SUPERLUMINAL NON-BARYONIC PARTICLES IN A 3D-BRANE UNIVERSE WITHOUT EQUIDISTANCE POSTULATE.

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Abstract. The 3D-brane universe model is an alternative non-Einsteinian theory of gravity. The initial version of this theory uses the so-called equidistance postulate. In the current second version of the theory this postulate is omitted. In the present paper non-baryonic particles are studied within the second version of the theory. As an example their revolution around a Schwarzschild black hole is considered.

1. Introduction.

The initial version of the 3D-brane universe model was built in the series of papers [1–6] (see also [7–11]). Its second version was started in [12]. In this version of the theory the gravitational field is described by a time-dependent 3D metric

$$g_{ij} = g_{ij}(t, x^1, x^2, x^3), \quad 1 \leqslant i, j \leqslant 3, \tag{1.1}$$

and by a time-dependent scalar function

$$g_{00} = g_{00}(t, x^1, x^2, x^3). \tag{1.2}$$

Here x^1, x^2, x^3 are comoving coordinates. Their definition in the standard cosmology can be found in [13]. In the 3D-brane universe model they are defined in [1] and [12]. Their definition is the same for both versions of the theory.

Through t in (1.1) and (1.2) we denote the so-called brane time. It is well-defined in the first version of the theory due to the following equidistance postulate.

Postulate 1.1. *Watches of any two comoving observers can be synchronized.*

Comoving observers are those observers whose comoving coordinates are constants:

$$x^1(t) = \text{const}, \qquad x^2(t) = \text{const}, \qquad x^3(t) = \text{const}\,.$$

In the initial version of the theory the brane time was defined as the proper time of any comoving observer. This definition was consistent due to the equidistance postulate 1.1. The equidistance postulate is excluded from the current version of the theory. Therefore the brane time is defined as the proper time of some particular

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comoving observer. The choice of that particular comoving observer is not unique. Replacing one comoving observer by another leads to a time scaling transformation

$$\tilde{t}=\tilde{t}(t), \qquad\qquad t=t(\tilde{t}). \tag{1.3}$$

In [12] it was shown that the equations of the gravitational field in the theory are invariant under the time scaling transformations of the form (1.3) provided the scalar function (1.2) is transformed as follows:

$$g_{00}=\left(\frac{d\tilde{t}}{dt}\right)^{2}\tilde{g}_{00}, \qquad\qquad \tilde{g}_{00}=\left(\frac{dt}{d\tilde{t}}\right)^{2}g_{00}. \tag{1.4}$$

The equations of the gravitational field in the second version of the 3D-brane universe model were derived in two ways in [12] and in [14]. The second way uses the Lagrangian approach with the Lagrangian density of the gravitational field

$$\mathcal{L}_{\text{gr}}=-\frac{c_{\text{gr}}^{4}}{16\,\pi\,\gamma}\,\sqrt{g_{00}}\,(\rho+2\,\Lambda), \tag{1.5}$$

where c_{gr} is the speed of gravitational waves and ρ is a scalar given by the formula

$$\begin{aligned}\rho=g_{00}^{-1}\sum_{k=1}^{3}\sum_{q=1}^{3}g^{kq}\,\nabla_{kq}\,g_{00}-\frac{g_{00}^{-2}}{2}\sum_{k=1}^{3}\sum_{q=1}^{3}g^{kq}\,\nabla_{k}\,g_{00}\,\nabla_{q}\,g_{00}\,-\\ -R-g_{00}^{-1}\sum_{k=1}^{3}\sum_{q=1}^{3}b^{k}_{q}\,b^{q}_{k}+g_{00}^{-1}\sum_{k=1}^{3}\sum_{q=1}^{3}b^{k}_{k}\,b^{q}_{q}.\end{aligned} \tag{1.6}$$

In this paper we do not write and discuss the Euler-Lagrange equations that follow from (1.5) and (1.6). Instead of it, we shall complement the action integral of the gravitational field with the action integral of a non-baryonic particle and derive dynamical equations for its motion in a way similar to that of [6].

