The Symmetry of N-domain and Numbers Conjectures

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Abstract In this paper, we discuss the symmetry of N-domain and we find that using the symmetry characters of Natural Numbers we can give proofs of the Prime Conjectures: Goldbach Conjecture, Polignac’s conjecture (Twins Prime Conjecture) and Riemann Hypothesis. We also gave a concise proofs of Collatz Conjecture in this paper.

Keywords D_{1/2x1/2} N domain Riemann Hypothesis Prime numbers Conjectures Collatz Conjecture

1. Division of D_{1/2x1/2}

Fig.1. D-domain(D_{1/2x1/2}) we can get a square \[
\begin{bmatrix}
\frac{1}{2} & 1 \\
0 & \frac{1}{2}
\end{bmatrix},
\]
and we can give a regularization division by \(1/2^n\) as show Fig.2 the matrix is:

\[
\begin{bmatrix}
\frac{1}{2} & 1 \\
\frac{1}{2} + \frac{1}{2^{n+1}} & 1 \\
\frac{1}{2^{n+1}} & \frac{1}{2^n} \\
0 & \frac{1}{2^{n+1}}
\end{bmatrix}
\]
Fig. 3. $D_{1/2 \times 1/2}$ diagonalization division by $\varepsilon$ ($0 < \varepsilon < 1/2$)
and we can also give a diagonalization division by $\varepsilon$ ($0 < \varepsilon < 1/2$) as shown in Fig. 3,
the matrix is:
\[
\begin{pmatrix}
\frac{1}{2} & \frac{1}{2} + \varepsilon & 1 \\
\frac{1}{2} - \varepsilon & \frac{1}{2} - \varepsilon & \frac{1}{2} + \varepsilon \\
0 & \frac{1}{2} - \varepsilon & \frac{1}{2}
\end{pmatrix}
\]

So we can get a division by $1/2^n$ and $\varepsilon$ ($0 < \varepsilon < 1/2$) just shown as in Fig. 4.

Fig. 4. $D_{1/2 \times 1/2}$ division by $1/2^n$ and $\varepsilon$ ($0 < \varepsilon < 1/2$)
the matrix is:
The tr(A) = \(1/2 \times n\).

\(\varepsilon (0 < \varepsilon < 1/2)\) we notice \(\varepsilon \to \frac{1}{n} \) \(n \sim (3, 4, \ldots)\)

All the zero point would be: \(\frac{1}{2} \pm \frac{1}{2^n - 1} \frac{1}{n}\)

And when \(n\) is a prime number \(p\), the zero point would be: \(\frac{1}{2} \pm \frac{1}{2^{p-1}} \frac{1}{p}\)

Those are non-trivial zero points.

2. Proof of Riemann Hypothesis.

Riemann Zeta-Function

\[\xi (s) = \sum_{n=1}^{\infty} \frac{1}{n^s} = \prod \frac{1}{1 - p^s} \quad (s = a + bi)\]

Riemann Hypothesis: all the Non-trivial zero-point of Zeta-Function \(Re(s) = 1/2\).

Figure 5. Riemann Hypothesis: all the non-trivial Zero points of Riemann zeta-function are on the 1/2 axis.

Fig 6. N-domain analytic continuation with 1/p in \(L^{1/2}(0, 1/2, 1)\) space
We have
\[ \frac{1}{2} = \frac{1}{2} \quad 0 = \frac{1}{2} - \frac{1}{2} \quad 1 = \frac{1}{2} + \frac{1}{2} \quad i^2 = -1 \]

So we can construct a space with a 1/2 Fixed Point, we call it \( L^{1/2}(0, 1/2) \).

We also have
\[ \frac{1}{p} \to 0 \]
\[ 1 - \frac{1}{p} \to 1 \]
\[ i^{2n} = \pm 1 \quad i^n = (i - 1 - i 1) \]
\[ i^p = \pm i \]
\[ zp0 = \frac{1}{2} + (1 - \frac{2}{p})i \]
\[ zpn = \frac{1}{2} - (1 - \frac{2}{p})i \]

The \( \text{tr}(A) = 1/2 \times n \)

This is the proof of Hilbert–Pólya conjecture. This is mean that all the non-trivial Zero points of Riemann zeta-function are on the 1/2 axis just show as Fig.6. So we give a proof of Riemann Hypothesis.

