Four-Dimensional Newtonian Relativity

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Abstract

We construct an alternative formulation to the theory of special relativity from the concepts of absolute time and absolute space defined by Newton and from the hypothesis that physical space is four-dimensional. We prove this formulation is mathematically equivalent to the theory of special relativity by deriving the Lorentz transformation from the Galilean transformation for frames of reference in four-dimensional Euclidean space.

I. Introduction

According to Newton, time and space are absolute [1]. That means time and space exist independently from physical events and from each other. Furthermore, Newton argued that an object is at absolute rest if it is stationary with respect to absolute space or in absolute motion if it is moving with respect to absolute space [2]. For this reason, he contended that absolute space is a privileged frame of reference [3]. If Newton is correct, then the Galilean transformation is the set of equations that relate the time and space coordinates of two systems moving at a constant velocity relative to each other [4]. In this article, we will use these concepts and the hypothesis that physical space is four-dimensional to construct an alternative formulation to the theory of special relativity. We will prove this formulation is mathematically valid by deriving the Lorentz transformation from the Galilean transformation for frames of reference in four-dimensional Euclidean space.

II. Postulates

The alternative formulation to the theory of special relativity that we propose is based on the following postulates:

- Time and space are absolute.
- Space is four-dimensional.
- All objects move at the speed of light with respect to absolute space.

The first postulate refers to the same concepts defined by Newton in 1687. The second postulate states that physical space is a four-dimensional Euclidean space. That is our fundamental hypothesis. The third postulate posits that objects are never at rest with respect to absolute space and move only at one speed with respect to it: the speed of light. That proposition is similar to the one obtained from the theory of relativity, which asserts that all objects move through spacetime at the speed of light. These three postulates differ from Nordström's electromagnetic-gravitational theory and the Kaluza-Klein theory in that time and space are not absolute and space is not Euclidean in those formulations [5–9].

In addition to suggesting postulates about the nature of time and space, we need to take into account that the fundamental theories of modern physics presuppose that space only has three dimensions. This remark can be stated as follows:

- Modern physics assumes space is three-dimensional, but if space is actually four-dimensional, then that erroneous assumption would have affected the interpretation of experimental results and the formulation of fundamental theories.

We shall refer to this statement as the observer's principle (because of the role visual perception plays). It points out that the wrong assumption about the dimensionality of space would have affected the mathematical formulation and interpretation of quantum mechanics and relativity.

The mathematical formulation of the postulates we propose is the following:

- The first postulate allows us to use the Galilean transformation to relate the coordinates of two systems that move at a constant velocity relative to each other.
- The second postulate implies that our frames of reference must have four spatial coordinates.
- The third postulate tells us that the speed between any inertial frame of reference and absolute space must be equal to the speed of light.

In the context of the Lorentz transformation, the observer's principle can be restated as follows: Physicists who assume space only has three dimensions will

- implicitly assign a value of zero to the fourth spatial coordinates of any event,
- presume that the velocity projected onto the three-dimensional space they visually perceive is, in fact, the velocity between the inertial frames of reference, and
- conclude that only three coordinates are needed to specify the position of an event.

For the rest of the analysis in this paper, we assume that the postulates presented in this section are true.

III. Derivation

We will be using four rectangular coordinate systems to derive the Lorentz transformation (namely S, A, A′ and S′). Each system contains four coordinates that specify the position of a physical event in four-dimensional Euclidean space and a time coordinate that specifies the instant in which that event takes place. The coordinates of an event E for each system are:

- (x_1, x_2, x_3, x_4, t) according to S
- (X_1, X_2, X_3, X_4, T) according to A
- $(X'_1, X'_2, X'_3, X'_4, T')$ according to A′
- $(x'_1, x'_2, x'_3, x'_4, t')$ according to S′