2. Action integral for a non-baryonic massive particle.

The action integral of the gravitational field is expressed through its Lagrangian, while its Lagrangian is expressed through the Lagrangian density (1.5):

$$S_{\text{gr}}=\int L_{\text{gr}}\,dt, \qquad\qquad L_{\text{gr}}=\int\mathcal{L}_{\text{gr}}\,\sqrt{\det g}\;d^{3}x. \tag{2.1}$$

The motion of a non-baryonic particle is described by the coordinate functions

$$x^{i}=x^{i}(t), \qquad i=1,\,\ldots\,,3. \tag{2.2}$$

Time derivatives of the functions (2.2) are components of its velocity vector $\mathbf{v}$:

$$v^{i}=\dot{x}^{i}(t), \qquad i=1,\,\ldots\,,3. \tag{2.3}$$

In (2.2) and (2.3) we assume x^1, x^2, x^3 to be comoving coordinates and t to be a brane time. The action integral for a non-baryonic particle is given by the formula

$$S_{\text{nb}}=-\int m\,c_{\text{nb}}^2\,\sqrt{g_{00}-\frac{|\mathbf{v}|^2}{c_{\text{nb}}^2}}\,dt. \tag{2.4}$$

This formula (2.4) generalizes the formula (2.4) from [6]. The constant m in (2.4) is the rest mass of a particle. The constant c_{nb} is a speed constant similar to c_{gr} in (1.5). The same constant for baryonic particles is denoted through c_{br}. In the standard Einstein's relativity all of these speed constants should be equal to the speed of light c_{el} (speed of electromagnetic waves). The 3D-brane universe model is a less restrictive theory. Here all of these speed constants a priori can be different.

Using the formulas (2.1) and (2.4), we write the total action of the gravitational field and a single non-baryonic particle as the sum

$$S=S_{\text{gr}}+S_{\text{nb}}. \tag{2.5}$$

The Lagrangian associated with the action (2.5) is also written as a sum:

$$L=L_{\text{gr}}+L_{\text{nb}}. \tag{2.6}$$

The Lagrangian of the gravitational field L_{gr} is taken from (2.1). The formula for the Lagrangian L_{nb} is derived from (2.4). Indeed, we have

$$L_{\text{nb}}=-m\,c_{\text{nb}}^2\,\sqrt{g_{00}-\frac{|\mathbf{v}|^2}{c_{\text{nb}}^2}}. \tag{2.7}$$

Applying the stationary action principle (see [15]) to the sum of action integrals (2.5), we get the following Euler-Lagrange equations:

$$-\frac{d}{dt}\Bigl(\frac{\delta L}{\delta v^i}\Bigr)_{\mathbf{g},\mathbf{b},\mathbf{x}}+\Bigl(\frac{\delta L}{\delta x^i}\Bigr)_{\mathbf{g},\mathbf{b},\mathbf{v}}=0. \tag{2.8}$$

The first term L_{gr} in the right hand side of (2.6) does not depend on the functions (2.2) and (2.3), while the second term L_{nb} in the right hand side of (2.6) is not an integral. Therefore the Euler-Lagrange equations (2.8) reduce to

$$-\frac{d}{dt}\Bigl(\frac{\partial L_{\text{nb}}}{\partial v^i}\Bigr)+\frac{\partial L_{\text{nb}}}{\partial x^i}=0. \tag{2.9}$$

The partial derivatives in (2.9) are easily calculated using (2.7). Indeed, we have

$$\frac{\partial L_{\text{nb}}}{\partial v^i}=\frac{m\,v_i}{\sqrt{g_{00}-\dfrac{|\mathbf{v}|^2}{c_{\text{nb}}^2}}}\,,\qquad \frac{\partial L_{\text{nb}}}{\partial x^i}=\frac{\displaystyle\sum_{r=1}^{3}\sum_{s=1}^{3}\frac{m}{2}\,\frac{\partial g_{rs}}{\partial x^i}\,v^r\,v^s-c_{\text{nb}}^2\,\frac{m}{2}\,\frac{\partial g_{00}}{\partial x^i}}{\sqrt{g_{00}-\dfrac{|\mathbf{v}|^2}{c_{\text{nb}}^2}}}\,. \tag{2.10}$$

