Introduction to the cubic ellipsoid nuclear model

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Abstract

This study suggests that the nuclear structure determines the atomic properties and proposes a geometric nuclear model to confirm this claim.
The model combines the advantages of the liquid drop, shell, collective and cluster models and can serve as a starting point to an effective field theory process.
The main goal is not necessarily to obtain more accurate results than existing models, but rather to raise the possibility of a tangible interpretation of nuclear and atomic physics and to explore different perspectives of this idea.
According to the model, the nucleus generally has an ellipsoidal shape, made up of a three-dimensional lattice of proton-neutron bonds (treated here as a cubic system) and nuclear shells populated by protons, which resemble the atomic shells of the periodic table.
The excess neutrons (those not paired with protons) are located in the nuclear envelope.
The model was first tested and confirmed on various nuclear phenomena and then its link to atomic physics was demonstrated and analyzed.
Its main achievements are:

- a nuclear geometry from which the periodic system is derived.
- its agreement with various nuclear and astrophysical phenomena.
- demonstrating the link between the nuclear structure and the atomic properties through the correlation of the nuclear geometry with the atomic covalent radius.
- the interpretation of atomic phenomena in the light of the model.

This article summarizes the main stages of the research to understand the concept as a whole. A detailed analysis and description of each research phase will be published in separate articles.

1 Introduction

The nucleus and the atom have a size difference of about five orders of magnitude and are governed by different forces. The nucleons are held by the (strong) nuclear force and the properties of the atom are derived from the solution of the Schrödinger equation, where the nuclear is treated as a point electric charge. Beyond that, the nucleus has almost no effect on the atom. Only for hydrogen there is an exact solution. In larger atoms there is no complete solution and corrections are added.
In nuclear physics the theory is incomplete. There is no nuclear potential, only approximations and estimates through models and instead of a quantum field theory to describe the nuclear force, an effective field theory is used.
This study claims that there is a direct connection between the nuclear structure and the atomic properties and presents a model that supports this idea.
This claim seems to contradict the prevailing view in physics that there is a separation between the two theories, which is why it is necessary to explain how this is possible and why this idea might still be viable.
The separation between the nuclear structure and that of the atom is neither self-evident nor intuitive. The assumption that the structure of the hydrogen atom can be extended to the structure of larger atoms is not necessarily true. In hydrogen atom, we assume that the electron moves like a free particle in the field of the nucleus and obtain excellent calculation results, but this does not prove that this approach necessarily corresponds to reality.

The purpose of this research is to investigate whether there is a tangible interpretation of nuclear physics using a nuclear model that reflects the atomic structure.
The research questions are whether it is possible to create such a model and, if so, whether this nuclear structure significantly determines the atomic properties.
According to the model, the nucleus has an ellipsoid shape with cubic proton-neutron bonds and reflects the properties of the periodic table. The development of the model is described in detail in the study itself.
The model was successfully tested on various nuclear and astrophysical phenomena and a correlation was found between the nuclear structure and the atomic properties.
The model makes nuclear and atomic physics quite trivial and obvious. It is simple and tangible, and aims to deliver a realistic presentation of the nucleus, it handles all nuclei, small and large, stable and unstable.
The model is preliminary, referring to a static nucleus (ignoring for instance vibration and rotation) and requires extensions and improvements; nonetheless, even in this simple state it provides good results.
Two questions must be asked. The first is why the model is necessary, when the physical theories are so strong and accurate. The reason for this is that there may be a fundamental flaw in the approach of existing physics and the concept offered by the model may lead to a more correct description of the nucleus. It is not a mere philosophical discussion, but a principled one, in search of a more correct way to present reality. Through this process the standard model and atomic physics might also be analyzed.
The second question is whether it is possible that such a model has not been tested and rejected in the past. There might be several answers to this question. The success of the Bohr model and the impressive achievements of quantum theory gave rise to the feeling that this was the right approach.
The belief that the quantum solution has nothing to do with anything called classical, and therefore there is no way to attribute a tangible form or interpretation to it, possibly distanced physicists from the interest in interpretation. The approach according to which the interpretation is secondary and not necessary, contributed to the outright rejection of different approaches to the problem. The interpretation is seen as a marginal issue that is left to the care of the philosophers and has no scientific significance. This point of view is well described by the phrase coined by David Mermin "shut up and calculate" that means that as long as it works well the precise nature of the problem is secondary.
We compare it as an example with the trajectories of the planets in the geocentric model. The contradictions of planetary motion were apparently resolved by epicycles. When the heliocentric model was chosen, epicycles were still used to maintain circular planetary motion. Only later were the elliptical orbits established. Possibly a similar process happened in nuclear and atomic physics and a fixation was created through the success of the corrections and adjustments and thus a logical model is perceived as naive, arbitrary and speculative.

2 Part one: the model and its mass formula

2.1 The requirements

Assuming the model holds, the requirements are derived from the experimental data, the structure of the periodic table and "physical common sense".
The requirements were chosen to allow the initial development of a simplified nuclear model.
<u>The nuclear properties</u>

- The nuclear shape should make sense from a physical point of view.
- The system of bonds between the nucleons is assumed to be homogeneous and periodic; this means that the nuclear density (the distance between two neighboring nucleons) is assumed to be (at least nearly) constant in all three dimensions and for all nuclei.

- In a stable nucleus a proton is connected only to neutrons, *p-n* bond, because it is assumed that the *p-p* bond has a too strong electric repulsion; otherwise, one might expect to observe a stable diproton, He_2^2, atom for instance.
- In a stable nucleus a neutron is preferably connected with protons, *p-n* bond, because it is assumed that the proton stabilizes the neutron and that the *n-n* bond alone (with no protons involved) is unstable; otherwise, the observation of a stable neutronium nucleus, n_2, would be expected.
- The spin of the nucleons shall be equally and symmetrically divided in the nucleus.
- The nuclei of all isotopes shall have the correct total nuclear spin.

Nuclear shells

- The nuclear shells shall be populated with protons similarly to the atomic shells (here referred to the rows of the periodic table) to justify the model assumption.
- The same holds for the orbitals and their population sequence.
- The nuclear proton distribution shall be equal for all isotopes of the same element to justify their identical chemical behavior.
- Pauli's exclusion principle must be fulfilled.

A comparison with experimental data

A theoretical mass formula suitable for the model (unlike the common semi-empirical one) shall be constructed to test the matching between the theoretical and experimental data.

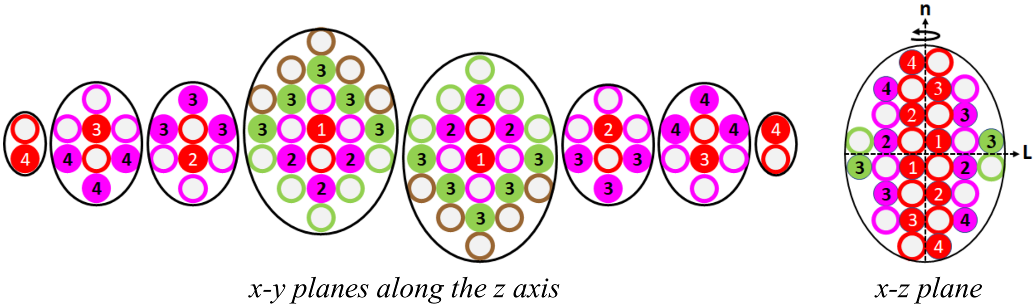
2.2 The model

From the above requirements a model was derived:

- The nuclear structure:
    - The shape of the nucleus is in general an ellipsoid, which is physically reasonable.
    - It consists of a cubic system of proton-neutron bonds with a constant distance between neighboring nucleons.
    - The excess neutrons, beyond those that are paired with protons, are in the envelope of the ellipsoid.
- Properties:
    - The population of the nuclear shells with protons are equal to the population of the atomic shells with electrons.
    - The nuclear principal quantum number, n, grows with the distance from the origin (the center of the nuclear ellipsoid).
    - The perpendicular distance of the nucleons from the z-axis (in the x-y-plane) depicts the angular momentum (and so the sub-orbitals).
    - The nucleons are equally distributed with protons and neutrons with spin-up or spin-down, except for only few nucleons, if their number is odd.
    - The nucleus possibly rotates around its main axis (the z-axis).
- The model attempts to assert the following:
    - A justification of the periodic table.
    - The correct nuclear population of protons and neutrons.
    - Reasoning why different isotopes of the same element have equal electronic properties.
    - The correct nuclear spin.
    - It agrees with Pauli's exclusion principle.

As an example, the x-y plane cross sections along the z-axis and the x-z plane cross section of the Krypton Kr_{36}^{82} nucleus are observed.

Figure 1: Kr_{36}^{82} nucleus



Legend: protons*: full circles according to the orbitals* ***S****,* ***P****,* ***D*** *($L = 0, 1, 2$).*

numbers*: principal quantum number n.*

neutrons*: hollow circles with colors according to their orbital.*

excess neutrons*, beyond the number equal to the protons (unpaired neutrons).*

2.3 The Mass formula

The mass formula was built in accordance with the theory of the model, unlike the semi-empirical one [7], [8], [9]: $m_{calc_x} = Z_x \cdot m_p + N_x \cdot m_n - \frac{(E_{b_x} - E_{c_x})}{c^2}$ (1)

- A_x: the atomic mass (number of nucleons) of the nucleus x.
- m_{calc_x}: the calculated mass of the nucleus x.
- Z_x: the atomic number.
- m_p: the mass of the proton.
- N_x: the number of neutrons ($N_x = A_x - Z_x$).
- m_n: the mass of the neutron.
- E_{b_x}: the total energy of the bonds between nucleons in the nucleus x.
- E_{c_x}: the total electric energy (between all protons) in the nucleus x.
- c: the speed of light.

$E_{b_x} = e_b \cdot n_{b_x}$: (2)

- e_b: the energy of a single nucleon-nucleon bond in the nucleus, assuming they are equal for all bond types and bonds in all nuclei.
- n_{b_x}: the number of nucleon-nucleon bonds in the nucleus x.

$$E_{c_x} = \frac{e^2}{4\pi\epsilon_0} \frac{1}{d_0} \left\{ \frac{1}{2} \sum_i^{Z_x} \sum_{j \neq i}^{Z_x} \frac{1}{d_{i,j}} \right\} = \frac{e^2}{4\pi\epsilon_0} \frac{1}{d_0} e_{c_x} \quad \text{where} \quad e_{c_x} := \frac{1}{2} \sum_i^{Z_x} \sum_{j \neq i}^{Z_x} \frac{1}{d_{i,j}} \qquad (3)$$

- d_0: the minimum distance between two neighboring nucleons (assuming all nuclei have the same cubic structure and distance between their nucleons).
- $d_{i,j}$: the unitless distance between the protons of the indices i and j measured in multiples of d_0: $d_{i,j} = \sqrt{(x_j - x_i)^2 + (y_j - y_i)^2 + (z_j - z_i)^2}$ (4)
- e_{c_x}: the unitless total electric energy of the nucleus (sum of the reciprocal distances).

The absolute relative error of the calculation for the nucleus x is:

$$rel\_err_x = \left| \frac{m_{calc_x} - m_{meas_x}}{Z_x \cdot m_p + N_x \cdot m_n - m_{meas_x}} \right| = \left| \frac{m_{calc_x} - m_{meas_x}}{mass\_defect_x} \right| \qquad (5)$$

- m_{meas_x}: the measured mass of the nucleus x.

- $mass\_defect_x$: $Z_x \cdot m_p + N_x \cdot m_n - m_{meas_x}$ is the mass defect of the nucleus x.
  Note: rel_err_x is represented here in percentage.

The mass formula depends thus only on the two variables:

- d_0: the minimum distance between two neighboring nucleons.
- e_b: the energy of a single nucleon-nucleon bond.