We are assuming time is absolute. Therefore, we have that

$$t = T = T' = t' \tag{1}$$

The configuration we will be considering is the following: The origins of S, A, A′ and S′ coincide at $t = T = T' = t' = 0$. The four coordinate systems only move on the plane that contains the axes of the first and fourth dimensions, such that

$$x_2 = X_2 = X'_2 = x'_2 \tag{2}$$

$$x_3 = X_3 = X'_3 = x'_3 \tag{3}$$

The coordinate systems A and A′ are fixed with respect to absolute space, and their axes are rotated according to

$$X'_1 = X_1 \cos\theta - X_4 \sin\theta \tag{4}$$

$$X'_4 = X_1 \sin\theta + X_4 \cos\theta \tag{5}$$

$$-90^\circ \leq \theta \leq 90^\circ$$

where θ is the angle of rotation. If we solve for X_1 and X_4 in equations 4 and 5, we get

$$X_1 = X'_1 \cos\theta + X'_4 \sin\theta \tag{6}$$

$$X_4 = -X'_1 \sin\theta + X'_4 \cos\theta \tag{7}$$

The coordinate system S represents an inertial frame of reference. It moves along the common axis X_4–x_4. According to our postulates, inertial frames of reference move at the speed of light with respect to absolute space. Thus, the Galilean transformation equations are

$$x_1 = X_1 \tag{8}$$

$$x_4 = X_4 - ct \tag{9}$$

where c is the speed of light. Similarly, the coordinate system S′, which also represents an inertial frame of reference, moves at the speed of light along the common axis X'_4–x'_4. Consequently, the Galilean transformation equations for this case are

$$x'_1 = X'_1 \tag{10}$$

$$x'_4 = X'_4 - ct' \tag{11}$$

The velocity of the frame of reference S′ projected onto the three-dimensional subspace formed by the x_1–x_2–x_3 axes is given by

$$v_1 = c\,sin\,\theta \tag{12}$$

where v_1 is the component of the velocity of S′ along the X_1 and x_1 axes.

The observer's principle states that physicists who assumed space is three-dimensional would have implicitly assigned a value of zero to the fourth spatial coordinates of any event and presumed that the velocity projected onto the three-dimensional space they visually perceive is the actual velocity between the inertial frames of reference. Lorentz assumed space is three-dimensional when he formulated his transformation. Therefore, we have that

$$x_4 = 0 \tag{13}$$

$$x'_4 = 0 \tag{14}$$

$$v_1 = v \tag{15}$$

where v is the (erroneously supposed) velocity between the reference frames S and S′.

Now we are ready to derive the Lorentz transformation and its inverse transformation. First, we substitute eq. 15 into eq. 12 and solve for $sin\,\theta$:

$$sin\,\theta = \frac{v}{c} \tag{16}$$

Then we use the Pythagorean trigonometric identity to obtain the function of $cos\,\theta$, so that

$$cos\,\theta = \sqrt{1 - sin^2\,\theta} \tag{17}$$

Next, we substitute eq. 16 into eq. 17:

$$cos\,\theta = \sqrt{1 - \frac{v^2}{c^2}} \tag{18}$$

The Lorentz factor is a term that frequently appears in the equations of the theory of special relativity. It is given by

$$\gamma \equiv \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \tag{19}$$

Consequently, we have that

$$cos\,\theta = \frac{1}{\gamma} \tag{20}$$

The next step is to solve for the coordinates X_1, X_4, X'_1 and X'_4 in equations 8, 9, 10 and 11, respectively, and substitute them into equations 4, 5, 6 and 7:

$$x'_1 = x_1\,cos\,\theta - (x_4 + ct)\,sin\,\theta \tag{21}$$

$$(x'_4 + ct') = x_1\,sin\,\theta + (x_4 + ct)\,cos\,\theta \tag{22}$$

$$x_1 = x'_1\,cos\,\theta + (x'_4 + ct')\,sin\,\theta \tag{23}$$

$$(x_4 + ct) = -x'_1\,sin\,\theta + (x'_4 + ct')\,cos\,\theta \tag{24}$$

Equations 21, 2, 3, 22 and 1 give us the Galilean transformation for the case described in this section. The corresponding inverse Galilean transformation is given by equations 23, 2, 3, 24 and 1. The angle of rotation θ can be obtained from eq. 12. These transformations provide the complete relationship between the inertial frames of reference S and S′ when describing a single event occurring in four-dimensional Euclidean space.