The function g_{00} is interpreted as a scalar function. Therefore its partial derivative in (2.10) is equal to its covariant derivative:

$$\frac{\partial g_{00}}{\partial x^i}=\nabla_i\,g_{00}. \tag{2.11}$$

It is known that $\nabla_i\, g_{rs} = 0$ and it is known that this covariant derivative is given by the following formula (see § 7 in Chapter III of [16]):

$$\nabla_i\, g_{rs} = \frac{\partial g_{rs}}{\partial x^i} - \sum_{q=1}^{3} \Gamma^q_{ir}\, g_{qs} - \sum_{q=1}^{3} \Gamma^q_{is}\, g_{rq}. \tag{2.12}$$

From $\nabla_i\, g_{rs} = 0$ and from the formula (2.12) we derive

$$\sum_{r=1}^{3}\sum_{s=1}^{3} \frac{\partial g_{rs}}{\partial x^i}\, v^r\, v^s = \sum_{q=1}^{3}\sum_{s=1}^{3} 2\,\Gamma^q_{is}\, v_q\, v^s. \tag{2.13}$$

Here Γ^q_{is} are the components of the metric connection associated with the metric (1.1). Due to (2.11) and (2.13) the formulas (2.10) are written as follows:

$$\frac{\partial L_{\text{nb}}}{\partial v^i} = \frac{m\, v_i}{\sqrt{g_{00} - \dfrac{|\mathbf{v}|^2}{c_{\text{nb}}^2}}}, \qquad \frac{\partial L_{\text{nb}}}{\partial x^i} = \frac{\displaystyle\sum_{q=1}^{3}\sum_{s=1}^{3} m\,\Gamma^q_{is}\, v_q\, v^s - \frac{m\, c_{\text{nb}}^2}{2}\,\nabla_i\, g_{00}}{\sqrt{g_{00} - \dfrac{|\mathbf{v}|^2}{c_{\text{nb}}^2}}}. \tag{2.14}$$

Applying (2.14) to the equations (2.9), we derive

$$\frac{d}{dt}\left(\frac{m\, v_i}{\sqrt{g_{00} - \dfrac{|\mathbf{v}|^2}{c_{\text{nb}}^2}}}\right) = \frac{\displaystyle\sum_{q=1}^{3}\sum_{s=1}^{3} m\,\Gamma^q_{is}\, v_q\, v^s - \frac{m\, c_{\text{nb}}^2}{2}\,\nabla_i\, g_{00}}{\sqrt{g_{00} - \dfrac{|\mathbf{v}|^2}{c_{\text{nb}}^2}}}. \tag{2.15}$$

The time derivative in the left hand side of (2.15) is transformed as follows:

$$\begin{aligned} &\frac{d}{dt}\left(\frac{m\, v_i}{\sqrt{g_{00} - \dfrac{|\mathbf{v}|^2}{c_{\text{nb}}^2}}}\right) = \frac{m\, \dot v_i}{\sqrt{g_{00} - \dfrac{|\mathbf{v}|^2}{c_{\text{nb}}^2}}} + \\ &+ \frac{m\, v_i}{\left(\sqrt{g_{00} - \dfrac{|\mathbf{v}|^2}{c_{\text{nb}}^2}}\right)^3}\,\frac{1}{2}\left(\frac{d}{dt}\left(\frac{|\mathbf{v}|^2}{c_{\text{nb}}^2}\right) - \dot g_{00} - \sum_{s=1}^{3} v^s\,\nabla_s\, g_{00}\right). \end{aligned} \tag{2.16}$$

Combining (2.15) and (2.16), we derive the differential equations for v_i:

$$\begin{aligned} &\frac{m\, \dot v_i}{\sqrt{g_{00} - \dfrac{|\mathbf{v}|^2}{c_{\text{nb}}^2}}} + \frac{m\, v_i}{\left(\sqrt{g_{00} - \dfrac{|\mathbf{v}|^2}{c_{\text{nb}}^2}}\right)^3}\,\frac{1}{2}\left(\frac{d}{dt}\left(\frac{|\mathbf{v}|^2}{c_{\text{nb}}^2}\right) - \dot g_{00} - \right. \\ &\left. - \sum_{s=1}^{3} v^s\,\nabla_s\, g_{00}\right) = \frac{\displaystyle\sum_{q=1}^{3}\sum_{s=1}^{3} m\,\Gamma^q_{is}\, v_q\, v^s - \frac{m\, c_{\text{nb}}^2}{2}\,\nabla_i\, g_{00}}{\sqrt{g_{00} - \dfrac{|\mathbf{v}|^2}{c_{\text{nb}}^2}}}. \end{aligned} \tag{2.17}$$