The implementation requires two preliminary calculation steps for each nucleus:

- Drawing the nucleus x and counting the number of nucleon-nucleon bonds n_{b_x}.
- Calculating the relative total electric energy of the nucleus e_{c_x}.

Note: the exact shape of the nucleus affects the results of the mass formula.

2.3.1 The mass formula calculation of the nuclei of full sub-orbitals

As mentioned above, the nuclear shape influences the mass formula results; this is caused through:

- The shape of the nuclear core, that consists of proton-neutron pairs and affects the electric charge distribution.
- The total arrangement of the nucleons, that determines the number of bonds in the nucleus and therefore affects the binding energy.

These points are mentioned to ensure a scientific approach and to avoid manipulation of data.
The nuclear core has a specific and unambiguous arrangement, so the electric energy seems to be well calculated.
Unlike this, the arrangement of the excess neutrons in the nuclear envelope can be achieved in various ways and this entails a modification of the number of bonds and as a result a change of the binding energy.
A question that may arise is how to avoid arranging the excess neutrons in the nuclear envelope in a way that is suitable for obtaining good results.
To address this, the nuclei of complete sub-orbitals are drawn and the excess neutrons are arranged in a consistent manner. The drawings of the nuclei of full sub-orbitals are shown in the appendix.
Only then the mass formula is calculated.
This process achieves the following (experimental data were taken from [1]):

- A consistent nuclear form is shown.
- All nuclei drawn according to this form are:
  - within the stability range of their isotopes, for elements smaller than $Z = 82$.
  - within the range of relative longer half-life, for radioactive elements, larger than $Z = 82$.
- The mass formula results, for the relative error of the eleven (11) nuclei larger than Argon, Ca_{20}^{40} till Ra_{88}^{218} are: $max < 3\%$, $mean < 1\%$, $std.dev < 0.7\%$.

This is within reasonable range [7], [10]. The calculation parameters were found as:

- $d_0 = 1.62 \pm 0.03\ fm$ the minimum distance between two neighboring nucleons.
- $e_b = 5.72 \pm 0.03\ MeV$ the energy of a single nucleon-nucleon bond.

this seems to be within range as well [5].
Through $d_0 \approx (r_n + r_p)$ a rough estimate for the sum of the radii of the proton and neutron is achieved and the relative error is estimated:

- $r_n \approx 0.80\,fm$ *neutron radius* [3], $r_p \approx 0.84\,fm$ *proton radius* [4].
- $r_n + r_p \approx 1.64\,fm$
- Relative deviation for d_0: $\left|\frac{d_0 - (r_n+r_p)}{(r_n+r_p)}\right| = \left|\frac{1.62-1.64}{1.64}\right| = \left|\frac{0.02}{1.64}\right| \approx 1.5\%$ (6)

This is a byproduct of the mass formula calculation, which provides an unintended result that reinforces the model assumption.

2.3.2 Extending the mass formula calculation to the most abundant nuclei

At this stage isotopes of elements with larger abundant, from Lithium, Li_3^7 to Plutonium, Pu_{94}^{244}, are drawn (for several elements more than one isotope was taken) and the mass formula calculation is performed. Experimental data are taken from [1].
Unlike the section above regarding the nuclei with full sub-orbitals, here the precise arrangement of the nucleons, and especially of the excess neutrons, is only roughly assessed, based on the knowledge gained before and on common sense; this means that the results achieved must be taken with suspicious, yet the aim isn't the exact prediction of the nuclear masses, but the new point of view on physics and the phenomena, that the model tries to explain, and this is achieved here.
Nuclei till approximately Argon, Ar_{18}^{40} show larger relative errors than those of heavier nuclei; this phenomenon is known also for the common mass formula [7], [8], [9]. It is assumed here that this occurs in small nuclei either due to a variation in the distance d_0 between the nucleons or due to a shift from the cubic form.

The calculation parameters were found to be equal to those for the nuclei with full sub-orbitals. After an improvement process, that included several iterations to ensure consistency of the nuclear shape and learning of the patterns, the relative error results for 330 nuclei from Ar_{18}^{36} to Pu_{94}^{244} were calculated:

max.	mean	st. dev.	rel. err. $\leq 2\%$	$\leq 1\%$ *	$\leq 0.5\%$
2%	0.4%	0.4%	100%	93%	67%

* meaning that 93% of the nuclei have a relative error equal to or smaller than 1% etc.
It is emphasized at this point that the aim of the mass formula is mainly to ensure that the model makes sense from a physical point of view, and it does this, as can be seen.
The exact results are not critical because the mass formula is simplified and the improvement process may well contain errors, so these results are not given too much importance; the goal is to test the feasibility of the model via its ability to address nuclear phenomena such as:

- The nuclear charge radius.
- The radioactivity of heavy nuclei beyond $Z \approx 82$.
- The short half-life of nuclei larger than $Z \approx 104$.
- The nuclear fission and its more probable products.

Further studies shall show that the main influence comes from the nuclear core of the proton-neutron pairs, so the precise deployment of the excess neutrons in the envelope is less crucial.

3 Part two: the link between the nuclear geometry and the atomic properties

To confirm the model hypothesis, a direct link between the nuclear structure and the atomic properties must be shown.

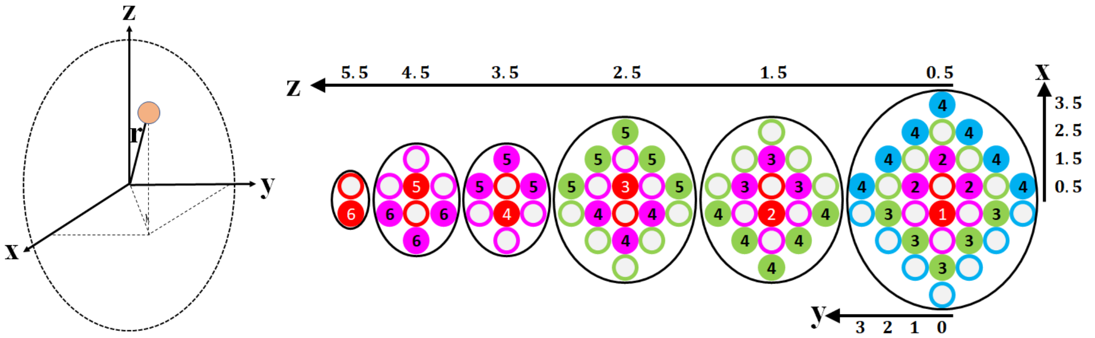
3.1 Nuclear covalent radius - definition and calculation

As a preparation step, two definitions are made:

- A "valance proton": the proton, that was last added to the current, in-filling process, sub-orbital of the nucleus.
- The "nuclear covalent radius": the relative geometric distances of the "valance proton" from the nuclear center.

The following illustrations demonstrate the way the "nuclear covalent radius" of the "valance proton is calculated from the nuclear geometry via $r_{x,y,z} = \sqrt{x^2 + y^2 + z^2}$

Figure 2: calculating the nuclear covalent radius



a proton in the nucleus

the ***x-y*** *planes along the nuclear* ***z*** *axis (upper half only)*
protons*: full circles according to the orbitals* ***S***, ***P***, ***D***, ***F***.
numbers*: principal quantum number.* ***neutrons****: hollow circles.*

Note: the variables x, y, z, refer to the distances of the protons from the nuclear center; due to the nuclear geometry, there is an apparent shift so that for instance the position of the central proton is $(x, y, z) = (-0.5, 0, 0.5)$ and not $(x, y, z) = (0, 0, 0)$ as might be intuitively expect.

3.2 A comparison between the atomic and nuclear covalent radii

The cubic ellipsoid nuclear model was first created in search of a connection between the nuclear structure and the atomic properties. Therefore, the pattern of the atomic covalent radius is sought to compare with the "nuclear covalent radius" of the corresponding nuclear sub-orbital.

Each atomic covalent radius must be associated with the correct "nuclear covalent radius".

It is easier to draw the radius of nuclei with a symmetrical shape, so only nuclei with even sub-orbitals are referred to here (s, p, d, f).

The atomic data is available for atomic number $Z \in [1, 96]$; the nuclei are thus:

Row 1: He_2^4

Row 2: Be_4^9, Ne_{10}^{20}

Row 3: Mg_{12}^{24}, Ar_{18}^{36}

Row 4: Ca_{20}^{40}, Zn_{30}^{70}, Kr_{36}^{72}

Row 5: Sr_{38}^{86}, Cd_{48}^{116}, Xe_{54}^{128}

Row 6: Ba_{56}^{132}, Yb_{70}^{168}, Hg_{80}^{202}, Rn_{86}^{214}

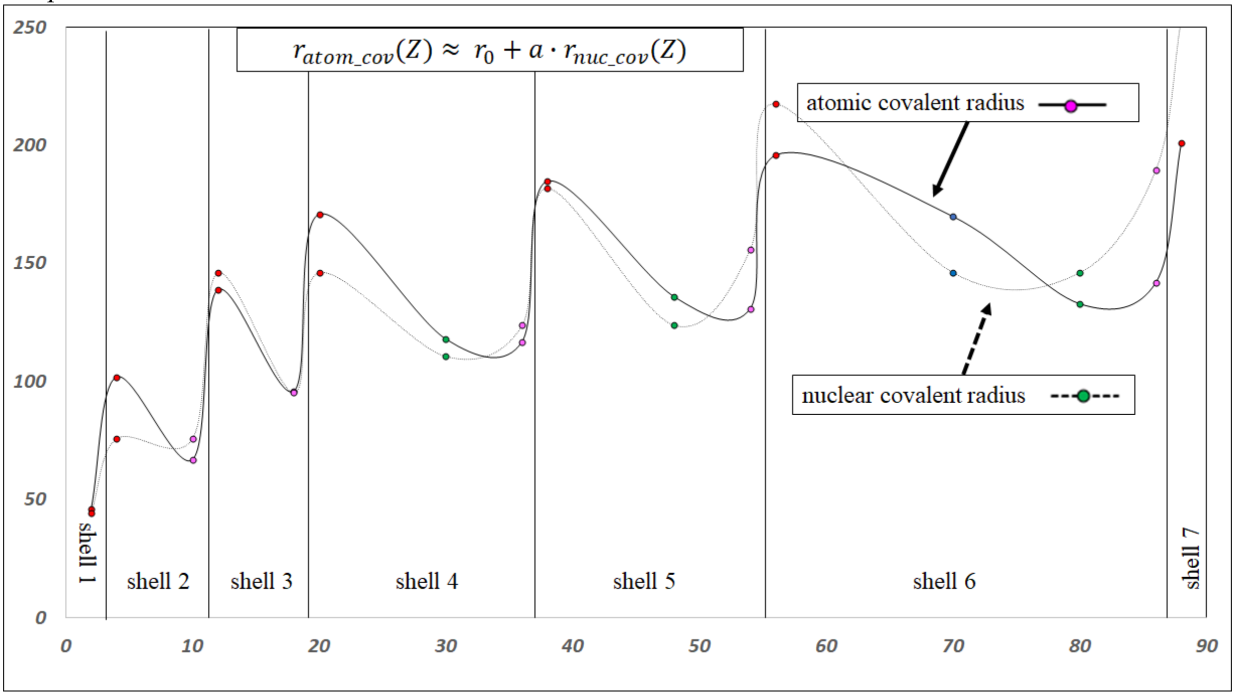
Row 7: Ra_{88}^{218}, No_{102}^{254}, Cn_{112}^{284}, Og_{118}^{296}

- Two curves are observed and compared:
- The atomic covalent radius, taken from [11].
- The nuclear covalent radius, calculated according to the illustrations above.

The comparison between the two curves is implemented in relative values, since the two curves refer to different sizes and units; the atomic covalent radius is given in

$[pm]$ or $10^{-12}m$, whereas the proton distance in relative values (the adjustment parameters have the values $r_0 \approx 40$, $a \approx 30$).

