

Unified Dark Energy and Dark Matter from Rute as Different Manifestations of Vacuum Energy

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Abstract

Dark energy and dark matter are both described as different manifestations of vacuum energy within the framework of Rute. It describes a gravitational inversion in a dimensional symmetry and interactions regulated by constraints. In this framework, the bare vacuum energy component exists in a gravitationally inert state where actual gravitational constant $G_o = 0G$, while real particles oscillates between this inert state and the active state where $G_o = 2G$. The background effect of neutrino substrates in the inert state makes $G_o > 0G$ in such state, causing the gravitation of virtual particles with positive pressure appearing as hot and cold dark matter. A non zero component of vacuum energy also spills into the gravitationally active state as a cosmological constant form of dark energy. This is due to an energy density constraint and speed limit asymmetry between the two gravitational states that limits the capacity of the inert state to contain all the vacuum energy component. This form of dark energy and dark matter, are both parameterized by a common asymmetry parameter Γ which is suppressed in deep gravitational wells. Rute predicts a testable Gravitational Wave Reheating (GWR) mechanism in which gravitational waves produce electromagnetic secondaries in the presence of negative pressure such as that from strong magnetic fields and even dark energy. This mechanism which may explain Fast Radio Bursts (FRBs) and Excess Radio Background (ERB) is also suppressed by the positive curvature of deep gravitational wells.

Keywords:- Rute – Gravitational Inversion – Constraints – Cosmological Constant – Dark Energy – Dark Matter – Neutrino Substrate – Speed Limit Asymmetry – Energy Density Constraint – Gravitational Wave Reheating.

1. Introduction

Dark energy has remained a mystery since its first discovery in 1998 [1,2]. Several independent observations including the Planck data indicates that dark energy constitutes about 68.3% [3] of the total energy budget of the universe and driving its accelerated expansion. This is in addition to the existing mystery of dark matter which appears to be some invisible but gravitating form of matter that clusters within and around galaxies. Dark matter makes up about 26.8%, while ordinary visible matter makes up 4.6% of the mass-energy content of the universe.

The prolonged mystery of dark energy and dark matter which now accounts for about 95% of the gravitating energy content of our universe, is not for want of efforts. Rather, it is an indication that their nature is deeply rooted in the way the universe actually works.

The simplest and most economical solution to explain dark energy already exists in the form of a cosmological constant term (Λ) earlier introduced by Einstein in his field equations and forms a key part of the standard model of cosmology known as the Λ CDM model (Lambda Cold Dark Matter). Λ readily comes from the expected gravitational effect of vacuum energy predicted within the framework of quantum field theory. The problem is the extremely large value of about 120 orders of magnitude expected from our understanding of quantum field theory compared to observation. This is the Λ problem [4].

Supersymmetry (SUSY) models readily provides a cancellation mechanism to reduce the bare Λ to a very small value but they suffer from excessive finetuning. Also, supersymmetric partners of the standard model required for cancellation have not been found. There are some string theory models in which vacuum energy is simply relaxed [5], as well as some that resorts to anthropic consideration [6]. There are some models that makes the spacetime metric insensitive to Λ [7,8].

While the efforts to resolve the Λ problem intensified, there had been several alternative approaches that avoids the difficult problem of Λ . Prominent among them are quintessence which involves a slowly evolving scalar field, modified gravity models, unification of dark energy and dark matter. See [9,10] for detailed review. For recent review, see [11,12].

The amount of theoretical and observational efforts that has been applied to understand dark energy and dark matter shows that they require new Physics beyond the existing standard model of cosmology and particle physics. One of such attempts is the Rute model earlier published in its nacent form in [13] focusing on dark energy and recently extended to include dark matter.

The Rute model, takes the speed limit asymmetry approach of resolving the Λ problem in which the bare vacuum energy component exists only in a gravitationally inert state where actual gravitational constant is zero. Real standard model particles oscillates between the gravitationally inert state and gravitational active state. A non zero vacuum energy density then emerges in the gravitationally active state as dark energy due to a key energy density constraint and a speed limit asymmetry.

It describes dark matter as a different manifestation of the bare vacuum energy component that depends on the background effect of neutrino substrates on the gravitational constant. Neutrinos

particularly cosmic neutrinos illuminates the background in the gravitationally inert state such that $G_o > 0G$, making virtual particles gravitate with positive pressure and appear as cold dark matter.

The model framework describes a gravitational inversion in a dimensional symmetry that doubles large spatial dimensions with microscopic partners while their interactions are regulated by constraints. These constraints such as speed constraint, energy density constraint, volume constraint, and force constraint, simply ensures a fixed absolute value of associated physical quantities across the various dimensions and states.

This paper is organized as follows. Section 2 discusses the key dimensional symmetry that doubles the number of large spatial dimensions with microscopic partners. It also discusses the speed constraint, the 2 on and off gravitational states, their speed limit asymmetry and density constraint, which is then applied in section 4 to solve the Λ problem. Section 3 discusses General Relativity in extra dimension S_0 and explore it unique features of gravitational inversion as well as the chain of causality in gravitation. It is shown that the expansion of the timelike S_0 dimension is equivalent to the curvature of our visible spatial dimensions S_1 .