Through $\dot{g}_{00}$ in (2.16) and (2.17) we denote the partial derivative of the scalar function (1.2) with respect to the time variable. The equality (2.16) is derived using (2.11). Apart from (2.11) below we need the following equality from [14]:

$$b_{ij} = \frac{\dot{g}_{ij}}{2\,c_{\text{gr}}} = \frac{1}{2\,c_{\text{gr}}}\,\frac{\partial g_{ij}}{\partial t} = \frac{1}{2}\,\frac{\partial g_{ij}}{\partial x^0}. \tag{2.18}$$

The second term in the left hand side of (2.17) comprises the time derivative of $|\mathbf{v}|^2$. We calculate this time derivative as follows:

$$\begin{gathered}
\frac{d(|\mathbf{v}|^2)}{dt} = \frac{d}{dt}\biggl(\sum_{r=1}^{3}\sum_{s=1}^{3} g_{rs}\,v^r\,v^s\biggr) = \sum_{r=1}^{3}\sum_{s=1}^{3}\frac{d(g_{rs}\,v^r)}{dt}\,v^s + \\
+\sum_{r=1}^{3}\sum_{s=1}^{3}\frac{d(g_{rs}\,v^s)}{dt}\,v^r - \sum_{r=1}^{3}\sum_{s=1}^{3}\frac{dg_{rs}}{dt}\,v^s\,v^r = \sum_{s=1}^{3} 2\,\dot{v}_s\,v^s - \\
-\sum_{r=1}^{3}\sum_{s=1}^{3}\frac{\partial g_{rs}}{\partial t}\,v^s\,v^r - \sum_{r=1}^{3}\sum_{s=1}^{3}\sum_{i=1}^{3}\frac{\partial g_{rs}}{\partial x^i}\,\dot{x}^i\,v^s\,v^r.
\end{gathered} \tag{2.19}$$

We transform (2.19) using (2.18), (2.3), and (2.13). This yields

$$\begin{gathered}
\frac{d(|\mathbf{v}|^2)}{dt} = -\sum_{q=1}^{3}\sum_{s=1}^{3}\sum_{i=1}^{3} 2\,\Gamma^q_{is}\,v_q\,v^s\,v^i + \\
+\sum_{s=1}^{3} 2\,\dot{v}_s\,v^s - \sum_{r=1}^{3}\sum_{s=1}^{3} 2\,c_{\text{gr}}\,b_{rs}\,v^s\,v^r.
\end{gathered} \tag{2.20}$$

Now we multiply (2.17) by v^i and sum up with respect to i running from 1 to 3:

$$\begin{gathered}
\frac{m\displaystyle\sum_{i=1}^{3}\dot{v}_i\,v^i}{\sqrt{g_{00}-\dfrac{|\mathbf{v}|^2}{c_{\text{nb}}^2}}} + \frac{m\,|\mathbf{v}|^2}{\biggl(\sqrt{g_{00}-\dfrac{|\mathbf{v}|^2}{c_{\text{nb}}^2}}\,\biggr)^3}\,\frac{1}{2}\biggl(\frac{d}{dt}\biggl(\frac{|\mathbf{v}|^2}{c_{\text{nb}}^2}\biggr) - \dot{g}_{00} - \\
-\sum_{s=1}^{3} v^s\,\nabla_s\,g_{00}\biggr) = \frac{\displaystyle\sum_{q=1}^{3}\sum_{s=1}^{3}\sum_{i=1}^{3} m\,\Gamma^q_{is}\,v_q\,v^s\,v^i - \frac{m\,c_{\text{nb}}^2}{2}\sum_{i=1}^{3} v^i\,\nabla_i\,g_{00}}{\sqrt{g_{00}-\dfrac{|\mathbf{v}|^2}{c_{\text{nb}}^2}}}.
\end{gathered} \tag{2.21}$$