Graph 1: the nuclear vs. atomic covalent radius

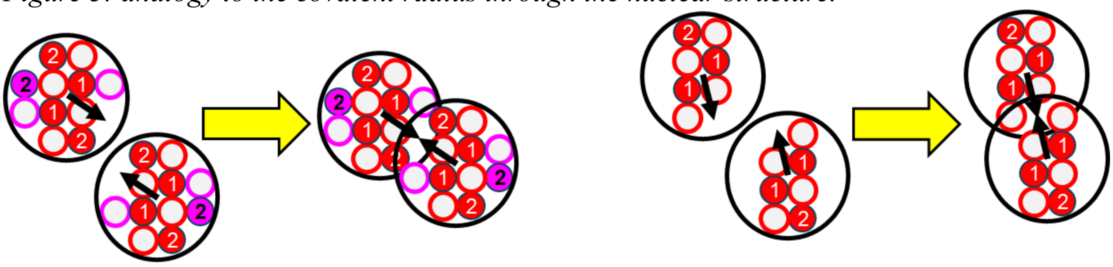


A seemingly correlation between the two curves is observed, although the atomic curve is taken from the experimental data and the nuclear covalent radius is a geometric property, measured according to the ellipsoid model; therefore, it is concluded that this implies a basic inherent link between these two entities.

3.3 The atomic covalent radius - geometric interpretation

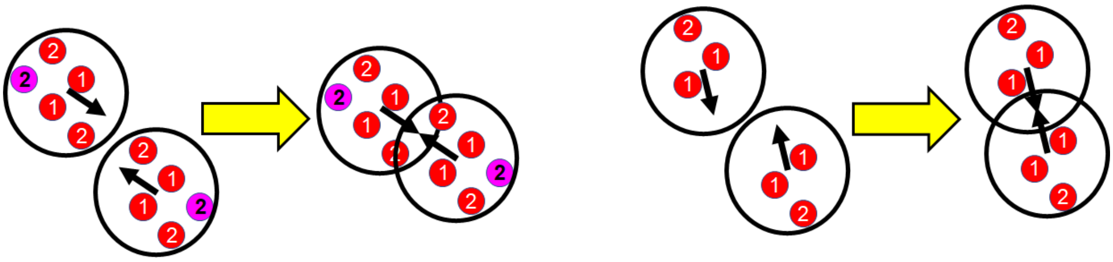
To visually explain the relationship between the valence proton and the atomic covalent radius (in the x-z plane), pairs of atoms are observed, first before they combine into molecules (to the left of a yellow arrow) and then as molecules (on the right); the black arrows depict the covalent radius, that is generally smaller than the most distant position on the nuclear surface. The covalent bond is assumed to occur so that the molecule strives to complete symmetry. To clarify which position in the atom is meant, the situation is first presented as if the connection occurs between the nuclei:

Figure 3: analogy to the covalent radius through the nuclear structure.



Then the same illustrations are used for the atom and it is treated as if the atomic shape is similar to the nuclear one (the electrons are set in the positions of the protons and the neutrons are removed).

Figure 4: the atomic covalent radius. From separated atoms to molecule.



The source of attraction is assumed to be the protons of the suborbital that is being filled (see also appendix: ionization energy); electrons from the corresponding suborbital partially move towards the bonding position due to the greater positive attraction there.
The mechanism raised here could mean also, that the angles of the chemical bonds between atoms are influenced by the nuclear structure. Another hypothesis is raised that there may be a connection between the atomic and nuclear shape.
Remark: the interpretation of the covalent radius, drawn here, is less crucial at this stage of the research. What matters is the correlation between the atomic and the nuclear covalent radius.

4 Conclusion

The model

The cubic ellipsoid nuclear model offers a perspective on nuclear and atomic physics that is different than the common one and tries to justify this idea via calculations.
The model claims to deliver a comprehensive image of the nucleus and is appropriate for small and large nuclei. It addresses the nuclear mass, shells, clustering and charge radius. It can handle the nuclear rotation and vibration. It handles various nuclear and astro-physical phenomena.
It does not seem to contradict the theory, but rather to expand its understanding and open new research directions of the nuclear theory and possibly also to other fields of physics.

The link between the nuclear structure and the atomic properties

A correlation is found between the nuclear geometric structure and the atomic covalent radius. If the model assumption is accepted, then the atomic and nuclear shape are expected to correlate with each other; this correlation is therefore expected between the valence electron and the valance proton.
It is emphasized that the covalent atomic radius is not necessarily the "furthest point" on the "atomic surface", but rather the point that matches the "valence proton" (the van der Waals radius, for example, probably corresponds better to the "furthest position" on the "surface" of the atom). This unexpected correlation is a strong hint for a link between the nuclear and atomic geometries and their properties and might be the proof to the cubic ellipsoid nuclear model.

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Appendix

The purpose of the research is to look at nuclear and atomic physics from a different perspective instead of improving the mass formula or obtaining more accurate results.
A main goal is to investigate the hypothesis that the nuclear structure determines the atomic properties, therefore different nuclear and atomic phenomena will be treated considering the model, to ensure that the model can handle them without contradictions.
The various studies aimed at validating the model are briefly presented here.
Not all of these issues are fully developed. Some describe only an idea, but some go into details through calculations derived from the model.
The topics are divided according to their fields:

- The nuclei with full sub-orbitals: the development of the stable nuclei of the full sub-orbitals shows a consistent pattern that strengthens the model assumption and thus reduces the possibility of an arbitrary theoretical idea.

<u>Nuclear phenomena</u>

- According to the model, the excess neutrons are located in the nuclear envelope. The charge radius data helps reinforce this idea; the model is used to interpret charge radius measurements.
- Radioactivity of heavy and super-heavy nuclei is explained by the electric energy in the center of the nucleus; this also leads to the estimation of the energy of a typical alpha decay.
- Nuclear fission: the mechanism, the expected products and the energy released are analyzed using the model.
- Nucleon spin: possible distribution of the spin of the nucleons in the nucleus; this issue is not of critical importance at this stage, but serves as a tool for comparison between our model and the common nuclear shell model.
- Nuclear magic numbers: a search for the shape of the nuclei is carried out to find a pattern that may explain why these nuclei are more stable. Like the spin, also this subject is only in a preliminary research stage.

<u>Astrophysics</u>

- The hypotheses of constant tangential velocity and minimum atomic size: additional ideas related to the model as a means of evaluating astrophysical phenomena and developing ideas about the nature of the atom.
- Neutron star and the TOV limit: using the nuclear model to assess the size of a neutron star and the limit beyond which it collapses into a black hole.
- White dwarf and the lower limit of atomic size: similar to the last paragraph, the size and limits of a white dwarf, beyond which a neutron star is formed, will be estimated; as a result, ideas about the nature of the atom are raised.
- Pulsar - the lower limit of the rotation period: the model is used to estimate the highest frequency a rotating neutron star can reach.

<u>Atomic physics</u>

- The first ionization energy: a formula is constructed to explain the experimental data and then the link to the model is explained.

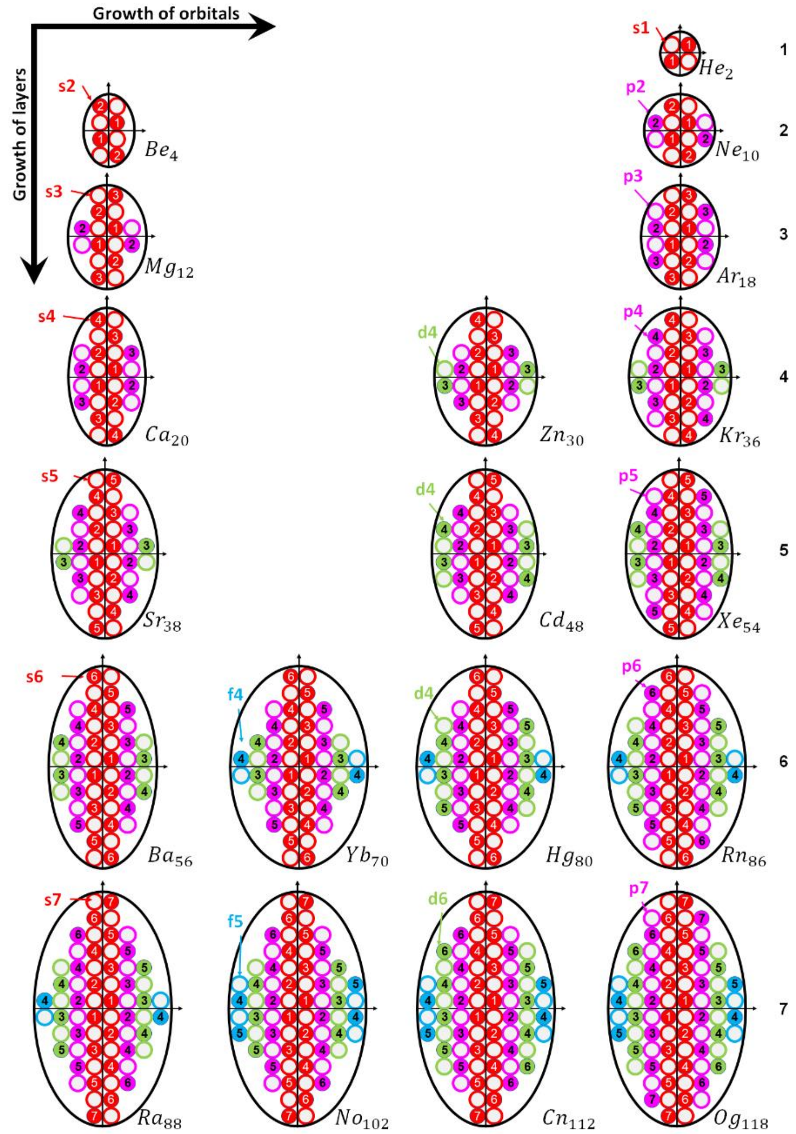
Electronic transition selection rules: under the model assumption, that there is a direct link between the nuclear structure and the atomic properties, the electronic transition rules seem to be obvious.

App. 1 The nuclei with full sub-orbitals

App. 1.1 The ellipsoids of the nuclei with full sub-orbitals of the periodic table

To better understand the model, the ellipsoids of the filled sub-orbitals are shown and arranged as they appear in the periodic table. The orbitals grow from left to right and the layers grow from top to bottom; the colored arrows refer to the last filled orbital.

Figure 5: Cross sections in the x-z plane of the ellipsoids of the full sub-orbitals



Legend: protons*: full circles according to the orbitals* ***S****,* ***P****,* ***D****,* ***F****.*
numbers*: principal quantum number.*
neutrons*: hollow circles with colors according to their orbital.*

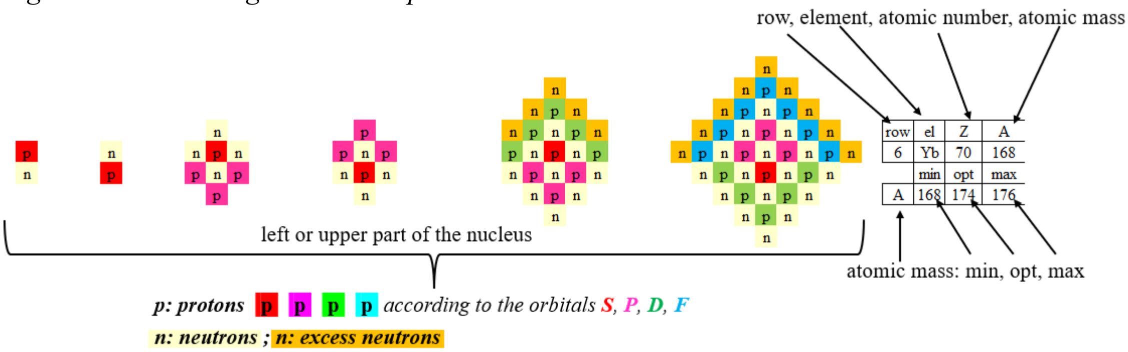
App. 1.2 The drawings of the nuclei of full sub-orbitals in a consistent form

In the following section the nuclei of closed sub-orbital S, P, D, F are shown in a consistent form that describes their development, so one can develop a feeling of the guidelines for a stable nucleus.
The fact that such a consistent structure can be generated and that the represented nuclei for each of the observed elements lie within the range of their stable isotopes (or long-lived ones in the case of heavy nuclei beyond Pb) may be a strong confirmation of the model.
It is emphasized again that first the nuclei of full sub-orbitals are created, in a consistent manner and only after that their mass formula is checked, meaning that it was not an iterative process, meaning that no data manipulation was done.
One can follow a clear pattern of nuclear growth with clear positions at which the excess neutrons are added.

