The twin fallacy

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Abstract

In this paper it is proposed that the clock or age difference predicted in the well known *twin paradox* thought experiment of special relativity is not a real effect, but only arises because proper clock rate changes when a real clock is transported to a state of higher kinetic energy have not been considered. The kinematic time dilation of SR given by the factor $\sqrt{(1 - v^2/c^2)}$ is cancelled exactly by an increase in proper clock rate by a factor $1/\sqrt{(1 - v^2/c^2)}$ that arises due to an increase of optical electron transition frequency when taking into account the relativistic mass increase of a moving atomic clock.

1 Introduction

Having stated the essence of this paper in the Abstract, I shall now discuss some of the background arguments and experiments pertaining to the issue. As far as the scientific community is concerned, the so-called *twin paradox* [1] was settled many years ago in favour of there being an age difference. The idea is that if a twin goes off somewhere at a relativistic speed and returns to the starting point, he or she will have aged less than the twin who was left behind. This prediction, derived using Albert Einstein's theory of special relativity SR [2], is important since it seems to suggest all sorts of (probably non-falsifiable) science fiction ideas, e.g. being able to get to vastly distant places in the universe in one's own lifetime, and back again. Understandably, many people historically have disputed the age difference, since SR is a theory that deals only with the symmetrical relative motion of inertial frames of reference, where neither frame is preferred, and so if a twin

X is not as old as twin Y, and twin Y is not as old as twin X, then logically they must be the same age. However, an asymmetry does occur in the thought experiment, making the frames distinguishable, and leading to the clock difference, but this is then not strictly within the limits of applicability of SR.

The issue has been discussed exhaustively over the past one-hundred and eighteen years since Einstein first mooted the idea in 1905, and it would therefore seem naive of me to raise the topic yet again. However, I shall propose in this paper that a fundamental step has been omitted from the conventional arguments, and that when this is taken into account, the kinematical clock difference predicted by SR disappears.

2 Background theory

2.1 Time dilation in SR

Rindler writes in his textbook on relativity [3] that "a standard clock A moving through the synchronized standard lattice clocks of an inertial frame of reference loses time steadily relative to those clocks."

This effect called kinematic time dilation occurs if two inertial frames of reference move relative to each other at an arbitrary speed v, where neither of the two frames is regarded as a preferred frame of reference, or as being at absolute rest, and the speed of light c is postulated to be invariant in all such inertial frames.

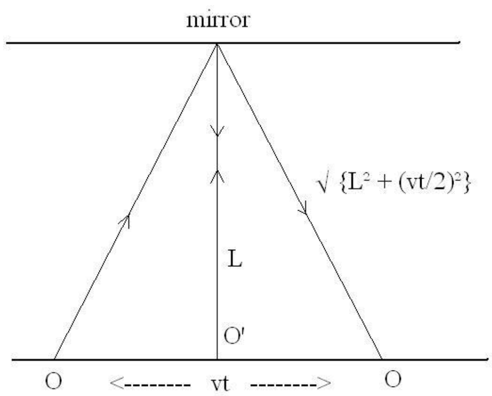

Figure 1: Illustrating kinematical time dilation with light signals

To obtain the relevant equations, consider an observer at rest in a coordinate frame O. Another frame O' passes by at a speed v, and when the origins coincide a light signal is sent (event 1) from there to a mirror a distance L away perpendicular to the direction of v. When the light signal returns to O' (event 2), the time interval between the events is recorded by a clock in the O' frame as dt'. Meanwhile O' has moved on relative to the O frame, and the light pulse returns to a point further on in that frame, having travelled a larger distance along the hypotenuse of the triangles. By postulating that the speed of light is the same in both frames, the following relationship is obtained from the diagram for the time intervals between two events viewed from the two frames:

$$dt' = \sqrt{1 - \frac{v^2}{c^2}}\ \ dt \tag{1}$$

dt' is a proper time interval, and from Equation 1 we see that it is always less than or equal to dt, the coordinate time interval on synchronized standard clocks in the observer frame O. If $v << c$ we may approximate this as

