

Four-Dimensional Newtonian Relativity

KENNARD CALLENDER

ORCID ID: 0000-0002-7303-4848

Santiago de Veraguas, República de Panamá

Abstract

An alternative formulation to the special theory of relativity was developed based on the concepts of absolute time and absolute space defined by Newton and on the hypothesis that physical space is four-dimensional. In order to prove this formulation is mathematically valid, the Lorentz transformation was derived from the Galilean transformation for frames of reference in four-dimensional euclidean space.

I. INTRODUCTION

ACCORDING to Newton, time and space are absolute [1]. This means that time and space exist independently from physical events and from each other. Furthermore, Newton argued that an object is either at absolute rest if it is stationary with respect to absolute space or in absolute motion if it is moving with respect to absolute space [2]. For this reason, he contended that absolute space is a privileged frame of reference [3]. If Newton is correct, then the Galilean transformation is the set of equations that accurately relate the time and space coordinates of two systems moving at a constant velocity relative to each other [4]. In this article we use these concepts, together with the hypothesis that space is four-dimensional, to develop an alternative formulation to the special theory of relativity. We prove this formulation is mathematically valid by deriving the Lorentz transformation from the Galilean transformation for frames of reference in four-dimensional euclidean space.

II. POSTULATES

THE alternative formulation of special relativity that we have developed is based on the following postulates:

- Time and space are absolute.
- Space is four-dimensional.
- All inertial frames of reference move at the speed of light with respect to absolute space.

The first postulate refers to the same concepts defined by Newton in 1687 [5]. The second postulate states that physical space is a four-dimensional euclidean space. This is our fundamental hypothesis. The third postulate posits that inertial frames of reference (which represent real physical objects) are never at rest with respect to absolute space and move only at one speed with respect to it, the speed of light. This proposition is similar to the result obtained from the theory of relativity which affirms that all objects move at the speed of light with respect to spacetime [6]. Lastly, these postulates differ from Nordstrom's theory and from the Kaluza-Klein theory in that time and space are not absolute and space is not euclidean in those formulations [7-11].

In addition to proposing these postulates, we need to take into account that the fundamental theories of modern physics have always assumed space is three-dimensional. This remark can be stated as follows:

- If space has four dimensions, but physicists have been assuming it has only three, then this would have had consequences that affect the theories they have formulated and the interpretation of the results from the experiments they have performed.

We shall refer to this statement as the observer's principle. It follows directly from the postulate that space is four-dimensional and from the fact that physicists (the observers) have been assuming space is three-dimensional based on their visual perception.

The mathematical formulation of the postulates we propose is the following:

- The first postulate allows us to use the Galilean transformation to relate the coordinates between frames of reference that move at a constant velocity relative to each other [12].
- The second postulate implies that the frames of reference we use must have four spatial coordinates.
- The third postulate tells us that the speed between any inertial frame of reference and absolute space must be equal to the speed of light.

The consequences that the observer's principle refers to can be represented mathematically. Specifically, we have that if a physicist assumes space has only three dimensions, he will

- implicitly assign a value of zero to the fourth spatial coordinates of any event,
- think that the velocity projected unto the three-dimensional space he visually perceives is in fact the velocity between the inertial frames of reference, and
- conclude that only three coordinates are needed to specify the position of an event.

For the rest of the analysis in this paper, we will be assuming that the postulates presented in this section are true.

III. DERIVATION

IN order to derive the Lorentz transformation, we are going to be using four rectangular coordinate systems: S, A, A' and S'. Each system contains four coordinates used to specify the position of a physical event in four-dimensional euclidean space and a time coordinate used to specify the instant in which that event takes place. The coordinates of an event E for each system are:

- (x_1, x_2, x_3, x_4, t) according to S
- (X_1, X_2, X_3, X_4, T) according to A
- $(X'_1, X'_2, X'_3, X'_4, T')$ according to A'
- $(x'_1, x'_2, x'_3, x'_4, t')$ according to S'

