Testing Special Relativity With an Infinite Arm Interferometer

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Abstract

The Michelson-Morley experiment and its resolution by the special theory of relativity form a foundational truth in modern physics. In this paper we propose an equivalent experiment involving a relativistic interferometer having infinite arms. Curiously, we find that lorentz contraction and time dilation are absent in our experiment leading to a conflict between special relativity and the symmetry of nature.

Keywords— Michelson-Morley, symmetry of nature, special relativity

1 Introduction

The paradigm shifting Michelson-Morley (MM) experiment [1] and its paradox of unequal path lengths has changed the way modern science interprets the nature of space and time. Let us investigate the geometry and sequence of events within a MM interferometer as follows:

1. We begin with the geometry of two flat triangles that are relevant to the discussions at hand.
2. Then we present a thought experiment involving ideal sinusoidal waves that travel, reflect and interfere with each other within the confines of a circular boundary. Further, we establish that our thought experiment is equivalent to an MM interferometer having infinite arms and moving through space under inertial rules.
3. Finally we propose a method to realise our thought experiment in order to arrive at a curious topic for discussion: Why are lorentz contraction and time dilation absent in our experiment?

2 Euclidean Geometry

On a flat surface, we draw any angle θ at origin Q bounded by two equal length line segments $QB = QB' = h$. We join points B and B' to points A and C such that the line segment AC is perpendicular to QB and centred at Q. We will restrict our arguments to the domain $x < h$. Fig. 1 illustrates.

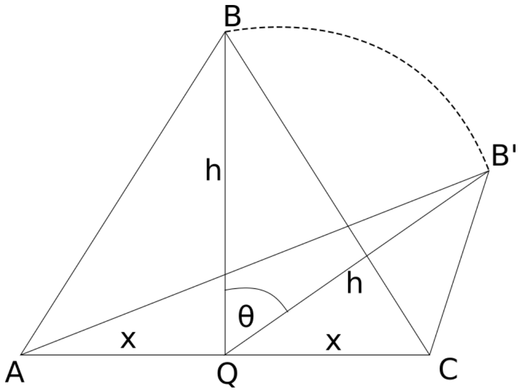

Figure 1: Triangles ABC and $AB'C$ rendered on a flat surface

From fig. 1, we establish the following geometric truths:

1. If $x > 0$, physical measurements will verify the theoretical statement $AB + BC \neq AB' + B'C$ is true for all $\theta \neq 0, \pi, 2\pi...$
2. Since h is constant, curve BB' will take the form of a circle as $0 \leq \theta \leq 2\pi$.

3 A Thought Experiment

Imagine an ideal homogeneous flat surface S1 enclosed by an ideal rigid boundary of geometrically circular shape (radius $= h$) and capable of transporting a travelling wave of the form,

$$\frac{1}{c^2}\frac{\delta^2 y}{\delta t^2} = \frac{\delta^2 y}{\delta x^2} \tag{1}$$

where the terms are as follows:

1. x represents the displacement of the measurement point from the origin of the wave measured along surface S1,
2. c represents the velocity of the wave measured along surface S1,
3. y represents the instant displacement of the wave measured perpendicular to surface S1.
4. t represents the time elapsed since the instant that the wave was created.

From directly above, we may project fig. 1 onto S1 without distortion such that the boundary of S1 is defined by curve BB', a circle of radius h about point Q. Now let us agree that surface S1 supports the geometry of fig. 1 over all $0 \leq \theta \leq 2\pi$ and $0 \leq x < h$.

We choose any point A on S1 and disturb the equilibrium causing an isotropic sinusoidal wave (wavelength $= \lambda$) to emanate from that point. As this primary wave expands, its wavefront will interact with S1's boundary generating innumerable secondary waves as it does so. Each reflection event along curve BB' generates its own isotropic wave and from physical measurements of fig. 1, we find that if $x \neq 0$ the statement $AB + BC \neq AB'_1 + B'_1C... \neq AB'_i + B'_iC$ is true (See fig. 2 which is a generalisation of fig. 1 over all $0 \leq \theta \leq 2\pi$). Let us invoke the following assumptions to debate the nature of the interference pattern at point C:

1. The wave we generate originates from a single point and comprises exactly one complete cycle of a sinusoidal travelling wave
2. λ remains constant in accordance with the law of conservation of energy [2]