Then we apply (2.20) to the time derivative of $|\mathbf{v}|^2$ in (2.21). As a result the equality (2.21) simplifies and reduces to the following one:

$$\begin{gathered}
\sum_{i=1}^{3}\dot{v}_i\,v^i = \sum_{q=1}^{3}\sum_{s=1}^{3}\sum_{i=1}^{3}\Gamma^q_{is}\,v_q\,v^s\,v^i + \frac{c_{\text{gr}}\,|\mathbf{v}|^2}{g_{00}\,c_{\text{nb}}^2}\sum_{r=1}^{3}\sum_{s=1}^{3} b_{rs}\,v^s\,v^r + \\
+\frac{\dot{g}_{00}\,|\mathbf{v}|^2}{2\,g_{00}} - \frac{c_{\text{nb}}^2}{2}\sum_{i=1}^{3} v^i\,\nabla_i\,g_{00} + \frac{|\mathbf{v}^2|}{g_{00}}\sum_{i=1}^{3} v^i\,\nabla_i\,g_{00}.
\end{gathered} \tag{2.22}$$

Now we substitute (2.22) into (2.20). As a result we get

$$\begin{aligned}\frac{d(|\mathbf{v}|^2)}{dt}&=-\frac{2\,c_{\text{gr}}}{g_{00}}\left(g_{00}-\frac{|\mathbf{v}|^2}{c_{\text{nb}}^2}\right)\sum_{r=1}^{3}\sum_{s=1}^{3}b_{rs}\,v^s\,v^r+\\&+\frac{\dot g_{00}\,|\mathbf{v}|^2}{g_{00}}-c_{\text{nb}}^2\sum_{i=1}^{3}v^i\,\nabla_i\,g_{00}+\frac{2\,|\mathbf{v}|^2}{g_{00}}\sum_{i=1}^{3}v^i\,\nabla_i\,g_{00}.\end{aligned}\tag{2.23}$$

The next step is to substitute (2.23) into the equation (2.17). This yields

$$\begin{aligned}\dot v_i-\sum_{q=1}^{3}\sum_{s=1}^{3}\Gamma^q_{is}\,v_q\,\dot x^s&=-\frac{c_{\text{nb}}^2}{2}\,\nabla_i\,g_{00}+\\+v_i\left(\sum_{s=1}^{3}\frac{v^s\,\nabla_s\,g_{00}}{g_{00}}+\frac{\dot g_{00}}{2\,g_{00}}\right.&\left.+\frac{c_{\text{gr}}}{c_{\text{nb}}^2\,g_{00}}\sum_{r=1}^{3}\sum_{s=1}^{3}b_{rs}\,v^s\,v^r\right).\end{aligned}\tag{2.24}$$

The left hand side of the equation (2.24) fits the definition of the covariant derivative of a covectorial field with respect to a parameter along a parametric curve, see (8.10) in § 8 of Chapter III in [16]). Therefore we can write (2.24) as

$$\begin{aligned}\nabla_t v_i=-\frac{c_{\text{nb}}^2}{2}\,\nabla_i\,g_{00}+v_i\left(\sum_{s=1}^{3}\frac{v^s\,\nabla_s\,g_{00}}{g_{00}}\right.&+\\\left.+\frac{\dot g_{00}}{2\,g_{00}}+\frac{c_{\text{gr}}}{c_{\text{nb}}^2\,g_{00}}\sum_{r=1}^{3}\sum_{s=1}^{3}b_{rs}\,v^s\,v^r\right)&.\end{aligned}\tag{2.25}$$

The equalities (2.3) can be written as differential equations

$$\dot x^i=v^i.\tag{2.26}$$

The equations (2.25) complemented with the equations (2.26) constitute the system of ordinary differential equations describing the motion of a non-baryonic particle in the gravitational field described by the metric (1.1) and the scalar function (1.2). These equations do not comprise the mass m at all. This observation can be interpreted as follows.