Section 4 discusses the emergence of dark energy due to a speed limit asymmetry and energy density constraint discussed in section 2, while in section 5 the key reheating prediction is discussed. Section 6 discusses the gravitation of virtual particles which appears as dark matter due to the background effect of neutrino substrates on the gravitational constant in the inert state. Summary and conclusion follows in section 7.

2. Dimensional Structure and Symmetry in Rute

In the Rute framework, the number of large spatial dimensions is described by the dimension number S_n such that,

$$S_n = [4n - 1]d \quad (1)$$

Where $n = 0, 1, 2, 3, \dots$

$n = 0$, corresponds to $S_n = -1d$ which is a timelike spatial dimension that invisible due to speed constraint discussed in section 2.1.

$n = 1$, corresponds to $S_1 = 3d$ which is our visible spatial dimensions.

$n = 2$, if it exists will correspond to $S_2 = 7d$ which will also be invisible due to a force constraint that restricts standard model particles to the visible spatial dimensions S_1 . Such invisible large dimensions however can be accessible by maximizing interactions in S_1 which minimizes the restricting interaction in S_2 according to equation 2.

$$F_P^2 = F_\mu^2 + F_\nu^2 \quad (2)$$

Where F_P is the Planck force, F_j is the resultant force a particle experiences in S_1 while F_μ is the restricting force of interaction in S_2 capable of making it inaccessible by standard model particles.

The dimensional symmetry in Rute doubles large spatial dimensions with microscopic dimensional partners with opposite dimension numbers such that the total dimension numbers of the universe is zero as illustrated in figure 1.

While the dimensional partner of S_0 , is the Planck size S_P dimension, the dimensional partner of S_1 is denoted S_α and the possible dimensional partner of S_2 will be denoted S_β .

The volume constraint implies that the large spatial dimensions such as S_1 and S_0 where inflated in the early universe at the expense of their dimensional partners S_α and S_P until they contracted to their microscopic sizes becoming invisible.

This paper focuses on the S_0 dimension and its microscopic dimensional partner S_P as well as their interactions with the visible spatial dimension S_1 .

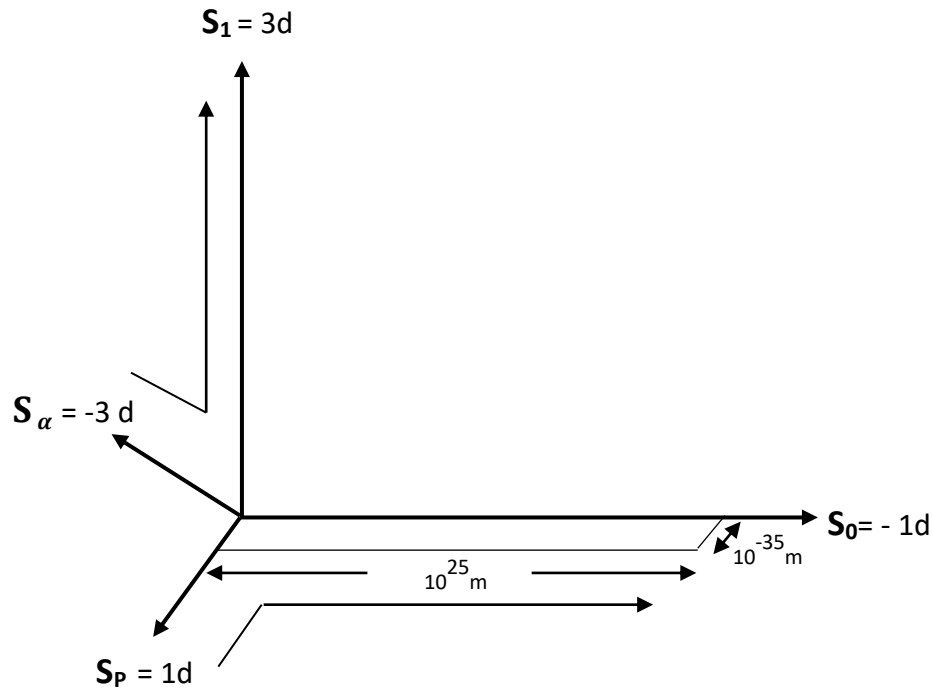


Figure 1: Dimensional symmetry in Rute showing the large spatial dimensions S_1 and S_0 and their microscopic dimensional partners. The volume constraint and gravitational inversion symmetry implies that S_P and S_α were contracted to their minimum microscopic sizes to inflate S_1 and S_0 in the early universe. Before then, they were all comparable in size. The gravitational inversion interaction is now between S_1 and S_0 as illustrated in figure 5 in section 3.1.

2.1 Speed Constraint and invisibility of S_0

Despite its cosmic size, the S_0 dimension is invisible due to its timelike behavior from the speed constraint illustrated in equation (2) and figure 2.

The speed constraint requires that a particle's velocity must always equal the maximum speed limit c in the $S_1 - S_0$ dimension.

Massless particles like photons have zero velocity component along S_0 , while massive particles and antiparticles travel in opposite directions along S_0 . This motion along S_0 is quantitatively equivalent to the passage of time such that,

$$c^2 = \mu^2 + v^2 \quad (3)$$

Where μ is particle velocity along S_0 , and v is particle velocity along visible spatial dimension S_1 .