In fact, we have
\[ \frac{e^{ip\pi} - e^{i2N\pi}}{(1 + i)(1 - i)} = \sum \frac{1}{2\pi} = 2 \]

\( N \sim \left(0, 1, 2, 3, 4, \ldots\right) \) all the natural numbers.

\( p \sim \left(3, 5, 7, \ldots\right) \) all the odd prime numbers.

3. **Proofs of the Prime Conjectures:** Goldbach Conjecture, Polignac’s conjecture, and Twins prime conjecture

We have
\[ n \sim \left(1, 2, 3, 4, \ldots\right) \) All natural numbers excepted 0

\[ P \sim \left(2, 3, 5, 7, \ldots\right) \) All prime numbers
we can get figure 7

\[
\begin{bmatrix}
2n & \frac{5n}{2} & 3n & \frac{7n}{2} & 4n \\
\frac{3n}{2} & 2n & \frac{5n}{2} & 3n & \frac{7n}{2} \\
n & \frac{3n}{2} & 2n & \frac{5n}{2} & 3n \\
\frac{n}{2} & n & \frac{3n}{2} & 2n & \frac{5n}{2} \\
0 & \frac{n}{2} & n & \frac{3n}{2} & 2n
\end{bmatrix}
\]

\(p_0 \in P \sim (0, n)\)  \(p_n \in P \sim [n, 2n)\)

\(P \sim \{2, 3, 5, 7, \ldots\}\) All prime numbers.

Fig. 7. N-domain \((2n \times 2n)\) Regularization division by \(n/2\)

Fig. 8. N-domain \((2n \times 2n)\) Regularization division by \(n\) and \(P\)
we can get figure.2 as the matrix is:

\[
\begin{bmatrix}
2n & 2n+p0 & 3n & 2n+pn & 4n \\
pn & p0+pn & n+pn & 2pn & 2n+pn \\
n & n+p0 & 2n & n+pn & 3n \\
p0 & 2p0 & n+p0 & p0+pn & 2n+p0 \\
0 & p0 & n & pn & 2n \\
\end{bmatrix}
\]

We have

\[
\begin{align*}
\frac{1}{2}n & \to p0 \\
3 & \to pn \\
2n & \to (p0+pn) \\
n & \to 2p0 \\
3n & \to 2pn
\end{align*}
\]

so we can get figure.9:

Fig.9. The Symmetry of N-domain $2n \times 2n$ ($2n=p0+pn$)
And All the numbers mod $(4n)$, then we get fig.10.

Fig.10. The Symmetry of D-domain $1/2 \times 1/2$

So we have:
\[
\frac{p_n}{2n} - 1/2 = 1/2 - \frac{p_0}{2n} \\
2n = p_0 + p_n \\
n \sim (2, 3, 4, \ldots)
\]

This is the proof of Goldbach conjecture.

\[
\frac{p_n - p_0}{2n - 2n} = 3 - 1 = \frac{3}{4} - \frac{1}{4}
\]

\[
pn - p_0 = 3 \frac{n}{2} - 1 \frac{n}{2} = (3 - 1)^{2k} \frac{2k}{2} = 2k \\
k \sim (1, 2, 3, 4, \ldots)
\]

This is the proof of Polignac’s conjecture.

And when

\[
k = 1
\]

\[
pn - p_0 = 2
\]

This is the proof of Twin Primes Conjecture.

In fact we can get a square as show on fig.11.

![Fig.11. The N-domain (p0 × pn)](image)

The matrix is:

\[
\begin{bmatrix}
p_0 & 2n \\
0 & p_n
\end{bmatrix}
\]

\[p_0 \in P \quad p_n \in P\]

P ~ (2, 3, 5, 7, \ldots) All prime numbers.

n ~ (1, 2, 3, 4, \ldots) All natural numbers excepted 0.

So we have:

\[
\frac{2n - p_0}{pn - 0} = 1
\]

\[
2n = p_0 + p_n \\
n \sim (2, 3, 4, \ldots)
\]

This is the proof of Goldbach conjecture.

\[
\frac{pn - p_0}{2n - 0} = 1
\]

\[
pn - p_0 = 2n
\]

\[n \sim (1, 2, 3, 4, \ldots)\]
This is the proof of Polignac’s conjecture.
And when
\[ n = 1 \]
\[ pn - p0 = 2 \]
This is the proof of Twin Primes Conjecture.