The nuclei of closed sub-orbitals according to their row in the periodic table

Row 1: He_2^4
Row 2: Be_4^9, Ne_{10}^{20}
Row 3: Mg_{12}^{24}, Ar_{18}^{36}
Row 4: Ca_{20}^{40}, Zn_{30}^{70}, Kr_{36}^{72}
Row 5: Sr_{38}^{86}, Cd_{48}^{116}, Xe_{54}^{128}
Row 6: Ba_{56}^{132}, Yb_{70}^{168}, Hg_{80}^{202}, Rn_{86}^{214}
Row 7: Ra_{88}^{218}, No_{102}^{254}, Cn_{112}^{284}, Og_{118}^{296}

The first two nuclei of closed sub-orbitals, He_2^4 and Be_4^9, are too small to show a specific pattern, so Ne_{10}^{20} opens the list.
A pattern is seen, and the calculation was done only subsequently, so no fit process was done to manipulate or improve the results.
The drawings describe cross sections of the nuclei in the x-y planes along the z axis.
For visibility only the upper (or left) half of the nuclei is shown.
Figure 6: determining the nuclear parameters

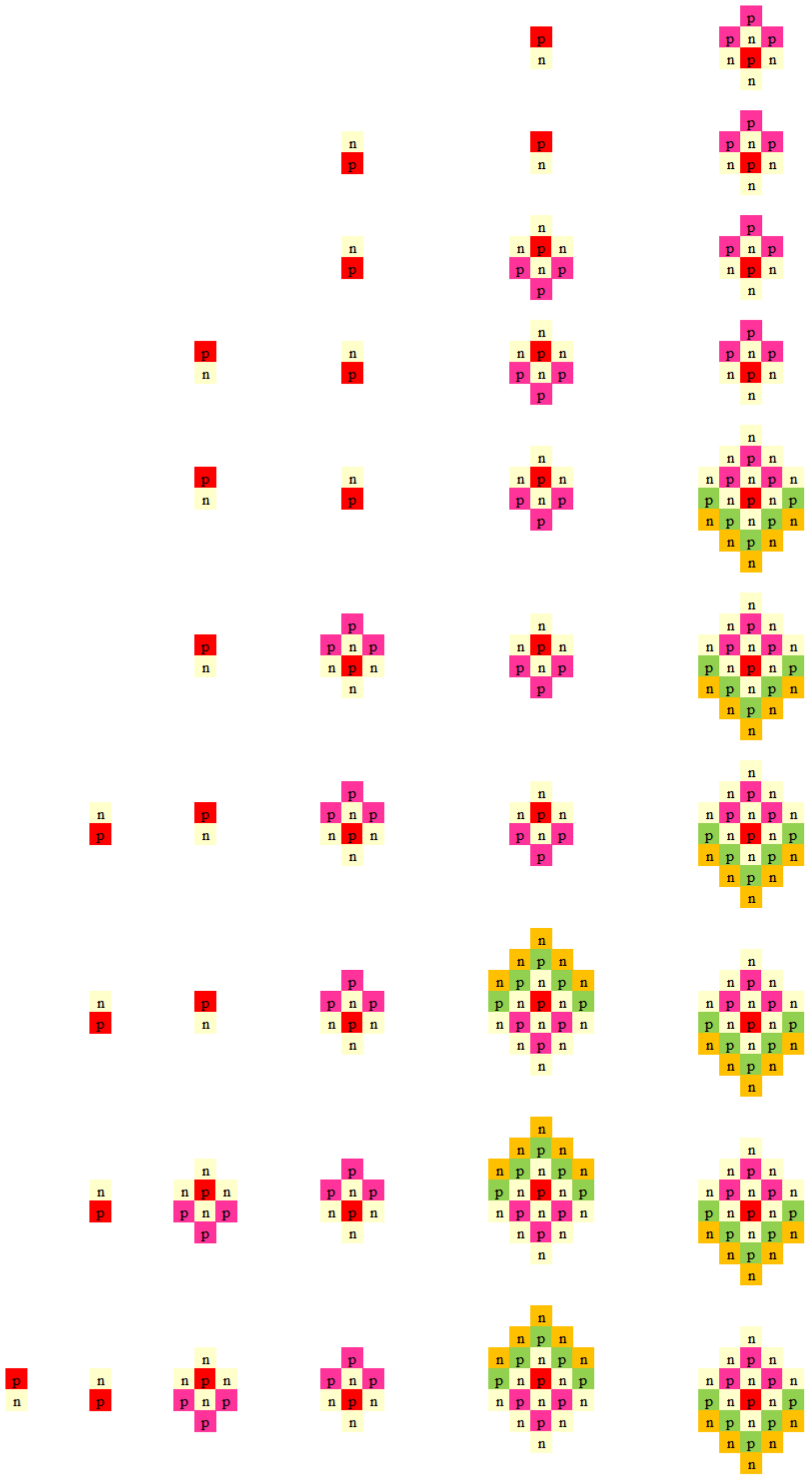


For the atomic mass four parameters are shown:

- A: the atomic mass of the nucleus referred to in the drawing.
- min: the minimum value for which the nucleus is stable or, for nuclei beyond Z=82, the minimum value for which the nucleus has a relative longer half-life.
- opt: the most abundant value.
- max: the maximum value (in a similar manner to the definition of the minimum).

All nuclei are found to be in the range: $min \leq A \leq max$. Data from [1].

Figure 7: the nuclei of full sub-orbitals



row	el	Z	A
2	Ne	10	20
	min	opt	max
A	20	20	22

row	el	Z	A
3	Mg	12	24
	min	opt	max
A	24	24	26

row	el	Z	A
3	Ar	18	36
	min	opt	max
A	36	40	40

row	el	Z	A
4	Ca	20	40
	min	opt	max
A	40	40	48

row	el	Z	A
4	Zn	30	70
	min	opt	max
A	64	64	70

row	el	Z	A
4	Kr	36	82
	min	opt	max
A	78	84	86

row	el	Z	A
5	Sr	38	86
	min	opt	max
A	84	88	88

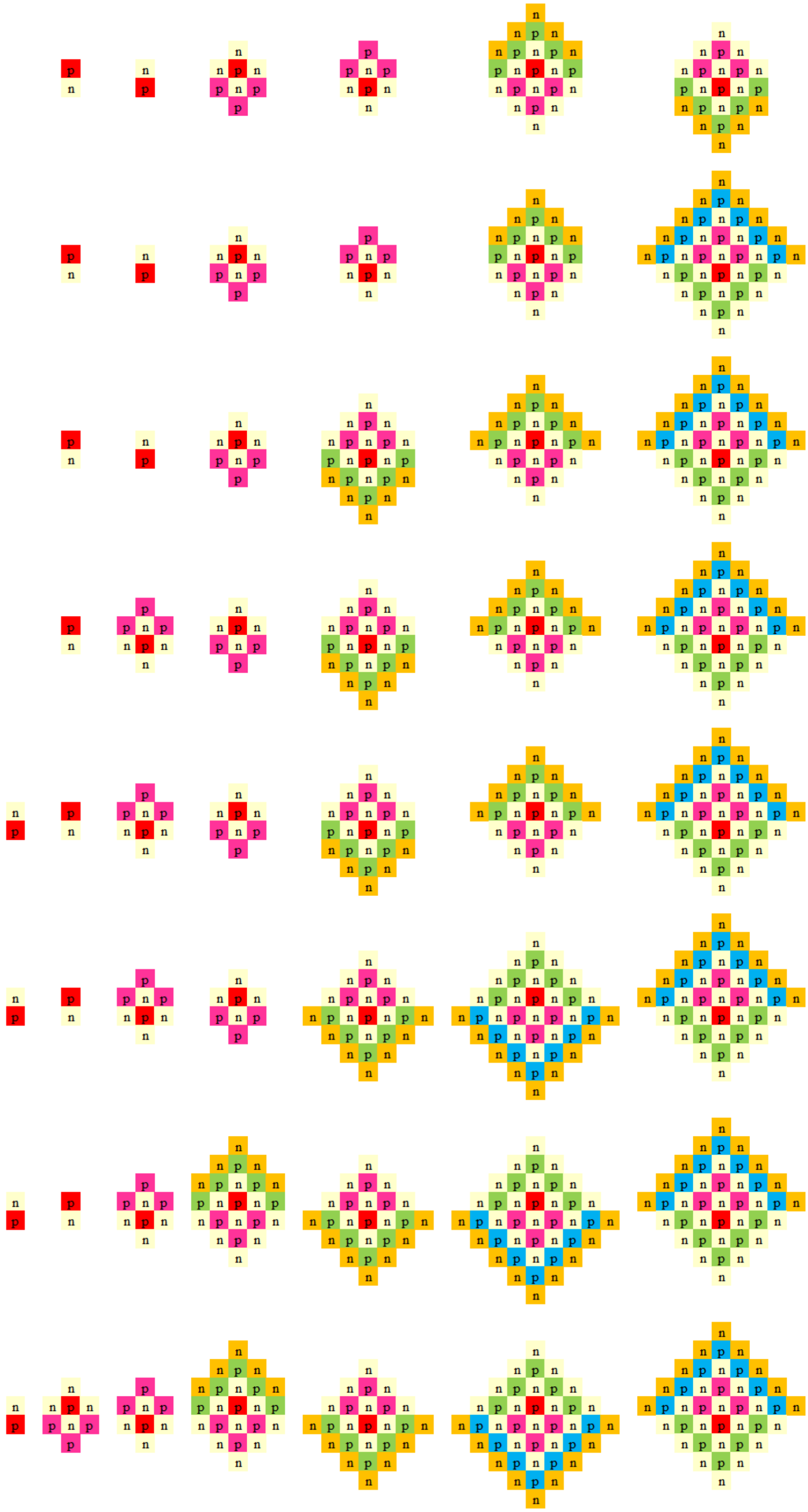
row	el	Z	A
5	Cd	48	116
	min	opt	max
A	106	114	116

row	el	Z	A
5	Xe	54	128
	min	opt	max
A	124	132	136

row	el	Z	A
6	Ba	56	132
	min	opt	max
A	130	138	138

p: protons p p p p *according to the orbitals* ***S, P, D, F***

n: neutrons ; n: excess neutrons



row	el	Z	A
6	Ba	56	132
	min	opt	max
A	130	138	138

row	el	Z	A
6	Yb	70	168
	min	opt	max
A	168	174	176

row	el	Z	A
6	Hg	80	202
	min	opt	max
A	196	202	204

row	el	Z	A
6	Rn	86	214
	min	opt	max
A	204	222	226

row	el	Z	A
7	Ra	88	218
	min	opt	max
A	212	226	230

row	el	Z	A
7	No	102	254
	min	opt	max
A	252	259	259

row	el	Z	A
7	Cn	112	284
	min	opt	max
A	278	283	286

row	el	Z	A
7	Og	118	296
	min	opt	max
A	294	294	296

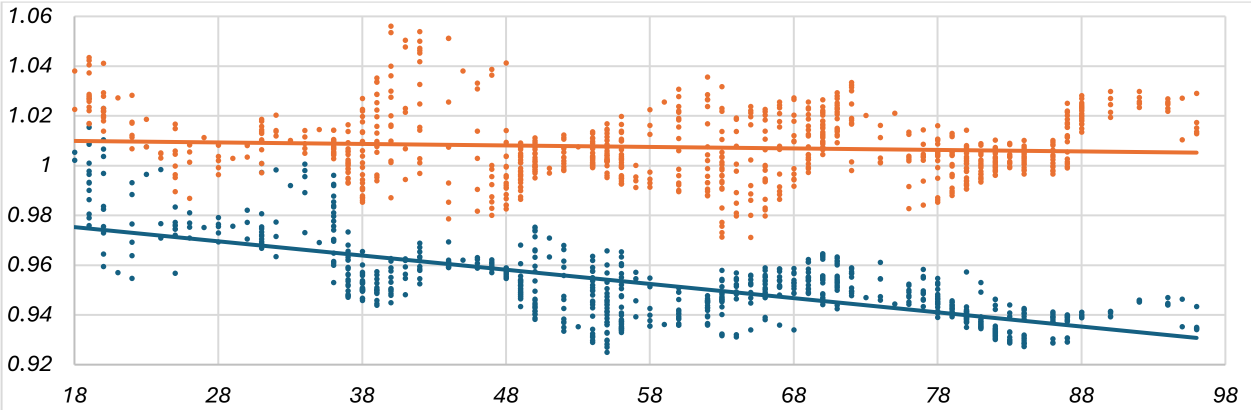
p: protons p p p p *according to the orbitals* ***S, P, D, F***

n: neutrons ; n: excess neutrons

App. 2 Nuclear phenomena

App. 2.1 The charge radius and the location of the excess neutrons in the nuclear envelope