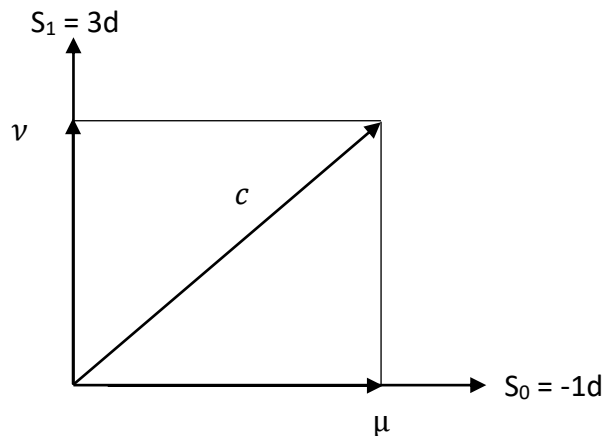


Figure 2: The speed constraint relationship between visible spatial dimensions S_1 and time like spatial dimension S_0 .

When $v \rightarrow 0$, for massive particles, $\mu \rightarrow c$ and conversely, when $v = c$ for massless particles, $\mu = 0$ to satisfy the speed constraint.

This suggests that time can be an emergent temporal dimension driven by the velocity of a particle along S_0 dimension. How fast time appears to pass for a massive particle can be equivalent to the ratio of its S_0 component of velocity μ to the speed limit c as,

$$\frac{1}{\gamma^*} = \frac{\mu}{c} \quad (4)$$

Where γ^* is the equivalent Lorentz factor for the S_0 dimension. Since

$$\mu^2 = c^2 - v^2 \quad (5)$$

$$\gamma^* = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \quad (6)$$

2.2 Speed limit Asymmetry

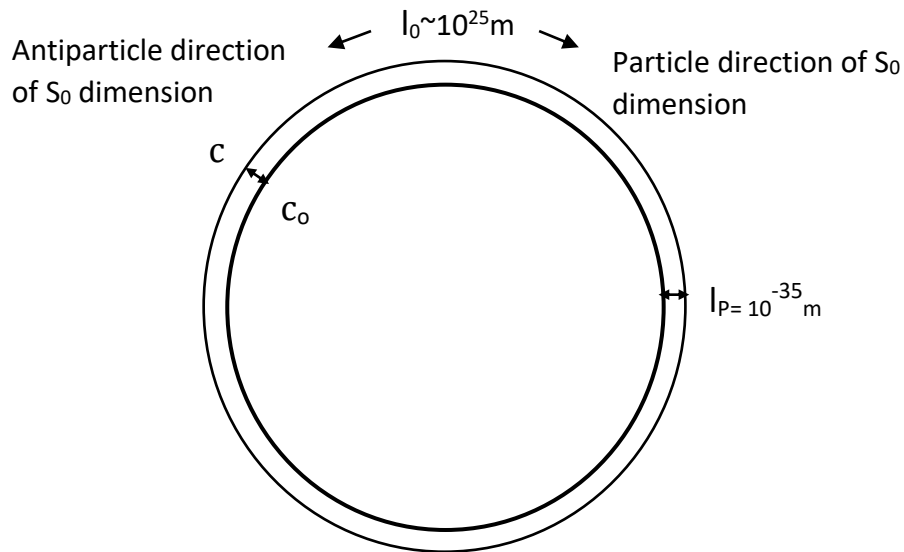


Figure 3: Ring structure of $S_0 - S_P$ dimensions with two unequal speed limit states C and C_0 , at the two ends of S_P dimension.

The asymmetry in speed limits c and c_0 is described by the asymmetry parameter Γ which is the ratio of the sizes of S_P and S_0

$$\Gamma = \frac{2\pi l_P}{l_o} \sim 10^{-60} \quad (7)$$

Where l_o is the size of S_0 and l_P is the Planck length which is also the size of the S_P dimension.

And

$$C_o = C[1 - \Gamma] \quad (8)$$

2.3 Gravitational State Oscillation

Standard model particles oscillates between the two speed states which are also opposite gravitational on and off states. This is such that for a particle of energy E , the state life time t_s for such particle is

$$t_s = \frac{4\pi\hbar}{\sqrt{E_P^2 - E^2}} \quad (9)$$

Where \hbar is the reduced Planck constant and E_P is the Planck energy

The oscillation of standard model particles between the two gravitational states $G_0 = 0G$ and $G_0 = 2G$, makes gravity discrete on microscopic spacetime scale. It however appears smooth on macroscopic spacetime scale with an average gravitational constant G .

2.4 Energy density Constraint

The energy density constraint essentially constrains the total energy density in a given volume of space to always equal the upper limit of the Planck density ρ_P .

ρ_P is only obtainable in the gravitational active state where the speed limit is c while the bare vacuum energy component exists in the inert state with lower speed limit and hence lower Planck density ρ_{P0}

$$\rho_P^2 = \rho_m^2 + \rho_{vac}^2 \quad (10)$$

Where ρ_{vac} is the vacuum energy density, and ρ_m is the total baryonic matter density.

The key significance of this is in the emergence of non zero Λ dark energy in section 4. However, such energy density constraint implies a Planck density limit to the density of black holes like that suggested in [14], where Planck stars replaces black hole singularities.

3. General Relativity in S_0 dimension

Solution of Einstein's field equations in 1+1 dimension is being used as a pedagogical tool [15, 16] and reveals some interesting properties of the equations in such dimensionality.