The instant in which an event occurs does not depend on the frame of reference it is measured from (time is absolute). This means that

$$t = T = T' = t' \quad (1)$$

We will be considering the case where the movement of these coordinate systems is restricted to the plane containing the axes from the first and fourth dimensions, such that

$$x_2 = X_2 = X'_2 = x'_2 \quad (2)$$

$$x_3 = X_3 = X'_3 = x'_3 \quad (3)$$

The coordinate systems A and A' are fixed with respect to absolute space, their origins coincide and their axes are rotated according to

$$X'_1 = X_1 \cos \theta - X_4 \sin \theta \quad (4)$$

$$X'_4 = X_1 \sin \theta + X_4 \cos \theta \quad (5)$$

$$-90^\circ \leq \theta \leq 90^\circ$$

where θ is the angle of rotation. If we solve for the coordinates X_1 and X_4 in equations 4 and 5, we get

$$X_1 = X'_1 \cos \theta + X'_4 \sin \theta \quad (6)$$

$$X_4 = -X'_1 \sin \theta + X'_4 \cos \theta \quad (7)$$

The coordinate system S represents an inertial frame of reference. It moves along the common axis X_4-x_4 . According to our postulates, inertial frames of reference move at the speed of light with respect to absolute space. Consequently, the Galilean transformation equations for this case are

$$x_1 = X_1 \quad (8)$$

$$x_4 = X_4 - ct \quad (9)$$

where c is the speed of light. Similarly, the coordinate system S' (which also represents an inertial frame of reference) moves at the speed of light along the common axis $X'_4-x'_4$. Thus, the Galilean transformation equations are

$$x'_1 = X'_1 \quad (10)$$

$$x'_4 = X'_4 - ct' \quad (11)$$

The velocity of the frame of reference S' projected unto the three-dimensional subspace formed by the x_1 - x_2 - x_3 axes is given by

$$v_1 = c \sin \theta \quad (12)$$

where v_1 is the component of the velocity of S' along the X_1 and x_1 axes.

The observer's principle states that, if a physicist assumes space is three-dimensional, he will implicitly assign a value of zero to the fourth spatial coordinates of an event and think that the velocity projected unto the three-dimensional space he visually perceives is in fact the velocity between the inertial frames of reference. Hendrik Lorentz assumed space is three-dimensional, therefore we have that

$$x_4 = 0 \quad (13)$$

$$x'_4 = 0 \quad (14)$$

and

$$v_1 = v \quad (15)$$

where v is the (erroneously supposed) velocity between the reference frames S and S' .

Now we are ready to derive the Lorentz transformation and its inverse transformation. First we substitute eq. 15 into eq. 12 and solve for $\sin \theta$:

$$\sin \theta = \frac{v}{c} \quad (16)$$

Then we use the Pythagorean trigonometric identity to obtain the function of $\cos \theta$, so that

$$\cos \theta = \sqrt{1 - \sin^2 \theta} \quad (17)$$

Next we substitute eq. 16 into eq. 17:

$$\cos \theta = \sqrt{1 - \frac{v^2}{c^2}} \quad (18)$$

The Lorentz factor is a term that frequently appears in the equations of the special theory of relativity. It is given by

$$\gamma \equiv \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \quad (19)$$

Therefore we have that

$$\cos \theta = \frac{1}{\gamma} \quad (20)$$

The next step is to solve for the coordinates X_1 , X_4 , X'_1 and X'_4 in equations 8, 9, 10 and 11 respectively, and substitute them into equations 4, 5, 6 and 7:

$$x'_1 = x_1 \cos \theta - (x_4 + ct) \sin \theta \quad (21)$$

$$(x'_4 + ct') = x_1 \sin \theta + (x_4 + ct) \cos \theta \quad (22)$$

$$x_1 = x'_1 \cos \theta + (x'_4 + ct') \sin \theta \quad (23)$$

$$(x_4 + ct) = -x'_1 \sin \theta + (x'_4 + ct') \cos \theta \quad (24)$$

Equations 21, 2, 3, 22 and 1 give us the Galilean transformation for the case described in this section. The corresponding inverse Galilean transformation is given by equations 23, 2, 3, 24 and 1. The angle of rotation θ can be obtained from eq. 12. These transformations provide the complete relationship between the inertial frames of reference S and S' when describing a single event occurring in four-dimensional euclidean space.