$$dt' = \left(1 - \frac{v^2}{2c^2}\right) dt \qquad [v << c]$$

and writing $\delta t/t = (dt' - dt)/dt$ we have the fractional time difference:

$$\frac{\delta t}{t} = -\frac{v^2}{2c^2} \qquad [v << c] \tag{2}$$

Furthermore the relative speed v is limited to c; the time dilation is independent of the direction of motion, since it appears squared in the expression, and it is symmetrical in the sense that if the frames are reversed, and the previous proper frame now becomes the frame with the synchronized coordinate clocks, the new coordinate clocks would run faster than a clock in the new proper frame. This is all embodied in what are called the Lorentz transformations.

Kinematic time dilation was first confirmed observationally in a qualitative way by Rossi and Hall in 1944 [4], as opposed to being just an interesting thought experiment, by the discovery that cosmic muons that had been created at a height of about 30 km at the top of the Earth's atmosphere are also detected at the Earth's surface. It is thought that primary cosmic rays, such as protons, reaching the Earth at near the speed of light, collide with an atmospheric molecule and are converted into secondary particles, including muons. In the laboratory in a particle collision experiment, a muon would have an average lifetime of 2.2 x 10^{-6} s. Travelling at near the speed of light, they would therefore decay within a distance of about 660 m, but from

Equation 1 we can understand that the muon lifetime in the coordinate frame (as observed by us) is very much greater than the lifetime in their proper frame, enabling them to survive a distance of 30 km, if their speed was in the region of 0.99995 c. In the meantime, the phenomenon of kinematical time dilation is routinely observed in decay products in particle accelerators, and the effect is also routinely corrected for in the global positioning system (GPS) [5].

2.2 Using the relativistic Doppler effect

The one-way trip of muons described above is not quite the same as the thought experiment in the twin paradox, where a twin makes a round-trip back to the starting point. I therefore want to ascertain how SR can apply to a two-way journey, and whether it makes any difference to the outcome. For this purpose, it is instructive to use the Doppler effect to discuss time dilation and provide insight for a round-trip. If the reader is familiar with this analysis, or wishes to proceed beyond it, the next two sections could be omitted entirely.

In SR, Einstein defines coordinate time by having synchronized coordinate clocks placed everywhere in the coordinate frame. To circumvent this practical issue (even though there is nothing wrong with it in principle), one can consider just one clock in each frame at the origin, and in principle use a telescope to observe the other clock that is travelling directly away or towards us, in which case we then have to take into account the Doppler effect of the light received. Since, in Einstein's theory, there is no background medium in which light propagates, i.e. the speed of light is invariant, and all inertial frames of reference are equivalent (Einstein's principle of relativity), we take the observer and source to be moving apart with a relative velocity v, but what we observe as the Doppler effect is always relative to the observer as stationary. This means that the observed frequency f of light (or clock rate) may be written as

$$f = \sqrt{1 - v^2/c^2} \left(\frac{1}{1 + v/c} \right) f_s = \sqrt{\frac{1 - v/c}{1 + v/c}} \, f_s \tag{3}$$

for a receding source of frequency f_s, and with the sign of v changed for an approaching source. This equation now takes into account both the Doppler effect and special relativity. We see that there is a redshift for a receding source (v positive, $f/f_s < 1$) and a blueshift for an approaching source (v negative, $f/f_s > 1$), both of first order in v/c, multiplied by a relativistic redshift of second order in v/c for both directions.