Theorem 2.1. *The inertial and passive gravitational masses of a non-baryonic massive particle are equal to each other.*

The definition of inertial mass and the definitions of active and passive gravitational masses are given in [17]. The same theorem 2.1 was formulated in [6] within the first version of our theory. It remains valid in the present second version too.

3. Legendre transformation and the energy function of a non-baryonic particle.

The Legendre transformation of a dynamical system is determined by its Lagrangian. In the case of the Lagrangian (2.6) it is given by the formulas

$$p_i=\left(\frac{\delta L}{\delta v^i}\right)_{\substack{g,\dot g,\mathbf{g}\\\mathbf{b},\mathbf{x}}},\qquad\beta^{00}=\left(\frac{\delta\mathcal{L}}{\delta\dot g_{00}}\right)_{\substack{g,\mathbf{g},\mathbf{b}\\\mathbf{x},\mathbf{v}}},\qquad\beta^{ij}=\left(\frac{\delta\mathcal{L}}{\delta b_{ij}}\right)_{\substack{g,\dot g,\mathbf{g}\\\mathbf{x},\mathbf{v}}}.\tag{3.1}$$

Since the Lagrangian L_{gr} does not depend on v^i and since the Lagrangian L_{nb} does not depend on b_{ij}, the formulas (3.1) can be written as

$$p_i=\Bigl(\frac{\partial L_{\mathrm{nb}}}{\partial v^i}\Bigr)_{\substack{g,\dot{g},\mathbf{g}\\ \mathbf{b},\mathbf{x}}},\qquad\beta^{00}=\Bigl(\frac{\delta\mathcal{L}_{\mathrm{gr}}}{\delta\dot{g}_{00}}\Bigr)_{\substack{g,\mathbf{g},\mathbf{b}\\ \mathbf{x},\mathbf{v}}},\qquad\beta^{ij}=\Bigl(\frac{\delta\mathcal{L}_{\mathrm{gr}}}{\delta b_{ij}}\Bigr)_{\substack{g,\dot{g},\mathbf{g}\\ \mathbf{x},\mathbf{v}}}.\tag{3.2}$$

The quantities p_i, β^{00}, and β^{ij} in (3.2) are the generalized momenta associated with the generalized velocities v^i, $\dot{g}_{00}$ and b_{ij}. The quantities β^{00}, and β^{ij} are already already calculated in [14], see the formulas (5.2) and (4.3) therein:

$$\beta^{00}=0,\qquad\qquad\beta^{ij}=\frac{c_{\mathrm{gr}}^4\,g_{00}^{-1/2}}{8\,\pi\,\gamma}\Bigl(b^{ij}-\sum_{k=1}^{3}b^k_k\,g^{ij}\Bigr).\tag{3.3}$$

The quantities (3.3) are not interested for us in this paper. The quantities p_i are also already calculated, see the formulas (2.10) above:

$$p_i=\frac{m\,v_i}{\sqrt{g_{00}-\dfrac{|\mathbf{v}|^2}{c_{\mathrm{nb}}^2}}}.\tag{3.4}$$

It is easy to see that p_i in (3.4) are components of a covector $\mathbf{p}$. This covector is called the momentum covector of a non-baryonic particle.

Now let's return back to the equations (2.15). Using the components of the momentum covector $\mathbf{p}$ from (3.4) and taking into account the equations (2.3), we can write the equations (2.15) as follows:

$$\dot{p}_i-\sum_{q=1}^{3}\sum_{s=1}^{3}\Gamma^q_{is}\,p_q\,\dot{x}^s=-\frac{m\,c_{\mathrm{nb}}^2\,\nabla_i\,g_{00}}{2\sqrt{g_{00}-\dfrac{|\mathbf{v}|^2}{c_{\mathrm{nb}}^2}}}.\tag{3.5}$$

The left hand side of the equation (3.5) fits the definition of the covariant derivative of a covectorial field with respect to a parameter along a parametric curve, see (8.10) in § 8 of Chapter III in [16]). Therefore we can write (3.5) as

$$\nabla_t p_i=-\frac{m\,c_{\mathrm{nb}}^2\,\nabla_i\,g_{00}}{2\sqrt{g_{00}-\dfrac{|\mathbf{v}|^2}{c_{\mathrm{nb}}^2}}}.\tag{3.6}$$