According to the liquid drop model an approximately constant value would be expected for the ratio between the nuclear charge radius $\boldsymbol{R_c}$ and the third root of $\boldsymbol{A}$, the atomic mass or the number of nucleons $\sqrt[3]{\boldsymbol{A}}$.
In this study, it is assumed that the nuclear core consists of *p-n* pairs and that the excess neutrons are located in the nuclear envelope. Therefore it is expected that the nuclear charge radius $\boldsymbol{R_c}$ will fit $\sqrt[3]{\boldsymbol{2 \cdot Z}}$ better than $\sqrt[3]{\boldsymbol{A}}$ (with $\boldsymbol{Z}$ the atomic number; the constant **2** is added to express the dependence on the nuclear core); this is actually the case.
The following graph confirms this via comparison between $\frac{\boldsymbol{R_c}}{\sqrt[3]{\boldsymbol{A}}}$ and $\frac{\boldsymbol{R_c}}{\sqrt[3]{\boldsymbol{2 \cdot Z}}}$ for more than 800 nuclei from Ar_{18} to Cm_{96} vs. $\boldsymbol{Z}$; for nuclei smaller than Ar_{18} the number of protons and neutrons is quite equal so there is no major difference between the two. Data from [2].



Graph 2 : charge radius. Comparison between $\frac{\boldsymbol{R_c}}{\sqrt[3]{\boldsymbol{A}}}$ *and* $\frac{\boldsymbol{R_c}}{\sqrt[3]{\boldsymbol{2 \cdot Z}}}$ *vs.* $\boldsymbol{Z}$ *for more than 800 nuclei.*

The following table summarizes the calculations of the above data.

Table 1: comparison between $\frac{\boldsymbol{R_c}}{\sqrt[3]{\boldsymbol{A}}}$ *and* $\frac{\boldsymbol{R_c}}{\sqrt[3]{\boldsymbol{2 \cdot Z}}}$

calculations	$\frac{R_c}{\sqrt[3]{A}}$	$\frac{R_c}{\sqrt[3]{2 \cdot Z}}$
slope	$-6 \cdot 10^{-4}$	$-6 \cdot 10^{-5}$
Δ = max – min	0.102	0.085
standard dev.	0.016	0.013

The excess neutrons that have less influence on the electric charge distribution and so different isotopes of the same element have the same chemical behavior.
This strengthens the model assumption.
Remark: an article on this subject shows how the nuclear geometry allows good estimates of the radii of the noble gases.

App. 2.2 The radioactivity of heavy and super-heavy nuclei

App. 2.2.1 The radioactivity hypothesis

According to the model, the mechanism that determines the radioactivity of heavy nuclei beyond Lead (Pb_{82}) is the electric energy that overcomes the binding energy (the strong nuclear force) between the nucleons

Instability is assumed to occur in the middle of the nuclear ellipsoid, where the electric energy reaches its maximum value; by the discussion of nuclear fission, this idea will be further used.

The calculations provide a rough prediction of nuclear stability. For nuclei larger than Lead (Pb_{82}) and up to approximately Rutherfordium (Rf_{104}) six nuclear bonds are required to keep the central protons stable.

The model hypothesis is that due to movements or fluctuations within the nucleus there is some probability that these six bonds are temporarily reduced to five bonds every certain period of time for a certain timespan; as a result the central proton becomes unstable, possibly ending with a radioactive emission; after several radioactive steps of this type the nucleus is transformed to Pb_{82}, where five bonds are sufficient to keep the central protons stable and radioactivity ends.

The probability for this to occur and the timespan this lasts, determines the half-life of the nucleus.

Or in other words, to explain the topic from a different perspective: the central proton has six bonds also for most nuclei smaller than Pb_{82} (due to the nuclear geometry) but for these nuclei only five nuclear bonds are required at most, and so even if one bond is missing for a short while, there is redundancy, so radioactivity doesn't occur; the probability for a simultaneous lack of two bonds is probably too low and so these nuclei are practically stable.

It is noted that also for nuclei beyond Lead, this phenomenon might have a low probability, leading by some of isotopes with $Z > 82$ to half-life in the range of even millions of years.

For nuclei beyond Rutherfordium (Rf_{104}) even six bonds are not enough to keep the central protons stable, meaning that they are inherently unstable, and therefore these nuclei have, in general, a short half-life of not more than hours, and usually much less.

Remarks:

- The focus here is to verify the feasibility of the model, so that only a very rough estimate of the process of radioactivity by heavy nuclei is given, without examining in depth the mechanism that governs it.
- Other mechanisms could also lead to instability, so that nuclei smaller than Rutherfordium (Rf_{104}) could also have a short half-life.

App. 2.2.2 The energy balance of an alpha decay

Our hypothesis is that the radioactive process of heavy elements, beyond $Z \approx 82$, occurs due to a single bond, with an energy of $e_{\boldsymbol{b}} \approx 5.7\ MeV$, that breaks for a short while in the center of the nucleus.

The energy balance of an alpha decay process:

- m_{init}: the mass of the initial nucleus.
- m_{end}: the mass of the end nucleus.
- m_{alpha}: the mass of the alpha particle.
- ΔE: the energy difference in the process.

and so: $\Delta E = \{m_{init} - (m_{alpha} + m_{end}\ )\} \cdot c^2$ (7)

The energy difference for several even nuclei with atomic number $90 \leq Z \leq 96$ was estimated with the result:

- $\Delta E \approx 5.1 \pm 0.4\ MeV$ as expected, if one bond is broken.

This might be a reinforcement to the model.
Typical energies for an alpha decay range from 3 to 7 MeV [42].
The energies smaller than 5.7 MeV can be explained due to energy losses during the process.
The larger energies might be explained by an extra energy that is available for nuclei that require more than six bonds, so the total energy of the particle can exceed 5.7 MeV, even if only one bond is broken.

App. 2.2.3 Maximum electric energy as a function of the number of nuclear bonds

According to the model mass formula the binding energy of the proton $\boldsymbol{k}$ in the nucleus $\boldsymbol{x}$ is:
$E_{\boldsymbol{b}_k} = e_{\boldsymbol{b}} \cdot n_{\boldsymbol{b}_k}$ where:

- $n_{\boldsymbol{b}_k}$ is the number of nucleon-nucleon bonds of the proton $\boldsymbol{k}$ in the nucleus.
- $e_{\boldsymbol{b}} = 5.72\ MeV$: (as found via the mass formula calculation) the energy of a single nucleon-nucleon bond in the nucleus (assuming they are equal for all bonds in all nuclei).

The electric energy of the proton $\boldsymbol{k}$ in the nucleus $\boldsymbol{x}$ is (*eq. (3)*):

$$E_{c_k} = \frac{e^2}{4\pi\epsilon_0}\frac{1}{d_0}\left\{\sum_{j\neq k}^{Z_x}\frac{1}{d_{k,j}}\right\} = \frac{e^2}{4\pi\epsilon_0}\frac{1}{d_0}\, e_{c_k} \quad \text{with} \quad e_{c_k} := \sum_{j\neq k}^{Z_x}\frac{1}{d_{k,j}}$$

- $d_0 = 1.62\ fm$: (as found via the mass formula calculation) the minimum distance between two neighboring nucleons (assuming all nuclei have the same structure of cubic bonds and the same distance between their nucleons).
- $d_{k,j}$: the unitless distance between the protons of the indices k and j measured in multiples of d_0 (*eq. (4)*): $d_{k,j} = \sqrt{(x_j - x_k)^2 + (y_j - y_k)^2 + (z_j - z_k)^2}$
- e_k: the unitless relative electric energy of the proton $\boldsymbol{k}$ in the nucleus (sum of the reciprocal distances).

The condition for proton bond stability is $E_{\boldsymbol{b}_k} \geq E_{\boldsymbol{c}_k}$; for this purpose, the maximum electrical energy of the proton k is analyzed depending on the number of its bonds:

$$(E_{\boldsymbol{b}_k} - E_{\boldsymbol{c}_k}) \geq 0 \quad \text{or} \quad (e_{\boldsymbol{b}} \cdot n_{\boldsymbol{b}_k} - \frac{e^2}{4\pi\epsilon_0}\cdot\frac{1}{d_0}\, e_{\boldsymbol{c}_k}) \geq 0$$

this means for a single bond:

$$\frac{e^2}{4\pi\epsilon_0}\cdot\frac{1}{d_0}\, e_{\boldsymbol{c}_k} \leq e_{\boldsymbol{b}}$$

Table 2: the maximum values of e_{c_k}

n_{b_k}	e_{c_k} max. value	E_b [Joule]	E_c [Joule]	E_b -E_c [Joule]
1	6.43	9.2E-13	9.2E-13	0.00
2	12.87	1.8E-12	1.8E-12	0.00
3	19.30	2.7E-12	2.7E-12	0.00
4	25.73	3.7E-12	3.7E-12	0.00
5	32.16	4.6E-12	4.6E-12	0.00
6	38.60	5.5E-12	5.5E-12	0.00

the maximum relative electric energy as a function of the number of nuclear bonds

This means that a proton with a single nuclear bond can sustain, at most, a relative electric energy of 6.43; a proton with two bonds, 12.87 and so on; a proton with six nuclear bonds can bear at most a relative electric energy of 38.60.

App. 2.2.4 Begin of instability of heavy nuclei - transition from five to six bonds

The transition from five to six bonds occurs in the region where nuclei radioactivity begins; this raises the following assumption:

- if six bonds are required for the stability of the central proton (to overcome the electric repulsion) and:
- if there is a certain probability that one bond breaks for a short while,
- then the nucleus is expected to be radioactive.

The half-life depends on the probability for the above to occur, meaning how often a bond is broken and for how long and on the processes that happen once a bond is broken.

nucleus	Z	max. half-life	bonds	max. e_{c_p}	
Pt	78	stable	5	31.30	
Au	79	stable	5	31.62	1.7%
Hg	80	stable	5	31.87	0.9%
Tl	81	stable	5	32.09	0.2%
Pb	82	stable	6	32.29	
Bi	83	y	6	32.40	
Po	84	y	6	32.73	
At	85	h	6	32.54	

deviation from the limit value 32.16

begin of instability

Table 3: the transition from five to six bonds required for the stability of the central proton.