3. Reflections are instantaneous and lossless

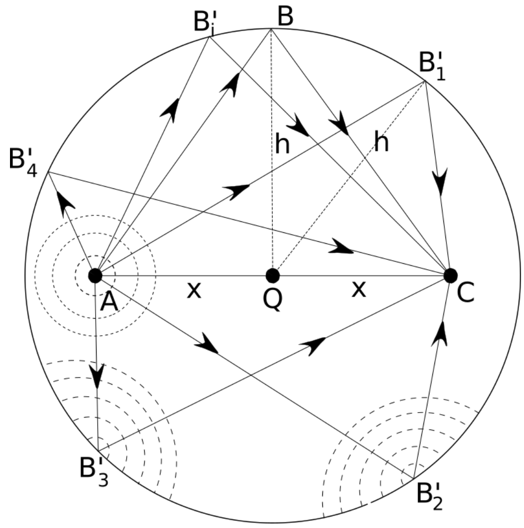

Figure 2: A single isotropic sinusoidal wave is emitted from point A and reflects from the circular boundary generating innumerable secondary wavefronts.

4 The Michelson-Morley Experiment

Now we turn to theoretical aspects of the MM experiment in order to establish it's equivalence with our thought experiment.

4.1 Frames of Reference

For the purpose of further discussion, we refer to fig. 1 and establish the following euclidean frames of reference:

1. A stationary reference frame I_0 centered at point Q.
2. A moving reference frame I_1 that translates from point A to point C with some constant velocity v relative to arbitrarily selected origin Q.

4.2 Geometry and Sequence of Events

Consider an MM interferometer [1] moving through space under inertial rules (see fig. 3). By fixing $\angle B_1'QB_2' = \pi/2$, line segments QB_1' and QB_2' form the arms of the interferometer. The arms are free to rotate about point Q and consequently each arm subtends its own angle θ measured from a perpendicular to line segment AC. Reference frame I_1 is fixed to the interferometric source and moves with constant velocity v from point A to point C.

The event cycle begins with the source at point A marking the simultaneous emission of a pair of photons (wavelength=λ). As the entire apparatus moves with some constant ($AQ = QC$) velocity v relative to origin Q along line segment AC, the photons are emitted at point A, reflect from mirrors B_1 and B_2 to finally arrive simultaneously (in phase with each other) at point C.

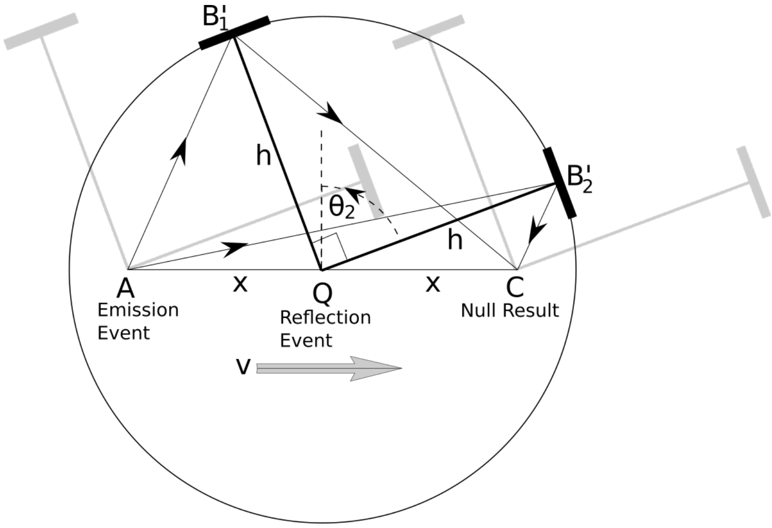

Figure 3: Geometry of the Michelson-Morley experiment depicting the general case $x \neq 0$. Equivalent to our thought experiment and identical to fig. 1, we find $AB_1' + B_1'C \neq AB_2' + B_2'C$ but yet we agree that the outcome is a null result at point C.

As is true in our thought experiment, it is straightforward to recognise that in one emission-reflection-result cycle of an MM interferometer and for all $0 \leq v < c$, the locus of all points in space where a reflection event can occur is a physical circle of radius h about origin Q. In terms of scope, our thought experiment is equivalent to one cycle of an MM interferometer having infinite arms (See fig. 2). It is also a well established fact of modern science [3] that the MM experiment presents a null result for all $0 \leq v < c$, where c represents the velocity of light.