In this section, the focus is on general relativity in -1+1 dimension which has some inverted features not seen 1+1 dimensionality because of the negative spatial dimension. The negative dimension number of the S_0 dimension gives it some timelike features such as appearance of positive energy density as negative energy density and the inversion of positive pressure to negative pressure.

Einteins field equations in S_0 can be expressed as

$$G_{\mu\nu}^* + \Lambda^* g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}^* \quad (11)$$

Where $G_{\mu\nu}^*$ is the equivalent Einstein tensor or the curvature terms in S_0 dimension. Λ^* is the equivalent cosmological constant and $T_{\mu\nu}^*$ is the stress energy tensor as seen in S_0 dimension.

In -1+1 dimensionality, just as in 1+1, the curvature term $G_{\mu\nu}^*$ is zero, and equation (11) becomes

$$\Lambda^* g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}^* \quad (12)$$

Which is essentially a linear algebraic equation and can be further decomposed into a spatial and a time component which is not feasible in 3 +1 dimensionality [16].

$$\Lambda^* g_{xx} = \frac{8\pi G}{c^4} T_{xx}^* \quad (13)$$

$$\Lambda^* g_{tt} = \frac{8\pi G}{c^4} T_{tt}^* \quad (14)$$

3.1 Features of General Relativity in S_0 dimension

The negative one dimensionality of S_0 confers on it some unique features such as:

A. Energy in S_1 = Energy everywhere in S_0

An energy density associated with a given Planck volume of 3d space S_1 appears as an equivalent energy density everywhere along S_0 dimension associated with it as illustrated in figure 4. This makes any form of energy density and positive pressure in S_1 appear as a cosmological constant in S_0 .

B. Positive Energy Density in S_1 = Negative Energy Density in S_0

Due to the negative dimensionality, any positive energy density in S_1 is inverted and appears as negative energy density which drives the positive cosmological constant expansion of S_0 .

C. Positive Pressure in S_1 = Negative Pressure in S_0

Any form of positive pressure in S_1 appears as Negative pressure in S_0 for the same reason of negative dimensionality. The result is a positive cosmological constant expansion of S_0 .

D. Negative Pressure in S_1 = Positive Pressure in S_0

The same negative dimensionality causes the inversion of negative pressure in S_1 appearing as positive pressure in S_0 driving its negative cosmological constant contraction.

E. Curvature term $G_{\mu\nu}$ in S_1 = $\Lambda^* g_{\mu\nu}$ in S_0

The inversion of roles also results in the curvature term in S_1 being equivalent to positive cosmological constant expansion S_0 as illustrated in figure 5 and equation (15).

F. Positive cosmological constant $(+\Lambda)$ in S_1 = $-\Lambda^* g_{\mu\nu}$ in S_0

A positive cosmological constant in S_1 is equivalent to a negative cosmological constant in S_0 as illustrated in figure 5 and equation (16).

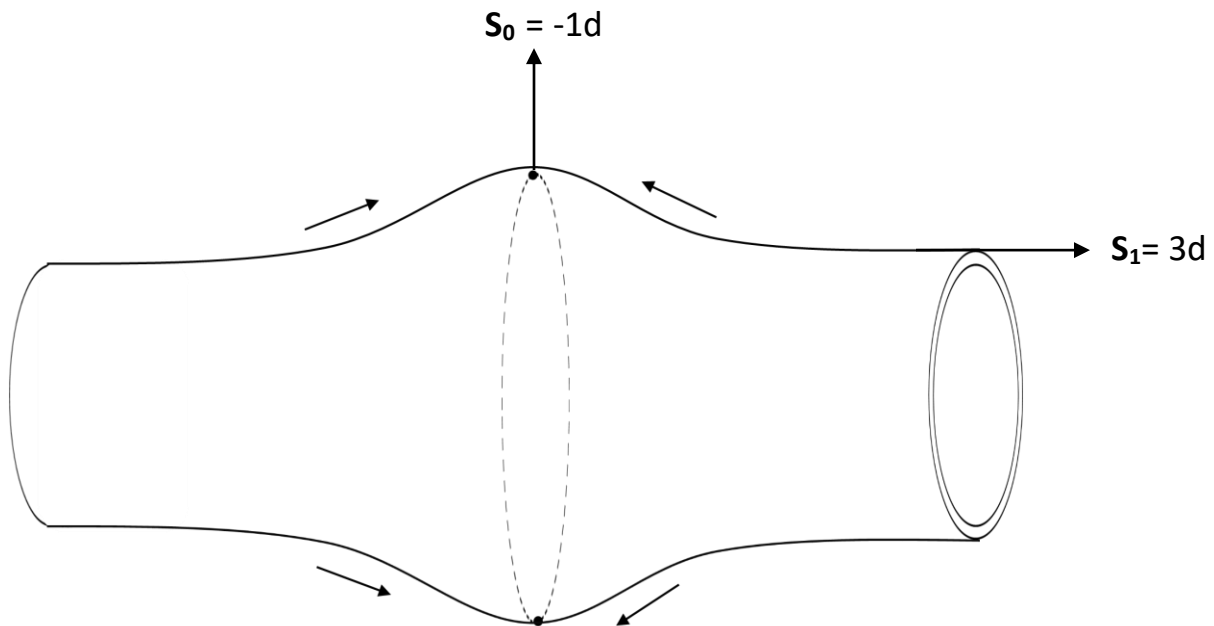


Figure 4: A gravitational well in S_1 is inverted into a gravitational hill with the expansion of S_0 by a test particle. The test particle is replicated about 10^{25} times everywhere along S_0 dimension like time.

