Is it Possible to Arbitrarily Slow Down Time in a Limited Volume With an Energy-Impulse Tensor Whose Components Can be Reduced Arbitrarily?

Part I:Introduction

Gianluca Perniciano*

Department of Physics of the University of Cagliari, Italy

Abstract

A solution is presented that describes a region of space, box or warp bubble, where time gets slowed down by an arbitrary factor, while reducing the components of the energy-impulse tensor by any chosen amount.

1 Introduction:

This papers investigates if it is possible, in the framework of general relativity, to slow down time by an arbitrary factor with respect to an observer in an inertial reference frame within a warp bubble, if the energy-impulse tensor has negative or positive components and if they can be reduced by an arbitrary value and a way to test this in a laboratory.

Note: all notations here are those used by Landau and Lifshitz in the second book ("The Classical Theory of Fields") of their well known Course of Theoretical Physics [2].

*email:g.perniciano@gmail.com

We start with the metric

$$ds^2 = a(x, y, z)\, dt^2 - b(x, y, z)^2\, dx^2 - b(x, y, z)^2\, dy^2 - b(x, y, z)^2\, dz^2 \qquad (1)$$

The differential proper time is:

$$d\tau = a(a, y, z)\, dt \qquad (2)$$

$a(x, y, z)$ is chosen in a simplified way (for a sphere with radius R and thickness $\Delta \ll 1$) as:

- 1)- $a(x, y, z) = 1$ for every $r > R + \frac{\Delta}{2}$
- 2)- $a(x, y, z) = 1$ for every r such that $R - \frac{\Delta}{2} < r < R + \frac{\Delta}{2}$ (cavity)
- 3)- $a(x, y, z) = A = constant \ll 1$ for $0 < r < R - \frac{\Delta}{2}$

then we get:

- 1)- $\tau = A t$ within the ball $0 < r < R - \frac{\Delta}{2}$,(zero initial conditions)
- 2)-Where $r = (x^2 + y^2 + z^2)^{\frac{1}{2}}$

t is the coordinate time in the exterior of the warp bubble (that is, proper time for observers whose reference frame is inertial, i.e., moving in a very weak gravitational field at a speed which is far smaller than the speed of light). τ tau is the proper time within the warp bubble (in our case).

When A<<1 the proper time becomes very small and it is like an hibernation of time within the warp bubble, we could call a chamber in this situation a "stasis chamber".

2 Appendix I:

We choose a spherically symmetric warp bubble with a shell having radius R and thickness

$$\Delta \ll 1$$

with the following convergence values , $r = (x^2 + y^2 + z^2)^{\frac{1}{2}}$.

for $b(x, y, z)$:

- 1)- $b(x, y, z) = 1$ for every r such that $r > R + \frac{\Delta}{2}$
- 2)- $b(x, y, z) \gg 1$ for every r such that $R - \frac{\Delta}{2} < r < R + \frac{\Delta}{2}$ (cavity wall)
- 3)- $b(x, y, z) = 1$ for every r such that $0 < r < R - \frac{\Delta}{2}$

3 The Einstein Tensor in contravariant form is:

:

$$G^{tt} = -\frac{1}{a(x,y,z)^2\, b(x,y,z)^4}\left(2\left(\frac{\partial^2}{\partial y^2} b(x,y,z)\right) b(x,y,z) - \left(\frac{\partial}{\partial x} b(x,y,z)\right)^2\right.$$

:

$$+ 2\left(\frac{\partial^2}{\partial x^2} b(x,y,z)\right) b(x,y,z) + 2\left(\frac{\partial^2}{\partial z^2} b(x,y,z)\right) b(x,y,z) - \left(\frac{\partial}{\partial y} b(x,y,z)\right)^2$$

$$\left. - \left(\frac{\partial}{\partial z} b(x,y,z)\right)^2\right)$$

$$G^{xx} = \frac{1}{b(x,y,z)^6\, a(x,y,z)}\left(2\left(\frac{\partial}{\partial x} a(x,y,z)\right)\left(\frac{\partial}{\partial x} b(x,y,z)\right) b(x,y,z)\right.$$

$$+\left(\frac{\partial^2}{\partial y^2} b(x,y,z)\right) a(x,y,z)\, b(x,y,z) + \left(\frac{\partial}{\partial x} b(x,y,z)\right)^2 a(x,y,z)$$

$$+\left(\frac{\partial^2}{\partial z^2} b(x,y,z)\right) a(x,y,z)\, b(x,y,z) + \left(\frac{\partial^2}{\partial y^2} a(x,y,z)\right) b(x,y,z)^2 - \left(\frac{\partial}{\partial y} b(x,y,z)\right)^2 a(x,y,z)$$

