Validation of the three tests of general relativity without metric tensor but using mass dependent vacuum refractive index

O. Loiselet, oplslt@mail.com
June 19, 2023

Abstract

General relativity forces us to consider a bending of space-time effect to validate gravitational lensing, gravitational red-shift and the correct value of mercury’s perihelion shift. By using Dicke’s formula and a Lagrangian which allows to compute mercury’s perihelion shift, we are able to reformulate general relativity in terms of refractive index dependent of the masses and we stay in the Euclidean space. We will present the Lagrangian in this document and we will try to investigate assumptions of its gravitational field dependency.

1 Introduction

The general relativity is an interesting theory to deal with gravitational problems. Its big prowess is to derive an accurate expression for the perihelion’s shift of Mercury, using several Taylor expansions [1], as a relativistic effect. This theory rather uses a complicated 4D space-time which is not compatible with quantum mechanics and this is an issue of consistency. Variable speed of light is an interesting alternative theory which can propose a simpler mathematical formalism, still in 3D Euclidean space, which considers simply that the refractive index of the vacuum is gravitational field dependent, as we can see in Einstein’s paper [2]. Let’s find out if, with a variable refractive index inside the relativistic Lagrangian for a test particle, we will find Newton’s law, perihelion’s shift, gravitational red-shift and gravitational lensing. Then we will try to provide a more general expression between the refractive index and the surrounding masses.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c_0$</td>
<td>speed of light in vacuum $c_0 \simeq 2.99 \times 10^8$ m/s</td>
</tr>
<tr>
<td>$c$</td>
<td>speed of light in presence of gravitational field</td>
</tr>
<tr>
<td>$n$</td>
<td>vacuum refractive index defined as $n = c_0/c$</td>
</tr>
<tr>
<td>$L_m$</td>
<td>modified relativistic Lagrangian in presence of gravitational field</td>
</tr>
<tr>
<td>$L_0$</td>
<td>relativistic free particle Lagrangian</td>
</tr>
<tr>
<td>$\Phi$</td>
<td>dimensionless gravitational potential</td>
</tr>
<tr>
<td>$G$</td>
<td>universal gravitational constant $G \simeq 6.67 \times 10^{-11}$ m$^3$ kg$^{-1}$ s$^{-2}$</td>
</tr>
<tr>
<td>$f(\Phi)$</td>
<td>speed of light dependency in gravitational field: $c = c_0 f(\Phi)$</td>
</tr>
</tbody>
</table>

Table 1: Notations
2 Slightly modified Dicke’s formula

Dicke proposed a formula in his paper to link the "refractive index" $n$ (or the product of relative permeability $\mu_r$ times permittivity $\epsilon_r$ of the medium) of the vacuum near a mass $M$ at the distance $r$ from an observer [3]:

$$n^2 = \mu_r \epsilon_r \simeq 1 + 2 \Phi$$  \hspace{1cm} (1)

with:

$$\Phi = \frac{GM}{c_0^2 r}$$  \hspace{1cm} (2)

with $c_0$ the speed of light in the absence of the mass (considered here as constant), $G$ the universal gravitation constant and $M$ a spherical mass at distance $r$ from the observer. In scattering optics or light deflection rays, we often approximate the refractive index to first order expression. Since we have:

$$\Phi \ll 1$$  \hspace{1cm} (3)

if the mass $M$ is not too big and we are not too close from it, we can state that this expression of the vacuum refractive index squared is a Taylor series of another unknown function. Taking the square root of equation (1) and Taylor expanding at first order we have the Einstein expression of refractive index [2]:

$$n \simeq 1 + \Phi$$  \hspace{1cm} (4)

In this article, which is in the frame of variable speed of light theories, we define as in optics the speed of light in vacuum $c$ by:

$$c = \frac{c_0}{n(\Phi)} \simeq \frac{c_0}{1 + \Phi} \simeq c_0 (1 - \Phi)$$  \hspace{1cm} (5)

where $c_0$ is the actual speed of light, i.e $c_0 \simeq 2.99 \times 10^8$ m/s, where there is no mass near the observer at space coordinates $r$.

2.1 Gravitational lens

This result is pretty straightforward since the squared refractive index in a medium with a mass $M$ located at distance $r$ is given at first order by Dicke’s formula:

$$n^2 = \mu_r \epsilon_r \simeq 1 + 2 \frac{GM}{c_0^2 r}$$  \hspace{1cm} (6)

So we can find the light deflection with this formula, using Fermat’s surface.