Before proceeding with the final steps of the derivation, we need to use eq. 1 to substitute t' for t and t for t' in equations 21, 22, 23 and 24:

$$x'_1 = x_1 \cos \theta - (x_4 + ct') \sin \theta \quad (25)$$

$$(x'_4 + ct) = x_1 \sin \theta + (x_4 + ct') \cos \theta \quad (26)$$

$$x_1 = x'_1 \cos \theta + (x'_4 + ct) \sin \theta \quad (27)$$

$$(x_4 + ct') = -x'_1 \sin \theta + (x'_4 + ct) \cos \theta \quad (28)$$

These equations (together with equations 1, 2 and 3) also provide a valid and adequate description of the relationship between the inertial frames of reference S and S' .

The mathematical consequences of the observer's principle are represented by equations 13, 14, 16 and 20. For this reason we substitute them into equations 25, 26, 27 and 28:

$$x'_1 = \frac{x_1}{\gamma} - vt' \quad (29)$$

$$ct = \frac{vx_1}{c} + \frac{ct'}{\gamma} \quad (30)$$

$$x_1 = \frac{x'_1}{\gamma} + vt \quad (31)$$

$$ct' = -\frac{vx'_1}{c} + \frac{ct}{\gamma} \quad (32)$$

The last step of the derivation is to solve for the coordinates x_1 , t' , x'_1 and t in equations 29, 30, 31 and 32 respectively:

$$x_1 = \gamma(x'_1 + vt') \quad (33)$$

$$t' = \gamma\left(t - \frac{vx_1}{c^2}\right) \quad (34)$$

$$x'_1 = \gamma(x_1 - vt) \quad (35)$$

$$t = \gamma\left(t' + \frac{vx'_1}{c^2}\right) \quad (36)$$

Equations 35, 2, 3 and 34 form the Lorentz transformation for inertial frames of reference that move relative to each other at a constant speed v along their common axis x_1 - x'_1 (also known as the Lorentz boost in the x_1 direction). That transformation is given by

$$x'_1 = \gamma(x_1 - vt) \quad (37)$$

$$x'_2 = x_2 \quad (38)$$

$$x'_3 = x_3 \quad (39)$$

$$t' = \gamma\left(t - \frac{vx_1}{c^2}\right) \quad (40)$$

Likewise, equations 33, 2, 3 and 36 form the corresponding inverse Lorentz transformation, which is

$$x_1 = \gamma(x'_1 + vt') \quad (41)$$

$$x_2 = x'_2 \quad (42)$$

$$x_3 = x'_3 \quad (43)$$

$$t = \gamma\left(t' + \frac{vx'_1}{c^2}\right) \quad (44)$$

Notice these equations (37–44) contain only three coordinates that specify the position of an event (instead of four). This is due to the fact that Hendrik Lorentz assumed space is three-dimensional when he formulated them, which is what the third mathematical consequence of the observer's principle predicted. This remark completes the derivation. The more general form of the Lorentz transformation can be obtained by extending the procedure presented here.

As a final note, we want to point out that the Galilean transformation derived in this section (given by equations 25, 2, 3, 26 and 1) and its

corresponding inverse (equations 27, 2, 3, 28 and 1) describe a single event. However, when the values of the fourth coordinates are set equal to zero (equations 13 and 14), then the resulting equations describe two events that occur at the same place but at different times. That would be the interpretation of this result from a mathematical perspective. From a physical perspective, this result is telling us that the effects from the Lorentz transformation (such as time dilation, length contraction and the constancy of the speed of light) are actually depth perception effects that are being interpreted as real effects because the fourth spatial dimension is not been taken into account. Other interesting remarks can be made about this result, but we will address them more profoundly in a future paper.