$$G_{\mu\nu} + \Lambda^* g_{\mu\nu} = 0 \tag{15}$$

and

$$\Lambda g_{\mu\nu} - \Lambda^* g_{\mu\nu} = 0 \tag{16}$$

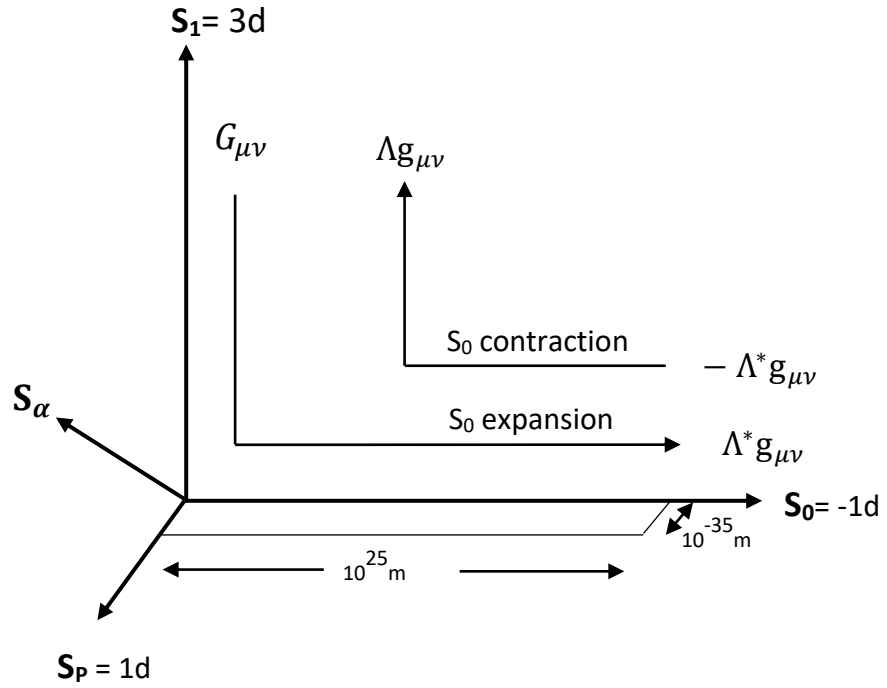


Figure 5: Gravitational inversion. A positive cosmological constant expansion of S_0 is equivalent to curvature of S_1 . Also, a negative cosmological constant contraction of S_0 is equivalent to a positive cosmological constant expansion of S_1 , making the multidimensional volume of the universe constant in a volume constraint.

3.2 Gravitational Chain of Causality

The inversion of gravitation discussed in the previous section suggests that fundamentally, gravity is the expansion or contraction of S_0 dimension. This then manifests in an inverted form as curvature or expansion of S_1 dimension as illustrated in figure 5.

The existence of gravitational on and off states acts as a filter in this gravitational chain of causality enabling real particles to gravitate while virtual particles are inert.

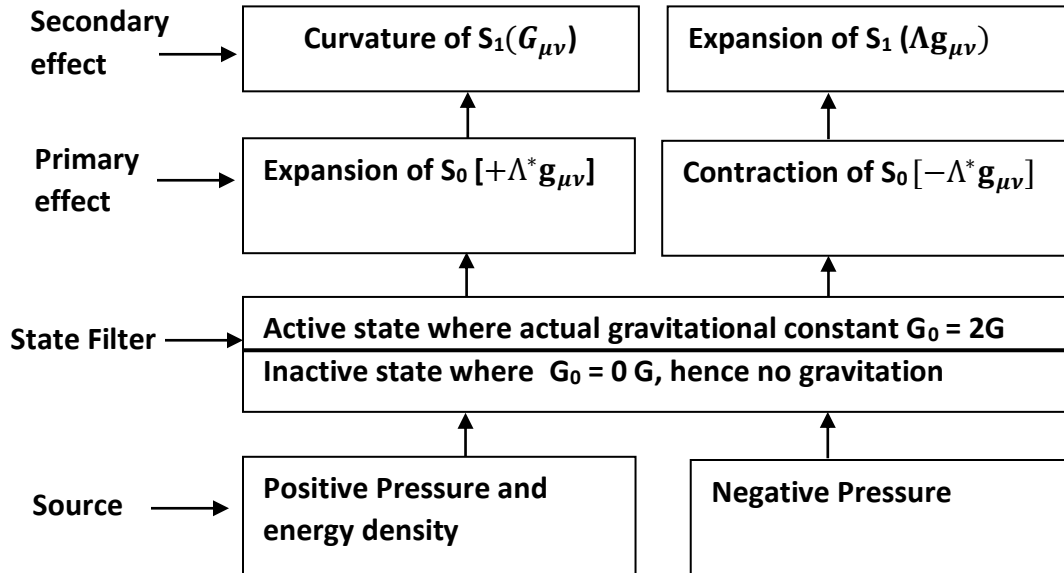


Figure 6: Gravitational chain of causality: gravity at a more fundamental level is about expansion or contraction of S_0 which manifests invertedly in S_1 as curvature or expansion respectively. Also gravitation of particles depends on if they are in gravitationally active or inert state.