2.2 Gravitational red-shift

Using equation (4), we can check gravitational red-shift. Let us take a plane wave $\Psi$ traveling in $x$ direction near a gravitational field, the amplitude is roughly given by:

$$\Psi \simeq e^{i k n(r)x}$$  \hspace{1cm} (7)

In two dimensions with $r = \sqrt{x^2 + y^2}$, $k = 2\pi \lambda_0$, $\lambda_0$ the wavelength in free space (with no mass). Let us define the variable wavelength $\lambda(r)$ by:
\[ \lambda(r) = \lambda_0 \frac{n(r)}{1 + \Phi(r)} \quad (8) \]

then we approximate it again to first order:

\[ \lambda(r) \approx \lambda_0 \left( 1 - \frac{GM}{c_0^2 r} \right) \quad (9) \]

Next we have the derivative with respect to \( y \) (perpendicular to the direction of propagation \( x \)):

\[ \partial_y \lambda = \lambda_0 \frac{yGM}{c_0^2 r_3} \quad (10) \]

and at the position \( x = 0 \) we have \((r = |y|)\):

\[ \partial_y \lambda|_{x=0} = \lambda_0 \frac{GM}{c_0^2 y^2} = \lambda_0 \frac{g}{c_0^2} \quad (11) \]

with \( g = GM/y^2 \), then for a small variation of \( \delta y \) we find the red-shift effect:

\[ \frac{\delta \lambda}{\lambda_0} = \frac{g}{c_0^2} \delta y \quad (12) \]

### 3 Expression of the Lagrangian which takes into account

**Newton’s gravity and relativistic corrections**

Shahid-Saless and K. Yeomans [4] provided a right expression for a Lagrangian to be able to find modified Newton’s equation of motion, which takes into account classical gravitational attraction and also the relativistic correction which permits to compute Mercury’s (and other planets) perihelion’s shift:

\[ L(r, \dot{r}) = mc_0^2 \sqrt{1 - 2\Phi(r) + 2\Phi(r)^2} - (1 + 2\Phi(r)) \frac{\dot{r}^2}{c_0^2} \quad (13) \]

Now we state that:

\[ 1 - 2\Phi(r) + 2\Phi(r)^2 \simeq \frac{1}{n^2} \quad (14) \]

at second order in \( \Phi \) (since we only know the first order of \( n^2 \)) and using Dicke’s relation (1), we can write a modified Lagrangian \( L_m \) as:

\[ L_m \simeq mc_0^2 \sqrt{\frac{1}{n^2} - n^2 \frac{\dot{r}^2}{c_0^2}} \quad (15) \]

then factorizing we get:

\[ L_m = mc_0^2 \sqrt{\frac{1}{n^2} \left( 1 - n^4 \frac{\dot{r}^2}{c_0^2} \right)} \quad (16) \]

then choosing (just a sign convention since Euler-Lagrange equations remain the same):
\[
\sqrt{\frac{1}{n^2}} = \frac{1}{n}
\]  
we obtain:

\[
L_m = -mc^2_0 n \sqrt{1 - n^4 \frac{\dot{r}^2}{c^2_0}}
\]

(18)

Since we defined \(c(r) = c_0/n(r)\) we can express the Lagrangian as:

\[
L_m = -mc^2_0 \sqrt{1 - \frac{\dot{r} c_0}{c_0^2}}^2
\]

(19)

which is really close to the actual relativistic Lagrangian \(L_0\) for a free body (in the case of no gravitational force \(\Phi = 0\) so \(c = c_0\)) of mass \(m\) defined by:

\[
L_0 = -mc^2_0 \sqrt{1 - \frac{\dot{r}^2}{c_0^2}}
\]

(20)

All the gravitational energy is contained inside the refractive index, or the variable speed of light \(c(r)\). With this expression, we do not need anymore the concept of bending space-time of general relativity and get rid-off the metric-tensor formalism. A similar idea has been found using a Hamiltonian formalism in Broekaert’s paper [5] with deeper consideration as the local invariant speed of light. The deep physical concept here is from Einstein and Mach: the relation between light and matter, more precisely speed of light and mass. This is a source/field relation as we can see in Maxwell equations for instance with current/magnetic field and electric charge/electric field. Now let us investigate an hypothetical form of \(n(\Phi)\), using elementary functions candidates, because we provided only the second order expression of the refractive index.