IV. CONCLUSION

IN this article we used the concepts of absolute time and absolute space defined by Newton and the hypothesis that physical space is four-dimensional to develop an alternative formulation to the special theory of relativity. We proved this formulation is mathematically valid by deriving the Lorentz transformation from the Galilean transformation for frames of reference in four-dimensional euclidean space. Based on this result, we concluded that the effects predicted by the Lorentz transformation are actually depth perception effects that are being misinterpreted by physicists because they have been assuming space has only three dimensions instead of four. Therefore, our final conclusion is that the alternative formulation to the special theory of relativity presented here could be considered as evidence in favor of the hypothesis that physical space is actually four-dimensional.

DEDICATION

This article is dedicated to the memory of my father, Dr. Lorenzo León Callender López, who always supported me and was there for me. Without him, this work would not have been possible.

REFERENCES

- [1] Newton, Isaac. *The Mathematical Principles of Natural Philosophy*. New-York, D. Adee, 1848, p. 77.
<https://www.loc.gov/item/04014428/>
- [2] Dainton, Barry. *Time and Space*. Second edition, McGill-Queen's University Press, 2010, p. 182.
<https://archive.org/details/timespace0000dain/page/182>
- [3] Dainton, Barry. *Time and Space*. Second edition, McGill-Queen's University Press, 2010, pp. 165–166.
<https://archive.org/details/timespace0000dain/page/165>
- [4] The Editors of Encyclopaedia Britannica. "Galilean Transformations". *Encyclopedia Britannica*, 2 June 2020.
<https://www.britannica.com/science/Galilean-transformations>
- [5] Rynasiewicz, Robert. "Newton's Views on Space, Time, and Motion". *The Stanford Encyclopedia of Philosophy (Spring 2022 ed.)*, edited by Edward N. Zalta.
<https://plato.stanford.edu/archives/spr2022/entries/newton-stm/>
- [6] Lincoln, Don. "What's the real reason you can't go faster than the speed of light?". *Big Think*, 17 Nov. 2022.
<https://bigthink.com/hard-science/real-reason-faster-than-light-speed-spacetime/>
- [7] Nordström, Gunnar. "On the Possibility of a Unification of the Electromagnetic and Gravitation Fields". *Modern Kaluza-Klein Theories*, edited by T. Appelquist et al., Addison-Wesley, 1987, pp. 50–56.
<https://archive.org/details/modernkaluzaklei0000unse/page/50>
- [8] Kaluza, Theodor. "On the Unity Problem of Physics". *Modern Kaluza-Klein Theories*, edited by T. Appelquist et al., Addison-Wesley, 1987, pp. 61–68.
<https://archive.org/details/modernkaluzaklei0000unse/page/61>
- [9] Klein, Oskar. "Quantum Theory and Five Dimensional Theory of Relativity". *Modern Kaluza-Klein Theories*, edited by T. Appelquist et al., Addison-Wesley, 1987, pp. 76–87.
<https://archive.org/details/modernkaluzaklei0000unse/page/76>
- [10] Klein, Oskar. "The Atomicity of Electricity as a Quantum Theory Law". *Modern Kaluza-Klein Theories*, ed. by T. Appelquist et al., Addison-Wesley, 1987, p. 88.
<https://archive.org/details/modernkaluzaklei0000unse/page/88>
- [11] Einstein, Albert, and P. Bergmann. "On a Generalization of Kaluza's Theory of Electricity". *Modern Kaluza-Klein Theories*, edited by T. Appelquist et al., Addison-Wesley, 1987, pp. 89–107.
<https://archive.org/details/modernkaluzaklei0000unse/page/89>
- [12] Arnold, Vladimir. *Mathematical Methods of Classical Mechanics*. Second ed., Springer-Verlag, 1989, pp. 3–6.
<https://archive.org/details/mathematicalmeth0000arno/page/3>