$$+\left(\frac{\partial^2}{\partial z^2} a(x,y,z)\right) b(x,y,z)^2 - \left(\frac{\partial}{\partial z} b(x,y,z)\right)^2 a(x,y,z)\Bigg)$$

$$G^{x\,y} = -\frac{1}{b(x,y,z)^6\, a(x,y,z)}\Bigg(\left(\frac{\partial^2}{\partial y\,\partial x} a(x,y,z)\right) b(x,y,z)^2 - \left(\frac{\partial}{\partial y} b(x,y,z)\right)\left(\frac{\partial}{\partial x} a(x,y,z)\right) b(x,y,z)$$

$$-\left(\frac{\partial}{\partial x} b(x,y,z)\right)\left(\frac{\partial}{\partial y} a(x,y,z)\right) b(x,y,z) - 2\left(\frac{\partial}{\partial y} b(x,y,z)\right)\left(\frac{\partial}{\partial x} b(x,y,z)\right) a(x,y,z)$$

$$+\left(\frac{\partial^2}{\partial y\,\partial x} b(x,y,z)\right) a(x,y,z)\, b(x,y,z)\Bigg)$$

$$G^{x\,z} = -\frac{1}{b(x,y,z)^6\, a(x,y,z)}\Bigg(\left(\frac{\partial^2}{\partial z\,\partial x} a(x,y,z)\right) b(x,y,z)^2 - \left(\frac{\partial}{\partial z} b(x,y,z)\right)\left(\frac{\partial}{\partial x} a(x,y,z)\right) b(x,y,z)$$

$$-\left(\frac{\partial}{\partial x} b(x,y,z)\right)\left(\frac{\partial}{\partial z} a(x,y,z)\right) b(x,y,z) - 2\left(\frac{\partial}{\partial z} b(x,y,z)\right)\left(\frac{\partial}{\partial x} b(x,y,z)\right) a(x,y,z)$$

$$+\left(\frac{\partial^2}{\partial z\,\partial x} b(x,y,z)\right) a(x,y,z)\, b(x,y,z)\Bigg)$$

$$G^{yy} = \frac{1}{b(x,y,z)^6\, a(x,y,z)} \left(2 \left(\frac{\partial}{\partial y} a(x,y,z) \right) \left(\frac{\partial}{\partial y} b(x,y,z) \right) b(x,y,z) \right.$$

$$+ \left(\frac{\partial^2}{\partial x^2} b(x,y,z) \right) a(x,y,z)\, b(x,y,z) + \left(\frac{\partial}{\partial y} b(x,y,z) \right)^2 a(x,y,z)$$

$$+ \left(\frac{\partial^2}{\partial z^2} b(x,y,z) \right) a(x,y,z)\, b(x,y,z) + \left(\frac{\partial^2}{\partial x^2} a(x,y,z) \right) b(x,y,z)^2 - \left(\frac{\partial}{\partial x} b(x,y,z) \right)^2 a(x,y,z)$$

$$\left. + \left(\frac{\partial^2}{\partial z^2} a(x,y,z) \right) b(x,y,z)^2 - \left(\frac{\partial}{\partial z} b(x,y,z) \right)^2 a(x,y,z) \right)$$

$$G^{yz} = - \frac{1}{b(x,y,z)^6\, a(x,y,z)} \left(\left(\frac{\partial^2}{\partial z\, \partial y} a(x,y,z) \right) b(x,y,z)^2 - \left(\frac{\partial}{\partial z} b(x,y,z) \right) \left(\frac{\partial}{\partial y} a(x,y,z) \right) b(x,y,z) \right.$$

$$- \left(\frac{\partial}{\partial y} b(x,y,z) \right) \left(\frac{\partial}{\partial z} a(x,y,z) \right) b(x,y,z) - 2 \left(\frac{\partial}{\partial y} b(x,y,z) \right) \left(\frac{\partial}{\partial z} b(x,y,z) \right) a(x,y,z)$$

$$\left. + \left(\frac{\partial^2}{\partial z\, \partial y} b(x,y,z) \right) a(x,y,z)\, b(x,y,z) \right)$$

$$G^{zz} = \frac{1}{b(x,y,z)^6\, a(x,y,z)} \left(2 \left(\frac{\partial}{\partial z} a(x,y,z) \right) \left(\frac{\partial}{\partial z} b(x,y,z) \right) b(x,y,z) \right.$$

$$+ \left(\frac{\partial^2}{\partial x^2} b(x,y,z) \right) a(x,y,z)\, b(x,y,z) + \left(\frac{\partial}{\partial z} b(x,y,z) \right)^2 a(x,y,z)$$

$$+\left(\frac{\partial^2}{\partial y^2} b(x,y,z)\right) a(x,y,z)\, b(x,y,z) + \left(\frac{\partial^2}{\partial x^2} a(x,y,z)\right) b(x,y,z)^2 - \left(\frac{\partial}{\partial x} b(x,y,z)\right)^2 a(x,y,z)$$

$$+\left(\frac{\partial^2}{\partial y^2} a(x,y,z)\right) b(x,y,z)^2 - \left(\frac{\partial}{\partial y} b(x,y,z)\right)^2 a(x,y,z)\Bigg)$$