Superheavy - very unstable nuclei - transition from six to more than six bonds

Similarly to the previous section it is assumed that:

- if more than six bonds are required for central proton stability
- then the nucleus is inherently unstable.

This occurs around $Z = 104$. The half-life is expected to be much shorter.

nucleus	Z	max. half-life	bonds	max. e_{c_p}	
Md	101	d	6	37.69	
No	102	m	6	37.91	1.8%
Lr	103	h	6	38.14	1.2%
Rf	104	m	6	38.37	0.6%
Db	105	h	6<	38.62	0.1%
Sg	106	m	6<	38.92	
Bh	107	m	6<	39.06	
Hs	108	m	6<	39.38	
Mt	109	s	6<	39.60	

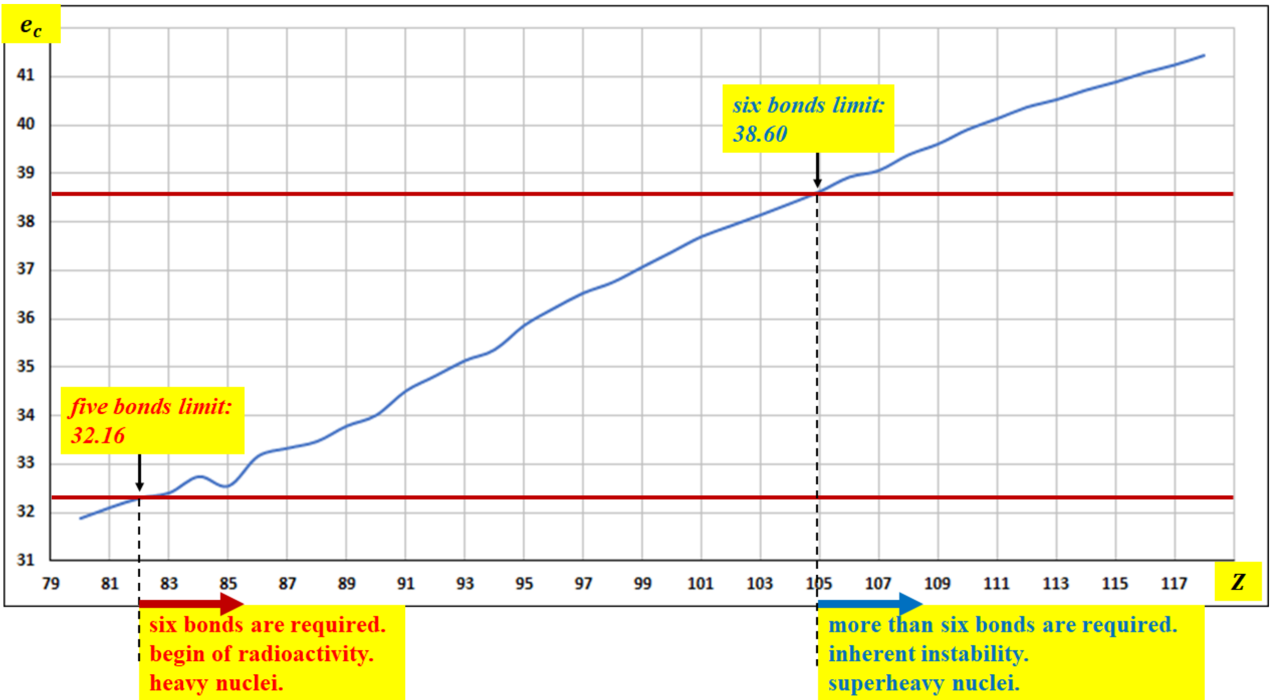
deviation from the limit value 38.60

begin of superheavy nuclei

Table 4: the transition from six to more than six bonds required for the stability of the central proton.

App. 2.2.5 Results: the number of bonds vs. the relative electric energy

The following graph illustrates the data from the above table.
The radioactivity is expected to begin around Lead (Hg_{80}) and the superheavy nuclei (those that are very unstable with short half-life) are expected to begin around Dubnium (Db_{105}).



Graph 3: the transition to heavy nuclei (six nuclear bonds) and superheavy nuclei (beyond six bonds)

App. 2.3 The nuclear fission

App. 2.3.1 The fission hypotheses

The following hypotheses are raised regarding the fission mechanism:

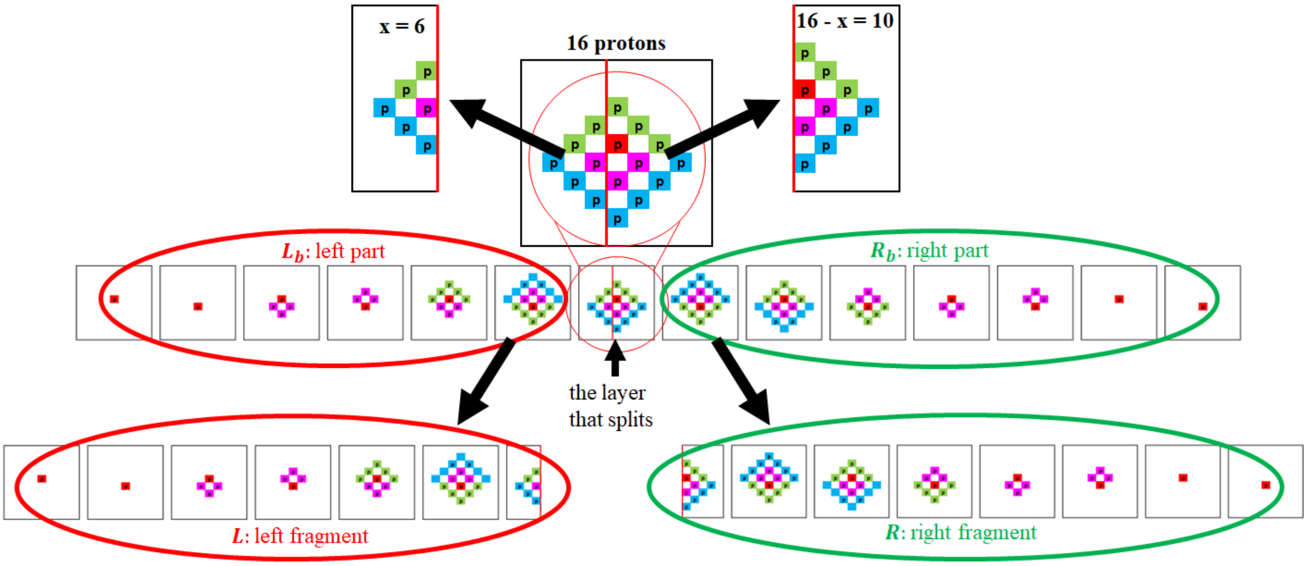
- A necessary but not sufficient condition for fission is that the nucleus is larger than Lead (Pb) and so has an unstable core.
- The splitting of the nucleus occurs in one of the two central (innermost) layers.
- The number of protons of each fission product (fragment) is the number of protons in the original nucleus from its outermost side till its fission point.
- For fissile material the number of neutrons must be a bit lower than the relatively more stable isotope of the nucleus; for example, for Uranium the more stable isotope is U_{92}^{238}, so the unstable isotope is smaller. The assumption here is that this lack of several neutrons enables some movement of the nucleons in the nucleus and so after radioactivity occurs in the center of the nucleus, a rearrangement of the inner parts enables the split and the creation of the fragments.

In the following sections the fission mechanism is described according to the model, the calculation of the size of the fragments is explained and examples are shown.

App. 2.3.2 The fission mechanism

The nuclear split occurs according to the model at one of the central nuclear layers (see illustration). For nuclei with an even number of protons it doesn't matter if the right or left center is selected as the one that splits, but for nuclei with odd number of protons, the two possibilities shall be considered separately, although this semantics is not crucial at this stage.

Figure 8: The fission and the definition of the fragments.



Definitions (see illustration):

- P : number of protons of the nucleus that undergoes fission.
- R_b: the number of protons of the right part of the nucleus till its center.
- L_b: as R_b, for the left part without its most inner layer (of 16 protons).
- x: the number of protons (out of 16) from the left side of the layer that splits.
- R: the number of protons of the right fragment.
- L: the number of protons of the left fragment.

calculate their values as follows:

- $P := \begin{cases} 2m+1 & P\ odd \\ 2m & P\ even \end{cases}$ with m integer
- $R_b := \frac{P}{2} = m$ (integer division)
- $L_b := \frac{P}{2} - 16 + remainder\left(\frac{P}{2}\right) = \begin{cases} m-16+1 & P\ odd \\ m-16 & P\ even \end{cases}$
- $R := R_b + 16 - x$
- $L := L_b + x$

and get that the sum of the fragments is equal, as required, to the total number of protons P:

- $R + L = (m + 16 - x) + \begin{cases} m-16+1+x \\ m-16+x \end{cases} = \begin{cases} 2m+1 & P\ odd \\ 2m & P\ even \end{cases} = P$

With $x \in [6, 16-6] = [6,10]$ and the most probable fission product are obtained. The process could be expanded to $x \in [1,15]$ to get additional potential fission products.

App. 2.3.3 Table of fission products (fragments)

The following table shows the results of the above calculation for the nuclei from Thorium (Th_{90}) to Fermium (Fm_{100}) with the x values $x \in [6,10]$ (and in each subsequent column the $16 - x$ values).
What is meant here are the immediate product pairs that arise from the fission, and not the end products of the full fission process, which may arise from further decay steps.

Table 5: the expected fission fragments from Th_{90} *to* Fm_{100} *for* $x \in [6,10]$

Nucleus	$x = 6$	10	$x = 7$	9	$x = 8$	8	$x = 9$	7	$x = 10$	6
Th_{90}	Sb_{51}	Y_{39}	Te_{52}	Sr_{38}	I_{53}	Rb_{37}	Xe_{54}	Kr_{36}	Cs_{55}	Br_{35}
Pa_{91}	Sb_{51}	Zr_{40}	Te_{52}	Y_{39}	I_{53}	Sr_{38}	Xe_{54}	Rb_{37}	Cs_{55}	Kr_{36}
U_{92}	Te_{52}	Zr_{40}	I_{53}	Y_{39}	Xe_{54}	Sr_{38}	Cs_{55}	Rb_{37}	Ba_{56}	Kr_{36}
Np_{93}	Te_{52}	Nb_{41}	I_{53}	Zr_{40}	Xe_{54}	Y_{39}	Cs_{55}	Sr_{38}	Ba_{56}	Rb_{37}
Pu_{94}	I_{53}	Nb_{41}	Xe_{54}	Zr_{40}	Cs_{55}	Y_{39}	Ba_{56}	Sr_{38}	La_{57}	Rb_{37}
Am_{95}	I_{53}	Mo_{42}	Xe_{54}	Nb_{41}	Cs_{55}	Zr_{40}	Ba_{56}	Y_{39}	La_{57}	Sr_{38}
Cm_{96}	Xe_{54}	Mo_{42}	Cs_{55}	Nb_{41}	Ba_{56}	Zr_{40}	La_{57}	Y_{39}	Ce_{58}	Sr_{38}
Bk_{97}	Xe_{54}	Tc_{43}	Cs_{55}	Mo_{42}	Ba_{56}	Nb_{41}	La_{57}	Zr_{40}	Ce_{58}	Y_{39}
Cf_{98}	Cs_{55}	Tc_{43}	Ba_{56}	Mo_{42}	La_{57}	Nb_{41}	Ce_{58}	Zr_{40}	Pr_{59}	Y_{39}
Es_{99}	Cs_{55}	Ru_{44}	Ba_{56}	Tc_{43}	La_{57}	Mo_{42}	Ce_{58}	Nb_{41}	Pr_{59}	Zr_{40}
Fm_{100}	Ba_{56}	Ru_{44}	La_{57}	Tc_{43}	Ce_{58}	Mo_{42}	Pr_{59}	Nb_{41}	Nd_{60}	Zr_{40}

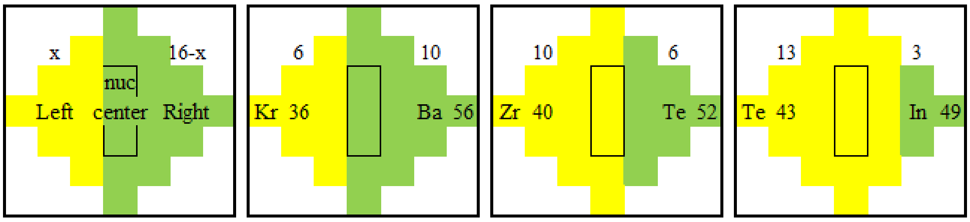
These results show the main fragments [13]; in order to get additional fragments x could be taken from a wider range (e.g. $x \in [3,13]$ or even $x \in [1,15]$).