Several experimental attempts have been made to confirm this expression for the relativistic Doppler effect, probably the first and most well-known historically being the Ives-Stilwell experiment of 1938 [6]. The effect that one really wanted to measure was the transverse Doppler effect, which is when the motion of light-emitting atoms is at right-angles to the direction of observation of light (i.e. in the crossover from advancing to receding). The longitudinal Doppler effect would then be suppressed, and we would only have the relativistic effect of kinematic time dilation. However, the authors remarked that it is nearly impossible to measure the transverse Doppler effect with respect to light rays emitted at right angles to the direction of motion of the rays. In their experiment they therefore measured the longitudinal effect which theoretically contains a slight asymmetry due to the second-order relativistic term, and claimed they had obtained a significant effect. Further experiments [6] seem to have confirmed the above formula, including Mössbauer rotor experiments and a direct transverse Doppler result.[1]

2.3 The twin paradox itself

Applying the above formula to a simple numerical case where the relative speed is three-fifths of the speed of light: $v = \frac{3}{5}c$, from Equation 3 we have $f_{rec} = \frac{1}{2}f_s$, and $f_{app} = 2f_s$. Note that the observer frequencies are reciprocals of each other depending on whether the source is receding or approaching. By realizing that a vibration frequency is equivalent to the ticking rate of a clock, this means a clock on the moving spaceship would appear to tick at half its normal (stationary) rate when receding from Earth, and double that rate when approaching. This would occur symmetrically, irrespective of which frame of reference was regarded as the moving frame. The relativistically calculated ratio (without the Doppler shift) is $\sqrt{1 - v^2/c^2} = \frac{4}{5}$ in this example, whether receding or approaching.

Next I shall calculate the time dilation for a spaceship leaving Earth and travelling to a planet 3 light years away. Using the numerical example above, with the speed of the spaceship constant at $v = \frac{3}{5}c$, the journey is calculated to take 5 years according to an observer O on Earth. O will thus see (through a telescope) a clock on the receding spaceship O' ticking at half the resting rate. Correspondingly, and symmetrically, an observer on the spaceship O' sees a clock on Earth O also ticking at half the resting rate. When the spaceship reaches the planet, light from the spaceship will have been delayed by 3 years on

[1] Unfortunately, I have not been able to obtain a copy of the original paper to assess its significance.

Earth. The observer on Earth will have therefore seen the spaceship's clock ticking at half its normal rate for a total of $5 + 3 = 8$ years, before the clock suddenly reverts to its normal ticking rate when the spaceship has landed on the planet. Ticking at half-rate for 8 years means that the spaceship clock will have advanced by 4 years during the flight, and thus the astronaut will have aged by 4 years while journeying to the planet. The Earth-based observer therefore concludes that while he or she has aged by 5 years, the astronaut will have aged by only 4 years.

To be sure about this inferred age difference, we now want the spaceship to return to its starting point on Earth, so that we can directly compare the times on the two clocks in the same spatial location. Consider now what occurs when the astronaut sets off immediately on his return journey. An observer on Earth had been watching the astronaut's clock ticking at half-rate for 8 years (appearing to age 4 years so far), then the astronaut's clock suddenly switches to double the normal rate (in accordance with the expectation due to the relativistic Doppler effect), whereupon - since he takes 10 Earth years for the complete journey - he arrives back on the Earth 2 years later, i.e. taking a total of 10 years for the round-trip according to the observer on the Earth. But the astronaut's clock had ticked at half-rate for 8 out of those 10 years and at double its resting rate for 2 of those 10 years, making a total of only $4 + 4 = 8$ years. Thus, if the astronaut and Earth-based observer had been twins, they would now be 2 years apart in age when they met again.

To complete the picture from the travelling twin's perspective, he/she would have seen the Earth clock ticking at half rate on the outward journey until reaching the planet after 4 years. So far, the Earth twin had aged only 2 years, but on reversing direction, the astronaut twin would immediately see the Earth twin ageing at double rate for the next 4 years, i.e. 8 years, before arriving back at Earth. In total, when they reconvened, the Earth twin would have aged $2 + 8 = 10$ years and the travelling twin only $4 + 4 = 8$ years, as before. Thus, there appears to be no inconsistency.