4.3 Conflict Resolution

The geometry of the MM experiment and its sequence of events present a paradox of unequal path lengths but only from the perspective of a stationary observer (reference frame I_0) i.e. in all experimental cases where $v \neq 0$. This conflict is traditionally resolved by the application of special relativity (SR). In order to reconcile the paradox of unequal path lengths, SR predicts the existence of measurable distortions in the structure of space and time known as lorentz contraction and time dilation. The magnitude of these effects is proportional to the lorentz factor [4] given by,

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \tag{2}$$

Equation 2 predicts that in cases where $v \approx c$, lorentz contraction and time dilation grow to infinite magnitudes. Both these effects have been confirmed to physically exist [5] [6].

5 Practical Implications

The thought experiment presented may be realised by utilising a pair of isotropic radio antennae placed within a reflective boundary of circular shape.

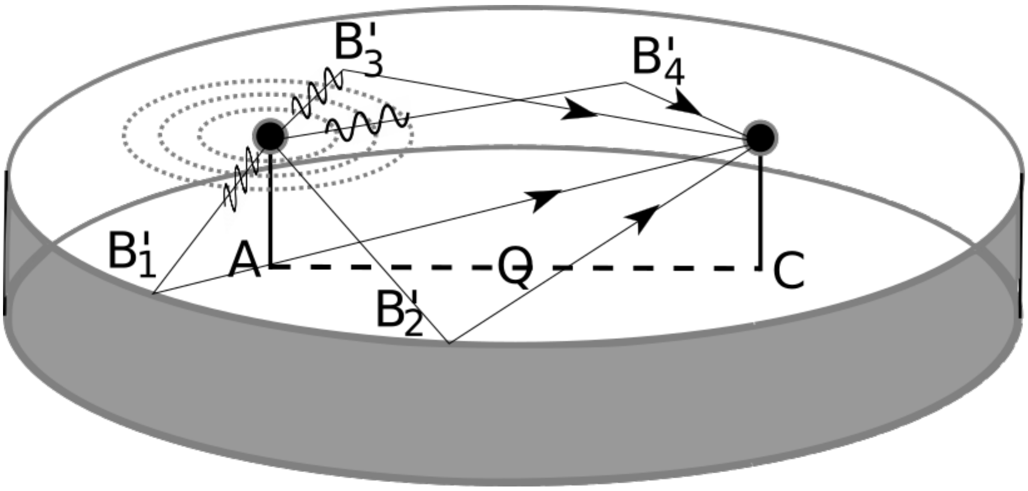

Figure 4: Two isotropic antennae placed diametrically opposite each other within a circular reflective boundary. When viewed from directly above, this physical setup is identical to fig. 2 and equivalent to a Michelson-Morley interferometer (having infinite arms) moving through space under inertial rules.

An isotropic source of electromagnetic waves is placed at some random point A within a circular shaped reflective boundary of arbitrary radius h. An isotropic receiver is placed diametrically opposite (point C). By energising the system, and according to the equivalency arguments of sec. 4 we have created an equivalent of the MM experiment with an interferometer having infinite arms. Now we invoke the symmetry of nature [7] to assert that this experimental setup must also render a null result at the receiver over all $0 \leq x < h$ or equivalently $0 \leq v < c$. Let us refer to this physical setup as the Infinite Arm Interferometer (IAI).

Since x and h in our thought experiment are equivalent to v and c in the MM experiment, the velocity of reference frame I_1 within the IAI with respect to origin Q can be expressed as a fraction of the speed of light equal to x/h. Both x and h can be obtained by physical measurements of the apparatus using a measuring rod.

When viewed from directly above, we note that within the IAI, $AB + BC \neq AB'_1 + B'_1C ... \neq AB'_i + B'_iC$ thereby presenting an equivalent paradox of unequal path lengths as observed in the MM setup. However, upon energising the system, common sense also tells us that independent of x (or equivalently v), the physically measured length AC will remain constant and the boundary of the system will remain a physical circle showing us that from the observational perspective of the stationary observer (reference frame I_0), lorentz contraction is absent.

6 Conclusion

Since lorentz contraction is absent, SR cannot be applied to reconcile the paradox of unequal path lengths presented by the IAI. This curious outcome leads us to two questions:

1. Given equivalent sequences of events within equivalent geometries, has nature abandoned her impartiality [7] and *preferred* to implement lorentz contraction in a two arm interferometer but **not** in an equivalent interferometer having infinite arms?
2. How do we reconcile the paradox of unequal path lengths presented by the IAI?

7 Statements and Declarations

The author has no competing interests to declare that are relevant to the content of this article. There are no data associated with this article.

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