Einstein Equation

$$G^{ik} = \frac{8\pi G}{c^4} T^{ik} \qquad [2]$$

If $b = b(x,y,z) \gg 1$ and $\partial b/\partial x^i \le b$, $\partial^2 b/\partial x^i \partial x^k \le b$, $x^i, x^k = x, y, z$, $i,k = 1,2,3$ in the cavity wall of the warp bubble or box the components of the impulse-energy tensor can be reduced by an arbitrary amount.

In the spherical coordinates, the metric (1) is, for a spherical chamber :

$$ds^2 = a(r)^2 dt^2 - b(r)^2 dr^2 - b(r)^2 r^2 d\theta^2 - b(r)^2 r^2 \sin(\theta)^2 d\phi^2$$

the value for a(r) and b(r) are given on pag 2 and 3

The components of the Einstein tensor are:

$$G^{tt} = -\frac{2\frac{d^2}{dr^2} b(r) b(r) r - \left(\frac{d}{dr} b(r)\right)^2 r + 4 b(r) \frac{d}{dr} b(r)}{a(r)^2 b(r)^4 r}$$

$$G^{rr} = \frac{2 b(r)\left(\frac{d}{dr} b(r)\right)\left(\frac{d}{dr} a(r)\right) r + \left(\frac{d}{dr} b(r)\right)^2 a(r) r + 2 b(r)^2 \left(\frac{d}{dr} a(r)\right) + 2 b(r)\left(\frac{d}{dr} b(r)\right) a(r)}{b(r)^6 r a(r)}$$

$$G^{\vartheta\vartheta}=\frac{(\frac{d^2}{dr^2}b(r))b(r)a(r)r+(\frac{d^2}{dr^2}a(r))b(r)^2r-(\frac{d}{dr}b(r))^2a(r)r+b(r)^2(\frac{d}{dr}a(r))+b(r)(\frac{d}{dr}b(r))a(r)}{b(r)^6r^3a(r)}$$

$$G^{\varphi\varphi}=\frac{(\frac{d^2}{dr^2}b(r))b(r)a(r)r+(\frac{d^2}{dr^2}a(r))b(r)^2r-(\frac{d}{dr}b(r))^2a(r)r+b(r)^2(\frac{d}{dr}a(r))+b(r)(\frac{d}{dr}b(r))a(r)}{b(r)^6r^3\sin(\vartheta)^2a(r)}$$

The components are now only four, which simplifieds the problem. Howerer, the energy density could be negative, and the three pressures must be (by assumtion) all positive in the warped region of the warp bubble.

As can be seen, time flows more slowly inside the warp bubble compared to outside of it. This means that the structure could be calibrated so that, for example, 5 minutes pass inside the warp bubble while a month passes outside in an inertial system or on Earth (which is nearly inertial, with low speeds compared to the speed of light and very weak gravitational fields). It would be as if an observer inside the bubble had traveled into the future. This could be interesting for potential experimental verification. Essentially, this is a machine for time travel into the future.

4 Conclusion:

This paper shows that is it possible to slow down time by an arbitrary factor up to approaching the "freezing" of time (hibernation of time) in a limited volume (a warp bubble or box) with respect to an inertial observer and to reduce the components of the energy-impulse tensor by an arbitrary value within the cavity wall of the warp bubble or box (with components which are partially negative, thus requiring esotic matter). The speed of light in the volume containing exotic matter is strongly slowed down (something that can be compensated) and this can be a problem depending on how much one wants to reduce the components of the energy-impulse tensor.

References

[1] Steven Weinberg, Gravitation and Cosmology, Principles and Applications of the General Theory of Relativity, John Wiley & Sons, New York, 1972.

[2] L. D. Landau & E. Lifschitz, Course of Theoretical Physics Series, book 2: The Classical Theory of Fields, Butterworth-Heinemann, 1980.

[3] Ray d'Inverno, Introducing Einstein Relativity, Clarendon Press, 1992.

[4] Hans Stephani, Relativity, Special and General, Cambridge University Press, 2004.

[5] Adler, Bazin, Schiffer, Introduction to General Relativity, International Student Edition, McGraw-Hill Kogakusha, LTD., 1975.

[6] Hans C. Ohanian, and Remo Ruffini, Gravitation and spacetime, W. Norton & Company, 1994.

[7] Van den Broeck, Chris (1999). "On the (im)possibility of warp bubbles". arXiv:gr-qc/9906050

[8] C. Van Den Broeck, Class. Quantum Grav. 16 (1999) 3973