Paul Langevin discussed this in 1911 and referred it as the twin paradox, but since it is not literally a paradox, it is often called a pseudo-paradox or apparent paradox. He explained the different ageing rates by stating that only the travelling twin undergoes an acceleration, this being the crucial asymmetry between the twins, which leads to the difference in age. The standard interpretation of the twin paradox found in many textbooks also states that the Earth and spaceship are not in a symmetrical relationship, i.e. the spaceship turns around and reverses its direction, so that the observers are not equivalent. This can be envisaged as if the travelling twin jumps off one inertial reference frame going outwards from Earth and jumps onto a different inertial reference frame approaching Earth, whereas the twin on Earth always remains on the same inertial frame of reference.

From the calculated example given above, it is clear that if one makes use of the Doppler effect to work out the clock rates, it is not even necessary to bring a discussion of either acceleration or the turn-around and return journey into the argument for explaining the ultimate difference in age of the twins.

So far, then, without adding any other ingredients to the analysis, it does seem that any controversy about an age difference has been settled beyond doubt, and that it is a real effect.

3 Clock synchronization

However, Einstein's thought experiments in SR contain the tacit assumption that clocks are ideal, meaning - as I see it - they do not change their "intrinsic" properties. The clocks are abstract and massless; they are the product of the imagination and occur only in the thought experiment. In practice, however, clocks are material objects with mass, and this means that when they are subject to some external physical influence, they could change their properties, including their ticking rate.

In SR, identical clocks are imagined to be placed throughout the coordinate frame. They have previously been synchronized to read the same time as a "master" clock at the origin O. Einstein took great trouble to define this procedure systematically by means of light signals. For example, a clock positioned x light seconds from the master clock would be seen at the origin ticking at exactly the same rate as the master clock, but with a time delay of x/c seconds, where c is the speed of light (space is isotropic).

In the twin experiment, the travelling clock A also needs to be synchronized to the master clock in the observer coordinate frame O before the travelling twin sets off with it on the journey in the O' frame. We can envisage doing this by placing the clocks next to each other at rest in frame O and doing the necessary adjustments to make them behave identically.

Then, the travelling twin sets off from rest with clock A on board, and accelerates up to the cruising speed v. In the twin thought experiment, gravity is not considered, but during the acceleration phase, work is done against the inertial mass being accelerated. The outcome is that a mass m increases according to Einstein's own formula derived using SR:

$$m = \frac{m_0}{\sqrt{1 - v^2/c^2}} \tag{4}$$

where m_0 is its rest mass and m the relativistic mass moving at speed v relative to the observer frame O.

This expression of mass-energy equivalence is known to be quantitatively correct, and used routinely, for example, in electron microscopy, where electrons are accelerated down an evacuated column by a high voltage. The electrons' mass increases with their kinetic energy, and this effect has to be taken into account when calculating de Broglie wavelengths for the purpose of analyzing electron diffraction measurements.

Einstein's abstract clocks in thought experiments, however, have no mass, and so the above consideration plays no part. But the real clocks we should be considering - atomic clocks, such as cesium beam clocks - do have mass, so let us consider for simplicity and convenience a hydrogen-like atom or ion with a single optically active electron as the vibration source in the clock. The electron energy levels E of such an atom may be written:

$$\Delta E = hf = E_2 - E_1 = \frac{Z^2 m e^4}{8h^2 \varepsilon_0^2} \left[\frac{1}{n_1^2} - \frac{1}{n_2^2} \right] \tag{5}$$

The frequency f, associated with a transition from an electron quantum state n_2 to n_1, is proportional to the electron mass m. Since all the quantities in the equation are constant for any particular atom acting as an atomic clock, except for the mass m of the optically active electron, then using Equation 4 the clock rate (or resonant frequency) at rest (f_0) compared with its value at speed v (f_v) is given by

$$f_v = \frac{f_0}{\sqrt{1 - \frac{v^2}{c^2}}} \tag{6}$$

The conclusion is now obvious. Due to the kinetic energy increase when a clock is taken to the O' frame from O, its ticking rate in the moving frame is increased by the factor $1/\sqrt{(1-v^2/c^2)}$. The kinematic time dilation of SR, by the factor $\sqrt{(1 - v^2/c^2)}$, then exactly compensates for this increase, such that the overall effect is zero kinematic clock difference, and no age difference in a twin experiment.