App. 2.3.4 Geometric assessment of the most probable fission products

Taking a different approach, the expected products according to the geometry of the central layer are assessed. At this example the central layer of Uranium is referred to, but the idea is general also for other nuclei.
First some rules are set:

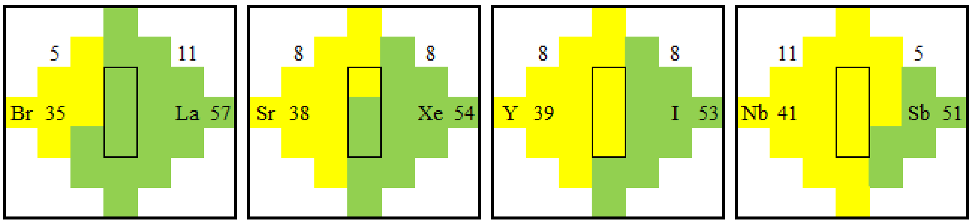
- The splitting will occur around the nuclear center. This means that the situation of $x \leq 6$ is less probable.
- One group of more probable splits includes those with a straight split line, meaning $x = 6, 10, 13$.
  This refers to the product couples:
  - $Kr_{36} - Ba_{56}$
  - $Zr_{40} - Te_{52}$
  - $Te_{43} - In_{49}$

Figure 9: nuclear fission. The split of the central layer.



The central layer of Uranium with a straight split. left part (yellow) and right part (green)

- The other group of more probable splits includes those that have no straight split line, but are located around the nuclear center.
  This refers to the product couples:
  - $Br_{35} - La_{57}$
  - $Sr_{38} - Xe_{54}$
  - $Y_{39} - I_{53}$
  - $Nb_{41} - Sb_{51}$



The central layer of Uranium with a split near its center.

Remark:

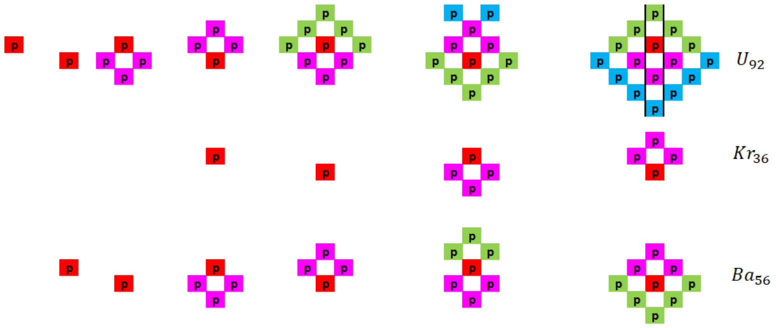
- This is only a rough estimation.
- This refers to the products that cause fission, not the potential end products after the immediate products have gone through radioactive decay stages.

App. 2.3.5 Fission products example

First, only the protons involved in the process are observed; Uranium products with a higher probability of appearing are selected [12] and are analyzed first according to the number of protons, assuming that the fission will occur in one of the two central layers, as predicted by the model. The area of the split in one of the center layers is marked with two lines (for clarity, only the upper half of the nucleus is shown).

- $U_{92} \Rightarrow Kr_{36} + Ba_{56}$

Figure 10: example to the Uranium fission; only protons

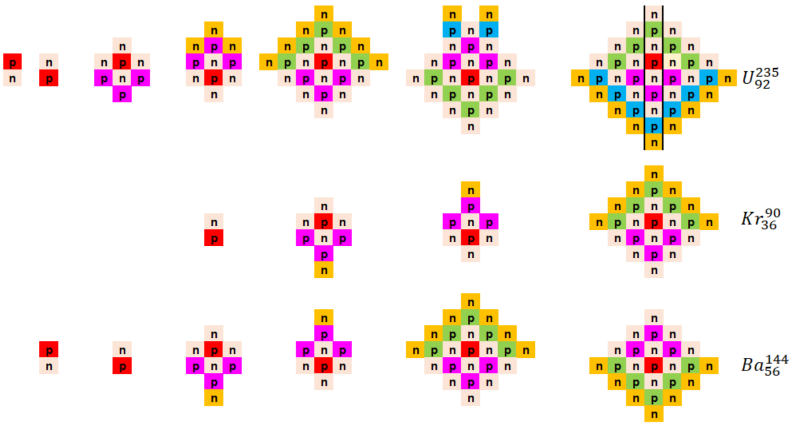


p: *protons according to the orbitals* ***S***, ***P***, ***D***, ***F***

At a next stage the above nuclei are considered as a whole [23]. The area of the split in one of the center layers is marked with two lines. The number of neutrons in the fission zone corresponds to the number of neutrons in the fission products, even though they were taken according to their location in the fission parts, determined in the previous step, where only protons were considered. That is, without adjusting the data, a correct number of neutrons was obtained in each product.
This is possibly another reinforcement for the model and the fission hypothesis.

$$U_{92}^{235} + n_0^1 \Rightarrow Kr_{36}^{90} + Ba_{56}^{144} + 2 \cdot n_0^1$$

Figure 11: example to the Uranium fission; including neutrons.



n: *neutrons (excess neutrons have a* ***background****)*

Remark: the observed nuclei don't have full sub-orbitals, so the precise arrangement of the excess neutrons is not known; yet because the two central layers are fully occupied and the nucleus is symmetric, it can be concluded how many neutrons will be in each fission product.

App. 2.3.6 The nuclear fission energy balance

Our hypothesis here is that the fission process occurs due to instability for a short while in the center of the nucleus.
The energy can be estimated by the number of bonds opened during the split.
The central layer contains 32 nucleons, that are connected to the 32 nucleons of the other central layer. Therefore, it is expected that the number of bonds opened during the fission will be n ≈ 32 bonds and so the fission energy is expected to be:

- $\Delta E \approx 32 \cdot 5.72 \text{ MeV} \approx 183 \text{ MeV}$.

To calculate the energy that causes the process, the energy balance is observed first, assuming the process involves only two product nuclei immediate as the split occurs.
With:

- m_{init}: the mass of the initial nucleus.
- m_{prod1}: the mass of the one product nucleus.
- m_{prod2}: the mass of the other product nucleus.
- ΔE: the energy difference in the process.

the energy is calculated to:

- $\Delta E = \{m_{init} - (m_{prod1} + m_{prod2})\} \cdot c^2$ (8)

Next this energy is estimated; the illustration below shows the upper half of U_{92}^{235}. To determine the expected products the following steps are taken:

- through the process described above the product elements are predicted.
- the neutrons of each product are added according to the fragmentation region.
- one neutron is subtracted from one of the products, so that the number of protons and neutrons at each product is either even or odd.
- one free neutron is added to the process to compensate the one subtracted above.
- the resulted energy is: $\Delta E = \{m_{init} - (m_{prod1} + m_{prod2} + m_{neutron})\} \cdot c^2$ (9)

Calculating this for Uranium U_{92}^{235} through the seven splits described above delivers the energy:

- $\Delta E \approx 184 \pm 6\ MeV$ (for one σ), which is equivalent to about 32 ± 1 bonds.

Remark: this is only a very rough estimate, yet it might be another reinforcement to the model.

App. 3 Astrophysics

App. 3.1 The hypotheses of constant tangential velocity and minimum atomic size

Two hypotheses are raised to support the following research steps.
The constant tangential velocity of bound electrons and nucleons
Hypothesis: the tangential velocity of nucleons in a nucleus is constant; this might be the case also for bound electrons in an atom.
To calculate the tangential velocity, the nuclear rotation is analyzed via its angular momentum:

- $L \approx \hbar \cdot l \approx p \cdot r = m \cdot v \cdot r = m \cdot (\omega \cdot r) \cdot r = m \cdot \omega \cdot r^2$ (10)

The model assumes that the orbital radius of the nucleus grows by a constant value, r_0, while moving from one orbital to its next neighbor:

- $r = l \cdot r_0$ with l: $\{0, 1, 2, 3\}$ for the orbitals L: $\{S, P, D, F\}$ (11)

and so:

- $L \approx \hbar \cdot l \approx m \cdot (\omega \cdot l \cdot r_0) \cdot l \cdot r_0$ (12)

this means for the tangential velocity:

- $v = \omega \cdot l \cdot r_0 = constant$ (13)

Note: from the nuclear shape according to the model, the radius is not equal for all protons of the same sub-orbital, but this is used as a rough assessment.
From the definitions of: $v_0 = \omega_0 \cdot r_0$, $\omega_0 = \frac{\hbar}{m \cdot r_0^2}$ and $\omega = \frac{\omega_0}{l}$ and using:

- $r_0 = d_0 = 1.62 \cdot 10^{-15} m$: the distance between neighboring nucleons.
- $m_p = 1.67 \cdot 10^{-27} kg$: the nucleon mass (For a rough estimate, the proton and neutron masses are considered to be the same).

the tangential velocity is calculated:

- $v_0 = \omega_0 \cdot d_0 \approx \frac{\hbar}{m_p \cdot d_0} = \frac{1.05 \cdot 10^{-34}}{1.67 \cdot 10^{-27} \cdot 1.62 \cdot 10^{-15}} \approx 3.8 \cdot 10^7 \ \frac{m}{s}$

The minimum volume of a bound electron
Hypothesis: the minimum volume occupied by a bound electron defines a lower limit for the atomic volume; or the same statement in the opposite direction: the minimum size of an atom is determined by the minimum volume occupied by a bound electron.
It is calculated by comparing the angular momentum of the electron and proton assuming a constant tangential velocity of the bound particles:

- $L_e \approx \hbar \approx L_p \ \rightarrow \ m_e \cdot r_e \approx m_p \cdot r_p \ \rightarrow \ r_e \approx \frac{m_p \cdot r_p}{m_e} \approx 1.5 \cdot 10^{-12} \ m$ (14)

which is in the range of its de-Broglie wavelength.
The meaning of this bound electron is assumed to be a minimum atomic size, before the proton and electron are fused to become a neutron; it is assumed that this is the case in the limit between white dwarfs and neutron stars, as discussed below.

App. 3.2 Neutron star and the TOV limit

To estimate the size and mass of a neutron star using an extension of the nuclear model, it is treated as if it is a large nucleus.
A basic particle in the star is assumed to have the mass of a nucleon m_p and a basic cell size is derived from the distance between two neighboring nucleons in the star:
$d = 2 \cdot r = d_0 \approx 1.62 \cdot 10^{-15}\ m.$
A cubic bond is expected, so the volume of the basic cell and its density are:

- $V_{cell} = d^3 \approx 4.3 \cdot 10^{-45}\ m^3$ cubic volume, unlike $V_{cell} = \frac{4 \cdot \pi \cdot r^3}{3}$.
- $\rho = \frac{m_p}{V_{cell}} \approx 3.9 \cdot 10^{17}\ \frac{kg}{m^3}$ this is within expected range [14].