The equations (3.6) complemented with the equations (2.26) constitute the system of ordinary differential equations describing the motion of a non-baryonic particle in the gravitational field described by the metric (1.1) and the scalar function (1.2). The right hand sides of the equations (3.6) are interpreted as the components of the force covector $\mathbf{F}$ acting upon a non-baryonic particle:

$$F_i=-\frac{m\,c_{\mathrm{nb}}^2\,\nabla_i\,g_{00}}{2\sqrt{g_{00}-\dfrac{|\mathbf{v}|^2}{c_{\mathrm{nb}}^2}}}.\tag{3.7}$$

The energy function of a non-baryonic particle is written using the components of its momentum covector $\mathbf{p}$ and the components of its velocity vector $\mathbf{v}$:

$$E=\sum_{i=1}^{3} p_i\,v^i-L_{\text{nb}}. \tag{3.8}$$

Applying (3.4) and (2.7) to (3.8), we derive

$$E=\frac{m\,|\mathbf{v}|^2}{\sqrt{g_{00}-\dfrac{|\mathbf{v}|^2}{c_{\text{nb}}^2}}}+m\,c_{\text{nb}}^2\sqrt{g_{00}-\frac{|\mathbf{v}|^2}{c_{\text{nb}}^2}}. \tag{3.9}$$

The formula (3.9) can be simplified. It reduces to

$$E=\frac{m\,c_{\text{nb}}^2}{\sqrt{g_{00}-\dfrac{|\mathbf{v}|^2}{c_{\text{nb}}^2}}}\,. \tag{3.10}$$

Due to (1.4) the quantity g_{00} in (3.10) can be reduced to $g_{00}=1$ at some particular point and at some particular instant of time if we apply the time scaling transformation (1.3) and pass to the proper time of the comoving observer at that particular point and that particular instant of time:

$$E=\frac{m\,c_{\text{nb}}^2}{\sqrt{1-\dfrac{|\mathbf{v}|^2}{c_{\text{nb}}^2}}}\,. \tag{3.11}$$

Due to (3.11) the constant c_{nb} is interpreted as the upper limit for the speed of a non-baryonic particle.

4. Circular revolution of a non-baryonic particle around a Schwarzschild black hole.

In [12] a Schwarzschild black hole was studied within the framework of the 3D-brane universe model using the spacial coordinates

$$x^1=\rho,\qquad\qquad x^2=\theta,\qquad\qquad x^3=\phi \tag{4.1}$$

and the time variable t associated with the comoving observer at infinity. Its gravitational field in the variables (4.1) is described by the scalar function

$$g_{00}=1-\frac{r_{\text{gr}}}{\rho} \tag{4.2}$$

and the diagonal three-dimensional metric g_{ij} with the components

$$g_{11}=\frac{1}{1-\dfrac{r_{\text{gr}}}{\rho}}\,,\qquad\qquad g_{22}=\rho^2,\qquad\qquad g_{33}=\rho^2\,\sin^2(\theta). \tag{4.3}$$

The nonzero components of the metric connection associated with the 3D metric (4.3) are given by the following formulas:

$$\begin{aligned}
&\Gamma^1_{11}=\frac{r_{\text{gr}}}{2\,\rho\,(r_{\text{gr}}-\rho)}, \qquad \Gamma^2_{12}=\Gamma^2_{21}=\frac{1}{\rho},\ \ \Gamma^1_{22}=r_{\text{gr}}-\rho, \qquad \Gamma^3_{23}=\cot\theta,\\
&\Gamma^1_{33}=(r_{\text{gr}}-\rho)\,\sin^2\theta,\ \ \Gamma^3_{13}=\Gamma^3_{31}=\frac{1}{\rho},\ \ \Gamma^2_{33}=-\frac{\sin(2\,\theta)}{2},\ \ \Gamma^3_{32}=\cot\theta.
\end{aligned} \tag{4.4}$$