In some ways my derivation and explanation of their being no twin age difference is also a thought experiment, but a thought experiment in which I additionally propose that atomic clocks increase their ticking rate at a higher kinetic energy, such that the kinetic time dilation of SR is exactly compensated for. This satisfies my own longstanding scepticism around the standard interpretation, and I now feel I understand why the usual interpretation always seemed to me to be flawed and unrealistic. Thus, when atomic clocks are involved as time-keepers, and they have been synchronized together, there will be no kinematic time dilation effect.

However, kinematic time dilation has undoubtedly been observed with regard to the lifetimes of cosmic muons, and other elementary particles. But muons are not atomic clocks, and their average lifetime is not described by an equation such as Equation 5. Secondly, the cosmic muons are created at high speed at the top of Earth's atmosphere, and are already in the moving O' frame, so there is no inertial acceleration up to the relative speed v, and then, if you take their average lifetime as a time interval, they could well behave like the ideal clocks of SR.

A difficult issue remains around those reports that claim to have measured kinematic time dilation with atomic clocks. Each of those experiments involved additional effects on clock rates, one of them being gravity, and so I shall briefly discuss this additional contribution to clock rates next.

4 Clock rate changes due to gravity

It has been well established that a clock taken to a higher gravitational potential will tick at a faster rate than a ground-based clock. This effect, called gravitational time dilation, can be explained within Einstein's general theory of relativity GR as a consequence of spacetime curvature [7], and the usual expression for clock rate in terms of time intervals dt at a radial distance r from Earth's centre may be written as

$$dt_r = \sqrt{1 - \frac{2GM}{c^2 r}}\, dt_\infty \tag{7}$$

This is interpreted to mean that a clock speeds up as it is raised in the gravitational field. Writing the gravitational potential as $U = -GM/r$, we have equivalently

$$dt_r = \sqrt{1 + \frac{2U}{c^2}}\, dt_\infty \tag{8}$$

Restricting ourselves to weak gravitational fields, this can be approximated as

$$dt_r = \left(1 + \frac{U}{c^2}\right) dt_\infty \tag{9}$$

Setting the zero of potential at the Earth's surface, instead, we may write

$$dt_h = \left(1 + \frac{U}{c^2}\right) dt_0 \tag{10}$$

where h is the height about ground level, and $U = gh$, if the gravitational field does not change significantly through that height difference. This is the generally used expression to describe and calculate gravitational time dilation for applications such as the GPS, for example.

Including the kinematic effect (Equation 2), one could then write approximately

$$dt_{h,v} = \left(1 + \frac{gh}{c^2} - \frac{v^2}{2c^2}\right) dt_{0,0} \tag{11}$$

From this expression, the atomic clocks on the GPS satellites at 20,000 km above the Earth's surface are *calculated* to run faster than those on the ground due to being at a higher gravitational potential, with a predicted gain of 45.8 μs per day, while the kinetic effect of the satellites travelling at a speed of 3,874 km/s would be predicted to run slower by 7.2 μs per day. In practice, as stated by Ashby [5] in his paper, the satellite clocks are synchronized with a master clock in Washington DC by adjusting the resonant frequencies by remote signals, and it would be difficult now to use this to test relativity, due to all the corrections that have to be incorporated to make the GPS work accurately.

5 Clocks in aeroplanes

Partly because it is of historical importance, I shall now discuss a very well known experiment where it was claimed that measured clock differences accord with the twin aging prediction of SR. About fifty years ago, Hafele and Keating [8] flew four cesium-beam atomic clocks on aeroplanes both east and west around the Earth, and then compared them with a master clock on the ground. The authors claimed that their results "provide an unambiguous empirical resolution of the famous clock paradox with macroscopic clocks".

