsilon$. We suppose the number of trials of obtaining the result $-1+\epsilon$ is equal to the number of trials of obtaining the result $+1-\epsilon$. We can depict experimental data r_1, r_2 as follows: $r_1 = +1-\epsilon$ and $r_2 = -1+\epsilon$. Let us write V as follows:

$$V = \sum_{l=1}^{2} r_l. \tag{4}$$

Note the following inequality:

$$(V \times V) \le 0. \tag{5}$$

Hence we have

$$(V \times V)_{\max} = 0. \tag{6}$$

On the other hand, we evaluate the value $(V \times V)$ and derive logically the other necessary condition. We suppose that the two operations Sum rule and Product rule commute with each other [18]. We have

$$(V \times V) \le \sum_{l=1}^{2}\sum_{l'=1}^{2} |r_l r_{l'}| = 4, \tag{7}$$

where we use $|r_l r_{l'}| = 1$. The above inequality is logically saturated when

$$r_l = r_{l'}, \tag{8}$$

because the following holds:

$$|r_l r_{l'}| = r_l r_{l'}. \tag{9}$$

And it is possible since we have

$$\begin{aligned} \|\{l|r_l = +1\}\| &= \|\{l'|r_{l'} = +1\}\|, \\ \|\{l|r_l = -1\}\| &= \|\{l'|r_{l'} = -1\}\|. \end{aligned} \tag{10}$$

Thus the above inequality is logically saturated. Clearly, we have the maximum for the value as

$$(V \times V)_{\max} = 4. \tag{11}$$

We cannot assign simultaneously the same two values ("1" and "1") or ("0" and "0") for the two suppositions (6) and (11). Thus, we are in the inconsistency.

In summary, we have been in the inconsistency when the first result is $+1-\epsilon$ by measuring the Pauli observable σ_z in the quantum state ρ_1, the second result is $-1+\epsilon$ by measuring the same Pauli observable σ_z in the quantum state ρ_2, and then $[\sigma_z, \sigma_z] = 0$.

III. THE INCOMPLETENESS OF A REAL EXPERIMENT

In a real experiment, a perfect detector is not feasible. There is an unforeseen effect that an imperfect detector does not count even though the particle indeed passes through the detector (the quantum efficiency). There is also an unforeseen effect that an imperfect detector counts even though the particle does not pass through the detector (the dark count). In this case, we increase measurement outcomes to $2N(\gg 1)$ and then we change such errors into trivial things. If we use the quantum predictions by $2N$ trials, then the inconsistency increases by an amount that grows linearly with $4N^2$. In fact, such an error of the number of particles becomes less and less important as we increase trials more and more by using the strong law of large numbers.

We introduce a value V which is the sum of $2N$ data in an experiment. The measured results of trials are either $+1-\epsilon$ or $-1+\epsilon$. We suppose the number of trials of obtaining the result $-1+\epsilon$ is N that is equal to the number (N) of trials of obtaining the result $+1-\epsilon$. We can depict experimental data $r_1, r_2, r_3, \ldots$ as follows: $r_1 = +1-\epsilon$, $r_2 = -1+\epsilon$, $r_3 = +1-\epsilon$ and so on. Let us write V as follows:

$$V = \sum_{l=1}^{2N} r_l. \tag{12}$$

Note the following inequality:

$$(V \times V) \le 0. \tag{13}$$

Hence we have

$$(V \times V)_{\max} = 0. \tag{14}$$

In the following, we evaluate the value of $(V \times V)$ and derive the other necessary condition. We suppose that the two operations Sum rule and Product rule commute with each other [18]. We have

$$(V \times V) \leq \sum_{l=1}^{2N} \sum_{l'=1}^{2N} |r_l r_{l'}| = 4N^2(+1-\epsilon)^2, \tag{15}$$

where we use $|r_l r_{l'}| = (+1-\epsilon)^2$. The above inequality is logically saturated when

$$r_l = r_{l'}, \tag{16}$$

because the following holds:

$$|r_l r_{l'}| = r_l r_{l'}. \tag{17}$$

And it is possible since we have

$$\begin{aligned} &\|\{l|r_l = +1-\epsilon\}\| = \|\{l'|r_{l'} = +1-\epsilon\}\|, \\ &\|\{l|r_l = -1+\epsilon\}\| = \|\{l'|r_{l'} = -1+\epsilon\}\|. \end{aligned} \tag{18}$$

Thus the above inequality is logically saturated. Clearly, we have the calculation result as

$$(V \times V)_{\max} = 4N^2(+1-\epsilon)^2. \tag{19}$$

We cannot assign simultaneously the same two values ("1" and "1") or ("0" and "0") for the two suppositions (14) and (19). Thus, we are in the inconsistency.

If we use the quantum predictions by $2N$ trials, then the inconsistency increases by an amount that grows linearly with $4N^2$. In fact, such an error of the number of particles becomes less and less important as we increase trials more and more by using the strong law of large numbers.

In summary, we have been in the inconsistency when the odd number results are $+1-\epsilon$ by measuring the Pauli observable σ_z in the quantum state ρ_1, the even number results are $-1+\epsilon$ by measuring the same Pauli observable σ_z in the quantum state ρ_2, and then $[\sigma_z, \sigma_z] = 0$.

IV. CONCLUSIONS

In conclusions, recently, Nagata and Nakamura have discussed a novel inconsistency within quantum mechanics when accepting we use the property of the Kronecker delta notation without extra assumptions about the reality of observables. Here, we have discussed a further strong novel inconsistency within quantum mechanics without the property of the Kronecker delta notation. Based on the argumentations, we have proposed an experimental accessible inconsistency in terms of imperfect sources and detectors.

In more detail, we have encountered an imperfect quantum state, the dark count, and the quantum efficiency, which cannot be avoidable from a real experimental situation. However, such an error of the number of particles has become less and less important as we increase trials more and more by using the strong law of large numbers. Our discussion has given in some sense the limitation of von Neumann's model for quantum measurement theories.

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The authors are in an applicable thought to ethical approval.

Competing interests

The authors state that there is no conflict of interest.

Author contributions

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