The relatively permanent existence of the bare vacuum energy component in the gravitationally inert state ensures that it does not gravitate. This partly solves the cosmological constant problem. The second part of the problem about the non zero value is addressed in the next section.

4. Emergence of non zero cosmological constant

The asymmetry in speed limit described in equation (8) between the two gravitational states also reflects in the values of their Planck densities such that

$$\rho_{P0}^2 = \rho_P^2 [1 - \Gamma^4] \quad (17)$$

Where ρ_{P0} is the lower Planck density in C_0 state and ρ_P is the Planck density in the C state.

The existence of the bare vacuum energy component in the lower C_0 state implies that

$$\rho_{vac} = \rho_{P0} \quad (18)$$

Which is less than the Planck density and the energy density constraint requires that the total vacuum energy density should be equal to the Planck density in the absence of matter.

Since the gravitationally inert lower speed state lack the capacity to contain all the vacuum energy components, a small component spills into the gravitationally active state as dark energy.

$$\rho_{DE} = \rho_P \Gamma^2 \quad (19)$$

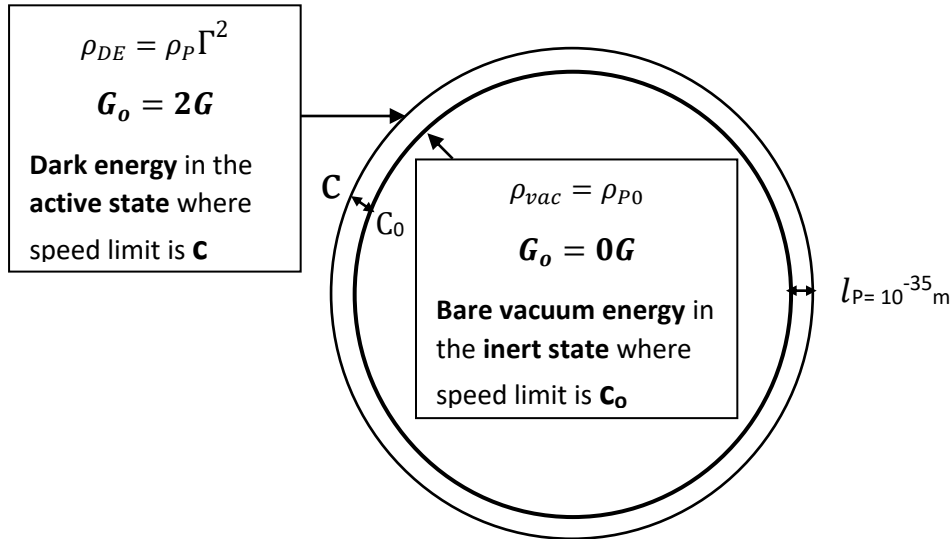


Figure 7: While the bare vacuum energy component occupy the gravitationally inert state C_0 to its limit, there has to be a component $\rho_P \Gamma^2$ in the gravitationally active state C to satisfy the energy density constraint so that the total vacuum energy density equals the Planck density ρ_P . The lower Planck density ρ_{P0} is associated with speed state C_0 .

And since Γ is suppressed with expansion of S_0 in the gravitational well of massive objects (equation (7) and figure 4), the density of this form of dark energy varies spatially according to this suppression. The deeper the gravitational well the more the suppression. And

$$\rho_{DE} = \int [\rho_P \Gamma^2] dV \quad (20)$$

5. Gravitational Wave Reheating (GWR) Prediction

The propagation of gravitational waves along the visible 3d space S_1 should naturally vibrate any extra spatial dimension such as the S_P and S_0 dimensions. Stretching of S_P dimension during such vibration will cause a squeeze out of vacuum energy as photons. This is the Gravitational Wave Reheating (GWR) mechanism of Rute.

However as illustrated in figure 7, the expansion of S_0 dimension in gravitational wells suppresses the possible stretching of S_P dimension resulting in threshold value of strain h and frequency f_g below which reheating cannot occur.