However, several people subsequently questioned the claims. Kelly was concerned about the accuracy of the results, and states that the experiment proved nothing, since the clock drifts were so large as to make the results unreliable, and the authors changed the results to confirm the theory [9]. In addition, Spencer and Shama re-analyzed Keating's raw data from the 1971 experiment [10], and found no significant time difference for the same results. Nevertheless, a re-enactment of the original experiment was made by the University of Maryland in

1976, and by the National Physical Laboratory in 1996, allegedly verifying the original results to a greater degree of accuracy than Hafele and Keating were able to achieve. Thus, credibility was restored to the measurements. Nevertheless, in my humble opinion, one could still pose the question of whether conclusions in general can ever be affected by the phenomenon of confirmation bias [11].

Although aeroplane speeds are nowhere near the speed of light, the experiment is interesting because it introduces additional features into the argument. For discussion purposes I shall therefore quote the findings shown in the original 1972 paper [8]. The time differences relative to the ground-based master clock are in units of nanoseconds. More than one cause of a time difference is indicated in the table. The first is a gravitational effect, as discussed in Section 4. Since the plane was flying at a higher gravitational potential than the master clock at ground level, the airborne clocks were *calculated* to gain time over the clock on the ground. The first column in the table shows the calculated prediction for this (applying Equation 10 above). The calculations are fairly similar in both directions, the difference being caused by the difference in heights flown and times spent on the two journeys.

	grav. (pred.)	kin. (pred.)	total (pred.)	measured
east	144 ± 14	-184 ± 18	-40 ± 23	-59 ± 10
west	179 ± 18	96 ± 10	275 ± 21	273 ± 7

As far as kinematic time dilation is concerned, the predicted outcome appears very curious at first sight (second column in the table). A change in sign for the time difference is indicated for the two directions of travel, eastwards and westwards, i.e. in the same direction as, and opposite to, the direction of rotation of the Earth.

In the framework of SR (in which only relative motion of two inertial frames of reference is considered), this result seems incorrect. The direction of travel should have no bearing on the outcome, since the time dilation appears as v^2 in Equation 1. One needs to realise, therefore, that the two frames being considered here (ground and aeroplane) are not really inertial frames: firstly they exhibit an acceleration with respect to each other, and secondly the motion is not linear. In other words, the criteria for Einstein's thought experiment - often called the standard configuration of SR - are not met, and one could simply reject the experiment as not a good test of SR.

However, the situation is more interesting than that, and the calculation and experimental results tell us a great deal. The asymmetry in the prediction arises due to the rotation of the Earth (and Hafele and

Keating understood this correctly). Although the master clock and aeroplane clock are not travelling at a constant velocity with respect to each other as the plane circumnavigates the Earth, they are both travelling at a constant velocity with respect to an underlying frame of reference whose origin is at the centre of the Earth and which is fixed in space, called here the centre of Earth inertial frame, CEI. Both the master clock and the aeroplane are accelerating in their rotational motion towards the centre of the Earth, but their tangential speeds with respect to the CEI frame can be regarded as constant (or can be averaged, in practice).

The speed u of a point on the Earth's surface at the Equator is about 436 m/s (or 1,570 km/h), and would be about 360 m/s at the latitude of Washington DC from where the aeroplanes were flown, while commercial aeroplane speeds relative to the Earth's surface are usually in the region of 200-250 m/s. Calling the aircraft speed with respect to the ground v (the same in either direction, for simplicity), the aircraft then has a speed $(u+v)$ travelling east and $(u-v)$ travelling west with respect to the CEI frame. We may then write approximately:

$$\frac{\delta t_e}{t_e} = -\left[\frac{(u+v)^2}{2c^2} - \frac{u^2}{2c^2}\right] + \frac{gh}{c^2} \;; \qquad \frac{\delta t_w}{t_w} = -\left[\frac{(u-v)^2}{2c^2} - \frac{u^2}{2c^2}\right] + \frac{gh}{c^2} \tag{12}$$

