

**Grand Unified Theory**  
**By the Einstein's equation**  
**Using as metric**  
**The four dimensional sphere**

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**Abstract:**

We critically review the Standard Model. We were coming to the conclusion, that the principal structure of the Standard Model is correct. But we found that the group structure of photons, electromagnetism, strong interaction, gravitation and weak interaction are exactly fit into the structure of the symmetry group  $S^4$  of order or dimension is four. We are able to incorporate electromagnetism and gravitation into Einstein's equation.

## **1 Introduction**

The history of the Standard Model started in the year 1954 [1], when Yang and Mills proposed their  $SU(2)$  gauge field theory to describe kernel physics of neutrons and protons.

1961 [2,3] have been proposed that the messenger fields, called gluons, in sum eight should be responsible for carrying color. There are there are eight independent types of gluons. It is believed that gluons carry mass.

A gauge theory eliminates degrees of freedom from the system. But it is necessary in order to adjust what should be measured. The symmetry group of the gauge field determines the solution.

For electromagnetism the gauge field is  $A_\mu = (A_x, A_y, A_z, \Phi)$ . A popular gauge is:

$$\text{div}\vec{A} = \frac{\partial\Phi}{\partial t}. \quad (1)$$

1979 was the year when S. Glashow, A. Salam and S. Weinberg were awarded with the Nobel Prize for discovering the electroweak theory (Glashow) and the combination of the Higgs [4-6] and electroweak theory [7-9]. The Standard Model is mathematically a multiplication of three groups:  $U(1) * SU(2) * SU(3)$ . One solves quantum field theory, by introducing gauge fields. A gauge field is an entity that is a degree of freedom of the given system.

1973 have Gross and Wilczek and too Politzer proposed a theory that should explain strong interaction. It has been named quantum chromodynamics [10,11].

This theory reveals color confinement, asymptotic freedom and chiral symmetry. If you try to separate two quarks, color confinement will be reached when the color charge is full with energy, that spontaneously a quark anti-quark is realised.

Asymptotic freedom is a phenomenon, that occurs when quark get very close together. The strength of their interaction force is increased.

The three unitary groups  $SU(2), SU(3)$  are local gauge groups. Thus they live in the quantum space, whereas  $U(1)$  is global. That means it is existing in spacetime.

$SU(2)$  is a two dimensional group,

$$[W^+, W^-] = Z, \quad (2)$$

which is equivalent to the three Pauli matrices. The W's and Z are the messengers of the weak interaction. The solution of the Pauli equation is  $S = e^{i\frac{1}{2}\vec{\alpha}\cdot\vec{\sigma}}$ . We are observing that the solution is a Dirac fermion, since the factor  $\frac{1}{2}$  is leading to a rotation of  $4\pi$ . A Dirac fermion is identified by its property that it is invariant under rotations of  $4\pi$  [12].

Quantum chromodynamics is identifying the messengers, named gluons, which are eight independently. They are carrying three different charges called color. And the particles building up QCD are the six quarks, u, p, d, s, b, t.

## 2 An alternative Higgs Theory and a grand unified theory

We depart from the metric:

$$x = \cos\theta\cos\varphi\cos\varrho, \quad (3a)$$

$$y = \cos\theta\cos\varphi\sin\varrho, \quad (3b)$$

$$z = \cos\theta\sin\varphi, \quad (3c)$$

$$ct = \sin\theta. \quad (3d)$$

Einstein's equation is well known and given by:

$$G_{\mu\nu} = 8\pi T_{\mu\nu}. \quad (4)$$

In [13] we revealed a Einstein equation that is expressing the evolution of the Universe. We presented an alternative evolution of the entropy.

Now it is possible to write down an Einstein equation that on l.h.s. is appearing the Higgs and on the r.h.s. our Universe appears where all four interaction fields are revealed.

It turns out, that the Universe, here in our proposal, is found that all four forces and the Higgs fit in the permutation group of four entities, i.e.:  $S^4$ .  $S^4$  is covering permutations. The different gauge fields are distinguished by permutations that are forming their group.

The Higgs is a scalar, thus it has full symmetry of the four dimensional rotation group. Electromagnetism contains of six axial vectors of order two.

The weak interaction is represented by three operations. The strong interaction is believed to be exchanged by eight gluons.

Gravity is too build up of six axial quantities. Those are objects, behaving like an angular momentum, but in four dimensions. Those objects are gravitons.

Below is shown the table of the permutation group,

$S^4$ .

Order (Dimension)	Signum	Number Elements	Permutations
1	+1	1	1
2	-1	6	(12);(13);(14) (23);(24);(34)
2	+1	3	(12)(34); (13)(24) (14)(23)
3	+1	8	(123)(132) (124)(142) (134)(143) (234)(243)
4	-1	6	(1234)(1243) (1423)(1432) (1324)(1341)

The signum -1 is revealing that the number of permutations is odd. +1 is the result, if the number of permutations is even. Every permutation can be decomposed into transpositions. A transposition is a cycle of length two, f.e., (13). For example (123) can be decomposed into (13)(12). The number of transposition is even, thus the object is not an axial vector.

Back to our equation (4). Like in [13], we receive:

$$H_0 = A_\mu + \sum_{i,j=1..3} [E_i, E_j] + \sum_{i,j=1..3} [B_i, B_j] + \sum_{\pi} \{(n, p)(e^-, \nu)\} + \sum_{a,b=1,..,8} [t^a, t^b] + \sum_{\mu\nu\sigma\theta} R_{\mu\nu\sigma\theta} \quad (5)$$

Derived from Einstein's equation, it is leading to an equation where all four forces are identified and they show principally what happens when the Higgs is decaying.

### 3 Discussion

We have briefly discussed the Standard Model. The Standard Model has been very successful in the past forty years. The question is: How does the initial Universe appear? And how are the four fundamental forces developed? The presented solution is based on unitary groups. The Standard Model assumes that the Higgs is going to break the fundamental symmetry.

Symmetry groups are revealing order of a system describing Nature. The main goal of modern science is finding patterns in Nature. Since we observe patterns everywhere. Patterns are appearing around us.

The symmetry groups that are the solution to the given problem are unitary. That means they are symmetry groups on the basis of quantum physics.

U(1) is believed to be the symmetry group of electromagnetism. SU(2) is the symmetry group of the weak interaction and SU(3) is the symmetry group of the strong interaction.

This is all compatible with our solution. But we are able to add two forces. First  $A_\mu$  the gauge field and the six generators of magnets and electric charges are together forming magnetic and electric field and are identified as the photon and electromagnetism. The six generators  $E_i, B_i$  are forming the electric and magnetic field. Moreover the weak interaction has a different description, but essentially it is compatible. And we found a representation of gravity. This is the Riemann tensor. Gravitation is represented by a six generators as been shown in the table above. Those are proposed to be gravitons.

(5) is our proposal for a unified theory of physics.

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