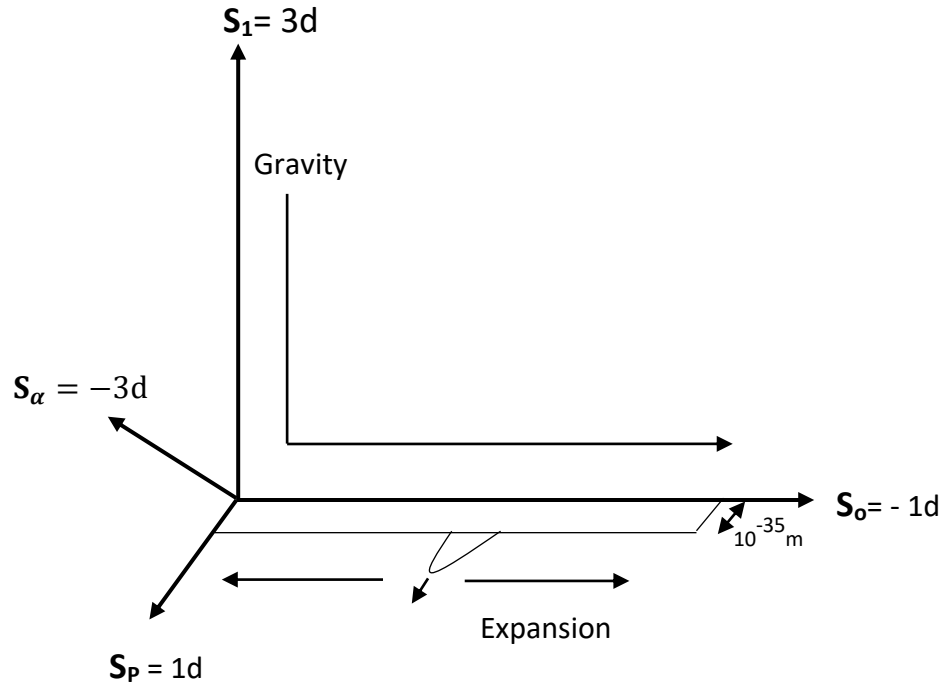


Figure 8: The expansion of the S_0 dimension in gravitational wells, suppresses the possible stretching of S_P dimension by gravitational waves oscillations.

If the S_P dimension stretches, it will reduce the value of the Planck density and the energy density constraint will squeeze out a component of vacuum energy as standard model photons.

5.1 Energy Scale of Reheat Photons

The maximum energy of the reheat photons that can be emitted in the resulting spectrum can be described by

$$E_{max} = h_0 E_P \quad (21)$$

Where E_P is the planck energy and h_0 is the effective strain for the S_P dimension. And

$$h_0 = \frac{hf_g - H_0^*}{f_g} \quad (22)$$

Where f_g is the gravitational wave frequency and h is the strain and both represent the threshold that has to be exceeded for reheating to occur.

H_0^* is the equivalent Hubble constant associated with each cubic planck volume describing the expansion rate of S_0 dimension.

H_0^* is proportional to the curvature of the visible 3d spatial dimension S_1 and hardens the S_P dimensions against gravitational wave vibration.

5.2 Magnetic Softening of the S_P Dimension

The magneto-curvature coupling aspect of general relativity was explored based on its tendency to flatten curvature and even dampen gravitational waves [17]. The key aspects of this coupling, is the vector nature of the magnetic field and the magnetic tension that tends to straighten the field lines against distortion by curved spacetime background.

Essentially the distortion of the field lines by curved background or a passing gravitational wave causes a backreaction from such magnetic field, flattening the curvature or ripples in the fabric of spacetime. This dampens the gravitational wave amplitude and energy, and can be seen as the hardening of the visible spatial dimensions against gravitational oscillation.

Within the Rute framework, the expansion rate H_0^* of the S_0 dimension is a measure of spatial curvature. Therefore the flattening of spacetime curvature suppresses H_0^* in equation (22), resulting in the softening of S_P dimension which becomes more sensitive to gravitational wave oscillation. Indeed with a flat geometry which implies $H_0^* = 0$, equation (22) becomes

$$h_0 = h \quad (23)$$

where the strain h_0 of S_P vibration becomes equal to the strain h of the incident gravitational wave resulting in maximum reheating.

In essence, a strong magnetic field like that obtainable around magnetars, softens the S_P dimension by lowering the threshold strain and frequency required for reheating to occur for incident gravitational waves.

Strictly from energy conservation perspective, the energy lost by gravitational waves passing through a strongly magnetized region has to be released in some form and Rute's GWR provides the mechanism for converting the gravitational wave energy into electromagnetic one. It is expected that the brief burst of electromagnetic waves radiated while the gravitational wave passes through a strongly magnetized region such as magnetars, encodes information about the incident gravitational waves. These burst of electromagnetic radiation can appear as Fast Radio Burst (FRBs) or even Gamma Ray Burst (GRBs) depending on source proximity or strain and frequency of the incident gravitational waves, making magnetars possible astrophysical gravitational wave detectors.

6. Dark Matter from Vacuum Energy

The bare vacuum energy component should always be gravitationally inert with its existence in the $G_0 = 0G$ state where the gravitational field is effectively switched off. However, in this framework, particle substrates of the standard model such as neutrinos affects their background while in the inert state such that $G_0 > 0G$.

Within such background in the inert state, neutrinos provides some gravitational illumination described by equation (24) that enables virtual particles to gravitate with positive pressure appearing as dark matter.

$$G_\nu = \beta \Gamma^2 G \quad (24)$$

Where G_ν is the induced gravitational constant in the inert state due to the background effect of neutrino substrates. The value of G_ν radially falls off as the inverse square law from each neutrino. β is a dimensionless constant and the value is particle dependent, while Γ is the same asymmetry parameter involved in the emergence of dark energy.

As mentioned earlier in previous sections, Γ suffers suppression within the deep gravitational wells of astrophysical objects. This in turn suppresses the substrate effect of neutrinos according to equation (24) and hence the near absence of dark matter effect in such wells.

This enables this substrate dependent form of dark matter to mimic Modified Newtonian Dynamics (MOND) form while also exhibiting some particle behavior of its neutrino substrates.

while hot dark matter is the form of dark matter that can be produced by relativistic neutrinos, cosmic neutrinos are unrelativistic. Such non relativistic cosmic neutrino substrates should be further slowed down by the Higgs like drag from its self induced gravitational interaction with virtual particles within its background. This enhances the clustering of cosmic neutrino substrates and the gravitating virtual particles in their background appear as cold dark matter.