where v and u are positive quantities, δt_e and δt_w are the time differences of the aeroplane clock with respect to the ground clock for each direction of flight, and I have included the contribution due to gravitational time dilation. This then gives

$$\frac{\delta t_e}{t_e} = \frac{(-\frac{1}{2}\,v^2 - uv + gh)}{c^2} \;; \qquad \frac{\delta t_w}{t_w} = \frac{(-\frac{1}{2}\,v^2 + uv + gh)}{c^2} \tag{13}$$

Hafele and Keating tell us that $t_e = 41.2$ hours; $t_w = 48.6$ hours, and so from their predictions in the table, by subtracting these simultaneous equations to give

$$\frac{\delta t_e}{t_e} - \frac{\delta t_w}{t_w} = -\frac{2uv}{c^2} \tag{14}$$

it is deduced that $v = 245\,m/s$, with $u = 360\,m/s$ (assumed), while adding the equations,

$$\frac{\delta t_e}{t_e} + \frac{\delta t_w}{t_w} = \frac{(2gh - v^2)}{c^2} \tag{15}$$

gives an average flight altitude $h = 9.0\,km$. These values have been reverse calculated here, and will no doubt differ somewhat from the actual values Hafele and Keating adopted. The reader should also be aware that the actual flight paths for circumnavigation were far from being a simple circular path at constant speed, but involved multiple landings and takeoffs, and zigzag paths from one airport to another (about which I have no exact details).

In summary, from the above reverse physics calculations, the three contributions to the overall time differences are: gravitation $\delta t/t \approx +1.0 \times 10^{-12}$, Earth's rotation $\pm 0.9 \times 10^{-12}$, the sign depending on direction, while kinematic time dilation makes the smallest contribution $\approx -0.33 \times 10^{-12}$.

The above analysis based on using the CEI frame as the coordinate frame and thinking in terms of motion relative to this frame, already incorporates an effect into the analysis, called the Sagnac effect, after Sagnac's discovery in 1910 that a light beam forced to travel in a circular orbit needs different times to make a revolution. The same principle applies to the orbiting of clocks on the aeroplanes, leading to predicted kinematical effects that are different in both magnitude and sign, depending on the direction. Due to the effect, the kinetic time delay changes sign depending on direction. The reader is referred to a review paper by Jonson (2009) [12].

If I were now to claim that kinematic time dilation played no part, we would have

$$\frac{\delta t_e}{t_e} = \frac{(-uv + gh)}{c^2} \quad ; \quad \frac{\delta t_w}{t_w} = \frac{(+uv + gh)}{c^2} \tag{16}$$

(to be compared with Equation 13 containing a contribution due to kinematical time dilation.) The question then remains whether this can be reconciled with Hafele and Keating's published clock differences, viz. $\delta t_e = -59\,ns$; $\delta t_w = 273\,ns$. Substituting these values into Equation 16 where kinematic time dilation has been omitted gives $v \approx 250\,m/s$ and $h \approx 6.5\,km$. We see that the (averaged) aircraft speed v is reproduced similarly to the analysis that included kinematic time dilation, but the (averaged) flight altitude h is adjudged to be lower.

6 Conclusion

I have tried to show here that, although relativistic kinematic time dilation is undoubtedly a real effect, if the clock measuring it is an atomic clock that has been transported from a stationary frame to a moving frame, the effect will be non-existent, because the proper clock rate increases by the same ratio as it decreases by kinematic time dilation. This means that if a human being, for example, whose metabolism is determined essentially by atomic physics, is subject to relativistic motion, this in itself will not affect the ageing process, and the idea of a twin age difference due to such an effect is nonsense. Gravity however does affect clock rates, and the proposal here that a kinetic energy increase has the same effect on a clock rate as a potential energy increase suggests there is an underlying principle in which a clock rate change is related to its energy change *per se*.

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