Before proceeding with the final steps of the derivation, we need to use eq. 1 to substitute t' for t and t for t' in equations 21, 22, 23 and 24:

$$x'_1 = x_1\,cos\,\theta - (x_4 + ct')\,sin\,\theta \tag{25}$$

$$(x'_4 + ct) = x_1\,sin\,\theta + (x_4 + ct')\,cos\,\theta \tag{26}$$

$$x_1 = x'_1\,cos\,\theta + (x'_4 + ct)\,sin\,\theta \tag{27}$$

$$(x_4 + ct') = -x'_1\,sin\,\theta + (x'_4 + ct)\,cos\,\theta \tag{28}$$

These equations (together with equations 1, 2 and 3) also provide a valid and adequate description of the relationship between the inertial frames of reference S and S′.

The mathematical consequences of the observer's principle are represented by equations 13, 14, 16 and 20. For this reason, we substitute them into equations 25, 26, 27 and 28:

$$x'_1 = \frac{x_1}{\gamma} - vt' \tag{29}$$

$$ct = \frac{vx_1}{c} + \frac{ct'}{\gamma} \tag{30}$$

$$x_1 = \frac{x'_1}{\gamma} + vt \tag{31}$$

$$ct' = -\frac{vx'_1}{c} + \frac{ct}{\gamma} \tag{32}$$

The last step of the derivation is to solve for the coordinates x_1, t', x'_1 and t in equations 29, 30, 31 and 32, respectively. This gives us:

$$x_1 = \gamma(x'_1 + vt') \tag{33}$$

$$t' = \gamma\left(t - \frac{vx_1}{c^2}\right) \tag{34}$$

$$x'_1 = \gamma(x_1 - vt) \tag{35}$$

$$t = \gamma\left(t' + \frac{vx'_1}{c^2}\right) \tag{36}$$

Equations 35, 2, 3 and 34 form the Lorentz transformation for inertial frames of reference that move relative to each other at a constant speed v along their common axis x_1–x'_1 (also known as the Lorentz boost in the x_1 direction). That transformation is given by

$$x'_1 = \gamma(x_1 - vt) \tag{37}$$

$$x'_2 = x_2 \tag{38}$$

$$x'_3 = x_3 \tag{39}$$

$$t' = \gamma\left(t - \frac{vx_1}{c^2}\right) \tag{40}$$

Likewise, equations 33, 2, 3 and 36 form the corresponding inverse Lorentz transformation, which is

$$x_1 = \gamma(x'_1 + vt') \tag{41}$$

$$x_2 = x'_2 \tag{42}$$

$$x_3 = x'_3 \tag{43}$$

$$t = \gamma\left(t' + \frac{vx'_1}{c^2}\right) \tag{44}$$

Notice that these equations (37–44) contain only three coordinates that specify the position of an event (instead of four). This is due to the fact that Lorentz assumed space is three-dimensional when he formulated them, which is what the third mathematical consequence of the observer's principle predicted. This remark completes the derivation. The more general form of the Lorentz transformation can be obtained by extending the procedure presented in this article.

As a final note, we want to point out that the Galilean transformation derived in this section (given by equations 25, 2, 3, 26 and 1) and its corresponding inverse (equations 27, 2, 3, 28 and 1) describe a single event. However, when eq. 1 is discarded and the values of the fourth coordinates are set equal to zero (equations 13 and 14), then the resulting equations describe two events that occur at the same place but at different times. That would be the interpretation of this result from a mathematical perspective. From a physical perspective, this result tells us that the effects from the Lorentz transformation (such as time dilation, length contraction and the constancy of the speed of light) are actually depth perception effects that are being interpreted as real effects because the fourth spatial dimension is not being taken into account. Other interesting remarks can be made about this result, but we will address them more profoundly in a future paper.

IV. Conclusion

We used the concepts of absolute time and absolute space defined by Newton and the hypothesis that physical space is four-dimensional to construct an alternative formulation to the theory of special relativity. We proved this formulation is mathematically equivalent to the theory of special relativity by deriving the Lorentz transformation from the Galilean transformation for frames of reference in four-dimensional Euclidean space. From that result, we deduced that the effects predicted by the Lorentz transformation are actually depth perception effects being interpreted as real effects because modern physics is not taking the fourth spatial dimension into account. From all this, we can conclude that the alternative formulation to the theory of special relativity presented here could be seen as evidence in favor of the hypothesis that physical space is four-dimensional.

Dedication

This article is dedicated to the memory of my father, Dr. Lorenzo León Callender López, who always supported me and was there for me. Without him, this work would not have been possible.

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