This substrate dependent dark matter tends to have a hybrid behavior between particle dark matter and MOND dark matter like the superfluid dark matter described in [18]. The details of its unique properties will be discussed in subsequent papers.

7. Summary and Conclusion

Rute provides an elegant framework for the resolution of the cosmological constant problem of dark energy and the nature of dark matter. In doing so it provides a deeper insight into the dimensional structure of spacetime and chain of causality involved in gravitation. Specifically, it places the bare vacuum energy component in a state where the gravitational field is switched off with actual gravitational constant $G_0 = 0G$, while real standard model particles oscillate between this gravitationally inert state and the active state.

Due to an energy density constraint and a speed limit asymmetry that limits the capacity of the inert state to contain all the vacuum energy component, a small component spills into the active state as dark energy. The asymmetry parameter emerges from a key dimensional symmetry and it is the ratio of the size of the spatial equivalent of time S_0 and its microscopic partner S_P .

The illuminating effect of neutrino substrates particularly cosmic neutrinos, induces non zero gravitational constant in the inert state, providing gravitation for virtual particles which appears as dark matter. This baryonic substrate dependent form of dark matter can exhibit a hybrid characteristics of particle dark matter and MOND form of dark dark matter like superfluid dark matter [18].

The reheating prediction in which gravitational waves produce electromagnetic secondaries with negative pressure from strong magnetic fields of magnetars and dark energy might be the sources of Fast Radio Bursts (FRBs) [19,20] and Excess Radio Background (ERB) [21].

In conclusion, the Rute framework offers new physics explanations for dark energy and dark matter as different manifestations of vacuum energy that can be tested and provides deep insights into the dimensional structure of spacetime and gravity.

8. References

- [1] Riess, A. G. *et al.*, 1998, Observational Evidence from Supernovae for an Accelerating Universe and a Cosmological Constant, *Astron. J.* **116**, 1009-1038, arXiv: astro-ph/9805201
- [2] Perlmutter, S. *et al.*, 1999, Measurement of Omega and Lambda from 42 High-Redshift Supernovae, *Astrophys. J.* **517**, 565, arXiv:astro-ph/9812133.
- [3] Ade, P. A. R. *et al.*, 2013, Planck 2013 results. XVI. Cosmological Parameters, arXiv:1303.5076[astro-ph.CO].
- [4] Carroll, S. M., 2001, The cosmological constant, *livingRev.Rel.*4:1, arXiv:astro-ph/0004075
- [5] Kachru, S. *et al.*, 2003, de Sitter Vacua in String Theory, *Phys. Rev.* **D68**, 046005, arXiv:hep-th/0301240.
- [6] Banks, T. *et al.*, 2001, On Anthropic Solutions of the Cosmological Constant Problem, *JHEP*, 0101, 031, arXiv:hep-th/0007206.

- [7] Arkani-Hamid, N., 2000, A small cosmological constant from a large extra dimension, Phys. Lett. **B480**, 193-199, arXiv:hep-th/000197.
- [8] Kachru, S. *et al.*, 2000, Self-tuning flat domain walls in 5-D gravity and string theory, Phys. Rev. **D62**, 045021, arXiv:hep-th/0001206,
- [9] Copeland, E. J. *et al.*, 2006, Dynamics of dark energy, Int. J. Mod. Phys. **D15**, 1753-1936, arXiv:hep-th/0603057.
- [10] Huterer, D., Shafer, D. L., 2018, Dark energy 2 decades after: Observables, Probes, Consistency Tests, Rep. Prog. Phys. **81** 016901
- [11] Oks E., 2021, Brief review of recent advances in understanding dark matter and dark energy, New Astronomy Reviews, **93**, 101632, arXiv:2111.00363[astro-ph.CO].
- [12] Arun K. *et al.*, 2017, Dark matter, dark energy and alternate models: a review, Advances in Space Research, **60**, 166-186, arXiv: 1704.06155[physics.gen-ph]
- [13] Umar K. A., Ayatunji B. G., Chizea F. D., 2019, Nature of Dark Energy, SSRG Int. J. App. Phys. **6 (3)**, 1-8
- [14] Rovelli, C., Vidoto, F., 2014, Planck Stars, Int. J. Mod. Phys. **D23**,1442026, arXiv:1401.6562
- [15] Boozer A. D., 2008, General relativity in (1+1) dimension, Eur. J. Phys. **29** 319-333
- [16] Mellinger R. 2012, Aspects of general relativity in (1+1) dimensions, Bulletin of the American Physical Society.
- [17] Tsagas C. G., 2001, Magnetic Tension and the Geometry of the Universe, Phys. Rev. Lett. **86**, 5421
- [18] Hossenfelder S., Mistele T., 2020, The Milky Way's rotation curve with superfluid dark matter, MNRAS **498**, 3484-349, arXiv:2003.07324[astro-ph.GA]
- [19] Zhang B., 2022, The Physics of Fast Radio Bursts, arXiv:2212.03972[astro-ph.HE]
- [20] Petroff E., 2021, Fast Radio Burst at the dawn of the 2020s, arXiv:2107.10113[astro-ph.HE]
- [21] Fialkov A., Barkana R., 2019, Signature of Excess Radio Background in the 21-cm global signal and power spectrum, arXiv: 1902.02438[astro-ph.CO]