The Schwarzschild black hole metric is a stationary solution of the gravitational field equations. Therefore we have the following relationships:

$$\dot g_{00}=0, \qquad\qquad b_{ij}=0 \ \text{ for } \ 1\leqslant i,j\leqslant 3. \tag{4.5}$$

Assume that a non-baryonic particle with the mass m revolves around a black hole in its equatorial plane with angular velocity ω. Then its revolution is

$$\rho(t)=\rho=\text{const}, \qquad\qquad \theta(t)=\frac{\pi}{2}=\text{const}, \qquad\qquad \phi(t)=\omega\,t. \tag{4.6}$$

Differentiating (4.6) with respect to t, we find the components of the velocity vector:

$$v^1=0, \qquad\qquad v^2=0, \qquad\qquad v^3=\omega. \tag{4.7}$$

The components of the velocity covector are produced by the standard formula

$$v_i=\sum_{k=1}^{3} g_{ik}\,v^k. \tag{4.8}$$

Applying (4.3) and (4.7) to (4.8) and taking into account (4.6), we get

$$v_1=0, \qquad\qquad v_2=0, \qquad\qquad v_3=\rho^2\,\omega. \tag{4.9}$$

The components of the acceleration covector are defined as follows

$$a_i=\nabla_t v_i=\dot v_i-\sum_{q=1}^{3}\sum_{s=1}^{3}\Gamma^q_{is}\,v_q\,\dot x^s. \tag{4.10}$$

Applying (4.4) and (4.9) to (4.10) and taking into account (2.26), we get

$$a_1=-\rho\,\omega^2, \qquad\qquad a_2=0, \qquad\qquad a_3=0. \tag{4.11}$$

Now we can apply the differential equation (2.25) describing the dynamics of the particle. Here are the gradient components of the scalar field (4.2) in it:

$$\nabla_1\,g_{00}=\frac{r_{\text{gr}}}{\rho^2} \qquad\qquad \nabla_2\,g_{00}=0, \qquad\qquad \nabla_3\,g_{00}=0. \tag{4.12}$$

Using (4.7) and (4.12), we derive

$$\sum_{s=1}^{3} v^s \, \nabla_s \, g_{00} = 0. \tag{4.13}$$

Due to (4.5), (4.13), and (4.10) the equation (2.25) reduces to

$$a_i = -\frac{c_{\text{nb}}^2}{2} \, \nabla_i \, g_{00}. \tag{4.14}$$

Applying (4.11) and (4.12) to (4.14), we derive the equality

$$-\rho \, \omega^2 = -\frac{c_{\text{nb}}^2 \, r_{\text{gr}}}{2 \, \rho^2}. \tag{4.15}$$

The gravitational radius of a black hole with the mass M is given by the formula

$$r_{\text{gr}} = \frac{2 \, \gamma \, M}{c_{\text{gr}}^2}. \tag{4.16}$$

The formula (4.16) can be found in § 100 of Chapter XII in [18] or in [19]: Substituting (4.16) into (4.15), we derive

$$\rho \, \omega^2 = \frac{c_{\text{nb}}^2 \, \gamma \, M}{c_{\text{gr}}^2 \, \rho^2}. \tag{4.17}$$

Despite the denominator in the force formula (3.7), the formula (4.17) coincides with the prediction of the classical Newton's theory of gravitation up to the factor $c_{\text{nb}}^2 / c_{\text{gr}}^2$. In the standard relativity both constants c_{nb} and c_{gr} are taken to be equal to the speed of light c_{el}. Our theory the 3D-brane universe model is different. It admits the option where these two constants c_{nb} and c_{gr} are different.

5. Conclusions.

The present paper extends the results of [6] to the second version of the 3D-brane universe model where the equidistance postulate 1.1 is excluded. Like in the previous version of the theory, if the inequality

$$c_{\text{nb}} > c_{\text{el}} \tag{5.1}$$

is fulfilled, then superluminal motion of a non-baryonic massive particle is possible.

The constant c_{nb} in (5.1) is not necessarily unique. There may be several sorts of non-baryonic matter each having its own speed constant c_{nb}.

6. Dedicatory.

This paper is dedicated to my sister Svetlana Abdulovna Sharipova.

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