4 Elementary functions that can fit with the exact expression of the refractive index

For mathematical simplicity, we assume that the refractive index should be monotonic with the surrounding gravitational potential. Moreover, we consider only elementary functions. With this restriction we can found expressions for \(n\) with, for examples, power law and exponential. Taking into account the second order development of \(1/n^2\) at equation (14) we can have for example for the most basic cases:

- Exponential: \(n(\Phi) = e^{\Phi}\), used by Krogh [6], Puthoff [7]

- Power law: \(n(\Phi) = \left(1 + p\Phi + \frac{p^2}{2} \Phi^2\right)^{1/p}, \ p > 0\), a parameter

Using these laws, we can express also the speed of light with the presence of mass \(M\) using \(c = c_0/n\):

- Exponential: \(c = c_0 e^{-\Phi}\)
- Power law: \[ c = c_0 \left(1 + p\Phi + \frac{p^2}{2}\Phi^2\right)^{-1/p}, \quad p > 0 \]

Let us generalize these expressions by simply states:

\[ c = c_0 f(\Phi) \]  \hspace{1cm} (21)

where \( f \) is an unknown function which fits this second order approximation (see equation (14)):

\[ f(\Phi)^2 \approx 1 - 2\Phi + 2\Phi^2 \]  \hspace{1cm} (22)

for \( \Phi << 1 \) (i.e low gravitational potential).

5 General static relation between light and masses: Poisson’s equation

We have determined previously a mass dependent velocity of light generally defined by several candidates functions \( f(\Phi) \) (equation (21)). Let us take the inverse of this function (assuming well-defined) to express it as:

\[ f^{-1}\left(\frac{c}{c_0}\right) = \Phi = \frac{GM}{c_0^2 r} \]  \hspace{1cm} (23)

We can take the Laplacian of this expression to obtain:

\[ \Delta f^{-1}\left(\frac{c}{c_0}\right) = \frac{GM}{c_0^2} \Delta \left(\frac{1}{r}\right) = -\frac{4\pi GM\delta(r)}{c_0^2} \]  \hspace{1cm} (24)

using the Laplacian’s Green’s function. So now we can generalize equation (5) with replacing the point-wise mass \( M \) by a general distribution by making the substitution \( M\delta(r) \to \eta(r) \), with \( \eta \) being the surrounding mass density around the observer to obtain:

\[ \Delta f^{-1}\left(\frac{c}{c_0}\right) = \Delta f^{-1}\left(\frac{1}{n}\right) = -\frac{4\pi G\eta(r)}{c_0^2} \]  \hspace{1cm} (25)

This gives a static source/field equation for the velocity of light, this is close to Einstein’s idea and Mach’s principle that relied the speed of light and the masses in the universe. Expression using Poisson’s equation in this topic is related to Sciama’s work \[8\] (even if it was applied specifically to Mach’s principle) and Broekaert’s paper \[5\] (with a much more complicated form which fit a gravitationally modified Lorentz transform). In this model, we still use a constant \( c_0 \). According to Mach or Dicke this constant could be linked to all the masses in the universe via the universal gravitational constant \( G \). To solve this equation, and try to derive \( c_0 \) in function of \( G \) (as in Mach’s principle), we need to define a boundary condition which is another big topic, so we prefer to stop here.

6 Conclusion

In this paper we have just considered that Dicke’s formula was a first order Taylor’s expansion of a squared quantity for refractive index and that the Shahid-Yeomans \[4\] expression provides
a second order expansion of inverse refractive index squared inside the square root of the Lagrangian. By doing so and slightly modifying the relativistic free particle Lagrangian (i.e we substitute $c$ by $c_0$ then we replace $mc^2$ by $mcc_0$, $c$ being mass dependent and $c_0 \approx 2.99 \times 10^8$ m/s the classical constant speed of light) we are able to prove gravitational lensing, gravitational red-shift, Newton’s gravity equation and relativistic perihelion shift correction for a test mass in a gravitational field at first order in $v/c$. All of these are found by general relativity, but here we do not consider a space-time curvature concept (the idea of Krogh [6]). Maybe this can be related to Lorentz ether’s concept to consider the direct effect from masses on light, without bending the space-time which modifies its trajectory.

7 Acknowledgment

I would like to address a special thanks to Alexander Unzicker and his community of followers and physicists for all their ideas in the comment section. It is clear that I am not an astrophysicist but the topic began to interest me in spring 2023, so I wanted to share thoughts with the community.

References