The gravitational pressure in the center of the star is $P = \frac{2 \cdot \pi \cdot G \cdot \rho^2 \cdot R^2}{3}$ [17] with R the star radius and the force on the central nucleon in the star is about $F = P \cdot S$, with $S = 4 \cdot \pi \cdot r^2$ the surface of the nucleon.
This leads to:

- $F = \frac{8 \cdot \pi^2 \cdot G \cdot \rho^2 \cdot R^2 \cdot r^2}{3}$ the force on the cell in the center of the star. (15)
- $R = \sqrt{\frac{3 \cdot F}{8 \cdot \pi^2 \cdot G \cdot \rho^2 \cdot r^2}}$ the star radius. (16)
- $M = \rho \cdot V = \frac{\rho \cdot 4 \cdot \pi \cdot R^3}{3}$ the star mass. . (17)

Based on data from [5] it is assumed that the maximum force a nucleon can bear is about:

- $F_{max} \approx [3.0, 4.0] \cdot 10^4\ N$ maximum tolerable force in the star center.

this means:

- $R \approx [1.3, 1.5] \cdot 10^4\ m$, $M \approx [3.6, 5.6] \cdot 10^{30}\ kg$

and with $M_{sun} = 2 \cdot 10^{30}\ kg$ the mass ratio is: $\frac{M}{M_{sun}} \approx [1.8, 2.8]$

which delivers a rough estimation to the maximum star mass, before it collapses to become a black hole; this is in the range of the Tolman-Oppenheimer-Volkoff limit [16].
To summarize the process:

- from the assumption of constant cubic arrangement, the density ρ is obtained.
- assuming the maximum force a nucleon can withstand is F_{max} the star radius R is obtained through the gravitational pressure P at the center of the star.
- via R the star volume V is calculated.
- the star mass M is calculated using the star volume and density.

Remark: the main assumptions here are that the neutron star is a kind of large nucleus with a constant density and a rough estimate of the maximum force a nucleon can bear.

App. 3.3 White dwarf and the lower limit of atomic size

The discussion of white dwarfs is done in a somewhat similar way to that of neutron stars above.
Hypothesis: there is a maximum pressure that an atom inside a white dwarf can withstand, beyond that it collapses, and its electron and proton fuse to form a neutron.
Using the constant tangential velocity assumption, the minimum cell radius of an atom is estimated to be in the range of $r_e \approx r_p \cdot \frac{m_p}{m_e} \approx 1.5 \cdot 10^{-12}\ m$.
Another assumption is that the star was created by light elements, so each basic cell consists of one electron, one proton and one neutron [18] meaning the atomic mass is about $A = 2$.
The star consists of atoms, so unlike the cubic arrangement in the neutron star, here the basic cell volume is assumed to be a sphere and not a cube.
Assuming a typical white dwarf radius and mass:

- $R \approx 1 \cdot 10^7\ m$ [18]
- $M \approx 0.5 \cdot M_{sun} = 1.0 \cdot 10^{30}\ kg$ [18]

it follows:

- $V = \frac{4 \cdot \pi \cdot R^3}{3} \approx 4.2 \cdot 10^{21}\ m^3$ white dwarf volume.
- $\rho = \frac{M}{V} \approx 2.4 \cdot 10^8\ \frac{kg}{m^3}$ white dwarf density.
- $V_{cell} = \frac{m}{\rho} = \frac{A \cdot m_p}{\rho} \approx 1.4 \cdot 10^{-35}\ m^3$ white dwarf basic cell volume.
- $r_{cell} = \sqrt[3]{\frac{3 \cdot V_{cell}}{4 \cdot \pi}} \approx 1.5 \cdot 10^{-12}\ m$ white dwarf basic cell radius.

The result for r_{cell} is in the range of r_e ; this strengthens the hypothesis, that there is a minimum atom size, beyond which the electron and proton are fused to form a neutron.

App. 3.4 The electron size and its spin as a rotation

Another issue raised from the last discussions is the spin of the electron. If it is treated as a rotation, then:

- $v = \omega \cdot r = \frac{S}{I} \cdot r \approx \left(\frac{\hbar}{2}\right) \cdot \frac{r}{m \cdot r^2} = \frac{\hbar \cdot r}{2 \cdot m \cdot r^2} = \frac{\hbar}{2 \cdot m \cdot r}$

The common value for the electron size is about $r_e \approx 3 \cdot 10^{-15}\ m$, leading to a contradiction:

- $v \approx 2 \cdot 10^{10}\ \frac{m}{s} \approx 100 \cdot c$ with a tangential velocity larger than the speed of light.

Assuming $r_e \approx 1.5 \cdot 10^{-12}\ m$, as done above, it follows:

- $v \approx 3 \cdot 10^{7}\ \frac{m}{s}$

This way the electron spin can be interpreted as a classical rotation.

App. 3.4 Pulsar - the lower limit of the rotation period

To analyze pulsars, it is assumed also here that a neutron star acts somewhat as a giant nucleus and as such maintains some of the nuclear properties; this shall be discussed now in the light of the model.
The elements formed in a star before undergoing supernova and transforming into a neutron star, are assumed to be mainly up to the fourth row of the periodic table.

The lower limit of the rotation period

In order to calculate the pulsar angular velocity, the assumption made above regarding the constant tangential velocity is used, in the ideal situation in which all nuclei lie parallel to each other and so their superposition leads to a maximal angular and tangential velocity.
Data of a pulsar:

- The minimum radius: $R \approx 1.5 \cdot 10^4\ m$ [21]
- the minimum rotation period: $T \approx 10^{-3}\ s$ [15]

the tangential velocity found above is:

- $v = \omega \cdot R \approx 3.8 \cdot 10^7\ \frac{m}{s}$

and a rough estimation for the angular velocity is calculated as:

- $\omega = \frac{v}{R} \approx \frac{3.8 \cdot 10^7}{1.5 \cdot 10^4} = 2.5 \cdot 10^3\ s^{-1}$

this implies a lower limit to the rotation period of pulsars:

- $T = \frac{2 \cdot \pi}{\omega} \approx 2.5\ ms$

which provides a rough estimate for the currently known data of about $T \approx 1.4\ ms$ [20].

The longer periods, or shorter angular velocities, are assumed to be due to a less parallel arrangement of the nuclei (or a smaller relative part with parallel nuclei) or of older pulsars that slowed down with time.
Remark: the constant tangential velocity hypothesis delivers only a rough estimate of the real velocity. If there was a factor of 2 in the calculation, then $v \approx 7.5 \cdot 10^7\ \frac{m}{s}$ and the results would be different, but the idea remains unchanged.

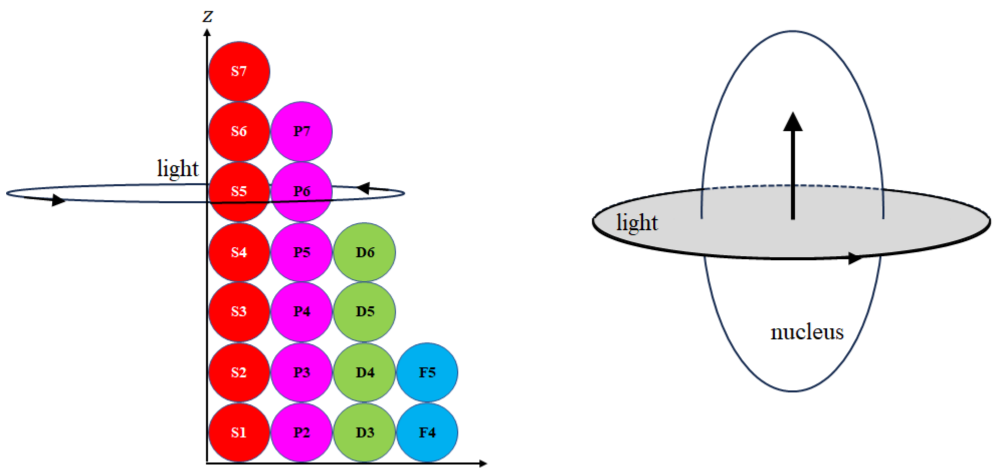
App. 4 Atomic physics

Note: the topic of this section (5.4.1) is at an early development stage and is brought up only to provide a rough idea or direction of how atomic phenomena can be explained by the cubic ellipsoid nuclear model.

App. 4.1 Electronic transition selection rules

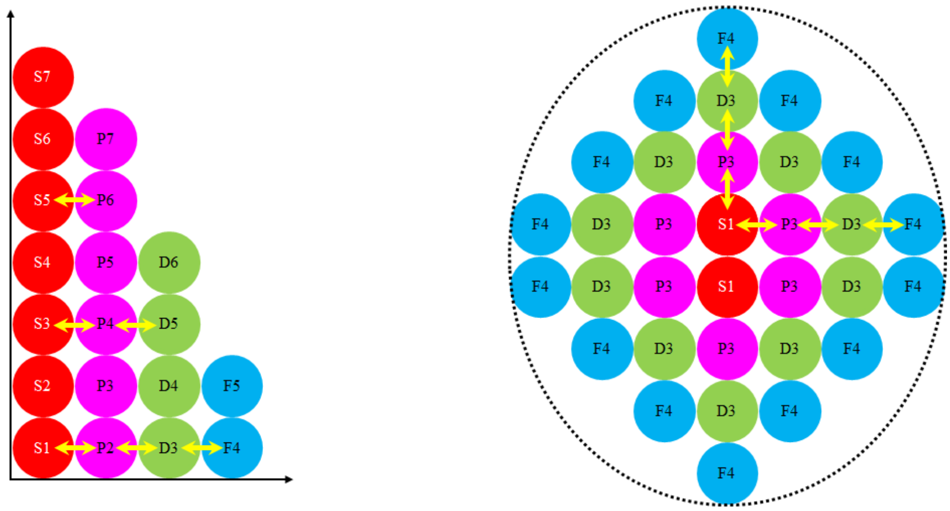
It is assumed that the atomic structure is somewhat similar to the nuclear geometry. According to the cubic nuclear model, the ionization energy depends on the number of electrons in the sub-orbital, so it is assumed that the transition of the electrons occurs by some completion of an orbital through the absorbed light, simulating a transition to the next orbital in the same plane, which is the x-y plane.

Figure 16: light absorption by the atom.



side view: light absorption occurs in the x-y-plane (perpendicular to the z-axis)

Figure 17: electronic transitions occur in the x-y plane as a result of light absorption.



side view: $\Delta n = \pm 1,\ \Delta L = \pm 1$
cross section in the x-z plane

top view: $\Delta n = \pm 1,\ \Delta L = \pm 1$
cross section in the x-y plane

From the above illustrations the following is taken:

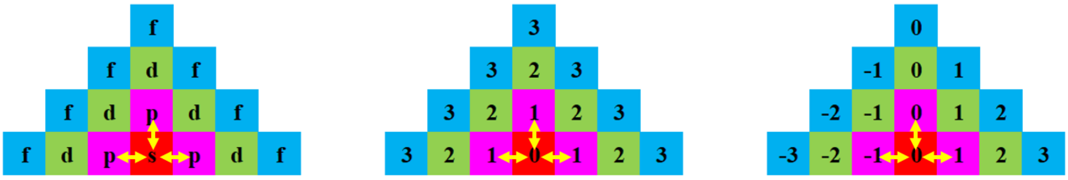
- Δn: each orbital in the x-y-plane has a principal quantum number larger than its predecessor.

- ΔL: moving toward the atomic envelope the angular momentum increases.
- Δm: the magnetic quantum number can be read through the symmetry of the model.

The following illustrations depict this through half an x-y layer to explain the rule for the magnetic quantum number as well.
The resulted selection rules are $\Delta n = \pm 1$, $\Delta L = \pm 1$, $\Delta m = 0, \pm 1$, as expected from the theory [24].

Figure 18: electronic transition rules.

orbitals *L: angular momentum* *m: magnetic quantum number*



$\Delta n = \pm 1$ $\Delta L = \pm 1$ $\Delta m = 0, \pm 1$

The conclusion is therefore that the selection rules are a simple consequence of the atomic structure and are essentially an inherent part of the model.