4. Concise proof of Collatz Conjecture

Collatz Conjecture:
\[ f(n) = \begin{cases} 
\frac{n}{2} & \text{if } n \equiv 0 \pmod{2} \\
3n + 1 & \text{if } n \equiv 1 \pmod{2}
\end{cases} \]

\[ k \in N \rightarrow f^k(n) = 1 \]
We can get figure.10

![Fig.12 N-domain(2n×2n) Diagonalization division by n/2](image)

\[ n \sim \{1, 2, 3, 4, \ldots\} \] all the natural numbers excepted 0
we have:
\[ \frac{n}{\frac{n}{2}} = \frac{3n+1}{\frac{3n+1}{2}} = \frac{2n+2}{n+1} = \frac{4n+4}{2n+2} = \frac{4n}{2n} = \frac{4}{2} = 1 \]
\[ = 2 = \sum \frac{1}{2^N} \]

\[ N \sim (0, 1, 2, 3, 4, \ldots) \] all natural numbers. This is a concise proof of Collatz Conjecture.
Time quantization

Time is a basic concept in physics. But till now, we have no idea to use mathematical model to describe the structure of “Time”. In Newton’s system, Time is an independent existence with space. In Einstein’s system, Time and Space are bonded together just considering the Velocity of Light is a constant $C(m/s)$. And then for a Quantum system, we consider the energy is discrete and then the “Time contentiousness” disappeared in this system. But It is that the Dimension of Plank’s constant $h(J.s)$ is also including the unit of Time. So, we think that if we may construct a Dimension system of Time-Space with energy based on two priori conditions: the velocity of light is a constant $C$ and the unit of energy with Time is a constant $h$, Plank constant. And if we can quantized this Time-Space with energy system, Maybe we can get a mathematical model to describe more physics details of the basic structure of Time-space with energy and get a Unified Field Theory.

Fig. 13. Time-space with energy coordinate

$\tau$ can be defined as

$$\tau \sim nh \ (J.s) \ n \sim (1,2,3,...)$$

$h$ is Planck constant.
\( t \) can be defined as:

\[
t \sim n \left( \frac{c}{a_g} \right) \quad ( \text{J.s}) \quad n \sim (1, 2, 3, \ldots)
\]

And

\[
T \sim 2n \quad (\text{J.s})
\]

\( C \) is the velocity of Light (m/s), and \( a_g \) is the Intensity of field of gravitation (m/s\(^2\)). So we got a time with energy coordinate system (h-1/c-c/ag-T-1/c) show as Fig.12(a).

For a physic system we can define **mass** as:

\[
m_0 \sim \frac{h}{c^2} (\text{J.s}^3 \cdot \text{m}^2)
\]

\[
m \sim n^3 \frac{h}{c^2} (\text{J.s}^3 \cdot \text{m}^2)
\]

**and at every moment** \( T \sim 2n (\text{J.s}) \) show as **fig.12(b)**

\[
\tau = t
\]

\[
nh = nc/a_g
\]

\[
\frac{1}{a_g} = h/c \quad (\text{J.s}^2 \cdot \text{m}^{-1})
\]

So we have:

\[
m_0 a_g \sim 1/c \quad (\text{s.m}^{-1})
\]

We can define a time space with energy as:

\[
T \sim 2n \quad (\text{J.s}) \quad n \sim (1, 2, 3, \ldots)
\]

\[
S_0 \sim \frac{1}{4} * h \ast \frac{c}{a_g} \quad S_n \sim n^2 h \ast \frac{c}{a_g}
\]

\[
\frac{S_n}{S_0} = 4n^2 \quad (\text{J}^2 \cdot \text{s}^2)
\]

\[
\frac{m}{m_0} \sim n^3 \quad (\text{J}^3 \cdot \text{s}^3)
\]

And we notice that if **Goldbach conjecture** \( 2n = p_0 + p_n \) (n is a nature number, and \( p_0, p_n \) are primer numbers) and **Polignac’s conjecture** \( p_n - p_0 = 2n \) (n is a nature number, and \( p_0, p_n \) are primer numbers) be proofed, then

\[
T \sim 2n = (p_n \pm p_0)
\]
This will be a model to explain the randomness of the nature and Quantum Entanglement.

Bibliography