Number Theory

Part 1: Transcendental Equations:

Solving Transcendental Equations using the \( \beta w \)-convergence formula

Abstract

The main purpose of inventing this paper is based on the general idea that an equation of this form can’t be \( a^x + b^x = c \) algebraically. In this question derived, the formula ((\( \beta w \)-convergence)) with mathematical proof can be used to solve such an equation with ease.

Since the formula is purely invented with my own approach, the article lacks references.

\[ \beta w \text{-convergence Formulae for Solving } a^x + b^x = c \]

Say, \( n \approx x \), then, \( n \)

\[ (a^x - a^n) + (b^x - b^n) = c - a^n - b^n \]

Factorizing, then

\[ a^n(a^{x-n} - 1) + b^n(b^{x-n} - 1) = c - a^n - b^n \]

Factorized, then

\[ (b^{x-n} - a^{x-n})(a^n b^n) = c - a^n - b^n, \text{ this is true if } n \approx x \text{ or } n = x \]

Dividing \( a^n b^n \) both sides, then

\[ (b^{x-n} - a^{x-n}) = \frac{c - a^n - b^n}{(a^n b^n)} \]

This can also be written as;

\[ \frac{b^x}{b^n} - \frac{a^x}{a^n} = \frac{c - a^n - b^n}{(a^n b^n)} \]

Back to the equation, \( a^x + b^x = c \),
\[ a^x = c - b^x \]

Therefore,
\[ \frac{b^x}{b^n} - \frac{(c - b^x)}{a^n} = \frac{c - a^n - b^n}{(a^n b^n)} \]
\[ \frac{a^n b^x - b^n c + b^n b^x}{a^n b^n} = \frac{c - a^n - b^n}{(a^n b^n)} \]

Multiplying both sides by \(a^n b^n\)
\[ a^n b^x - b^n c + b^n b^x = c - a^n - b^n \]

This can also be written as;
\[ a^n b^x + b^n b^x = c - a^n - b^n + b^n c \]

Factorizing,
\[ b^x(a^n + b^n) = c - a^n - b^n + b^n c \]

Where \(b^x\) will be;
\[ b^x = \frac{b^n c + (c - a^n - b^n)}{(a^n + b^n)} \]

Therefore,
\[ x = \frac{\log(b^n c + (c - a^n - b^n))}{\log(a^n + b^n)} \]

Similarly, following the same procedure,
\[ x = \frac{\log(a^n c + (c - a^n - b^n))}{\log(a^n + b^n)} \]

**Theorem**

I. When \( n \rightarrow x \), the closer \( n \) approaches \( x \), then accurate the answer until \( n = x \)

II. Meaning \( n_1 \) will be closer to answer than \( n_2, n_3 \) than \( n_2, ..., n_j \)
III. \( n_1 \) must be used to get \( n_2 \) to get \( n_3 \) ... until \( n_i = x \)

IV. Using \( \beta w \)-convergence formula, \( k \) can be used to calculate the first value of \( n \)

V. \( k \) can be any value assumed. as long the \( c - a^n - b^n > 0 \), or equal to 0

VI. Here in all calculations, I have taken \( b^x > a^x \); however, whichever the case, it does not interfere with the calculations. One can also use \( a^x > b^x \)

VII. The larger the value of \( k \), the more calculations would be needed, but the closer the value \( k \) to \( n \) fewer calculations would be needed.

VIII. The same formula calculates the first value of \( n \)

\[
n = \frac{\log \left( \frac{b^k c + (c - a^k - b^k)}{a^k + b^k} \right)}{\log b}
\]

IX. \( x = \frac{\log \left( \frac{b^k c + (c - a^n - b^n)}{(a^n + b^n)} \right)}{\log a} \) requires less calculation than \( x = \frac{\log \left( \frac{a^n c + (c - a^n - b^n)}{(a^n + b^n)} \right)}{\log a} \) to find the accurate answer.

\( \beta w \)-convergence formulae for Solving \( b^x - a^x = c \)

Following the same rule & procedure

\( n \to x \), then

\[
(a^x - a^n) + (b^x - b^n) = c + a^n - b^n
\]

Factorizing, then

\[
a^n(a^{x-n} - 1) + b^n(b^{x-n} - 1) = c + a^n - b^n
\]

However, if \( n \to x \), where \( n = x \), then \( a^n(a^{x-n} - 1) + b^n(b^{x-n} - 1) = c + a^n - b^n \)

Can be written (factorized) as;

\[
(b^{x-n} - a^{x-n})(a^n b^n) = c + a^n - b^n
\]

Dividing \( a^n b^n \) both sides, then
\( (b^{x-n} - a^{x-n}) = \frac{c + a^n - b^n}{(a^n b^n)} \)

This can also be written as;

\[
\frac{b^x}{b^n} - \frac{a^x}{a^n} = \frac{c + a^n - b^n}{(a^n b^n)}
\]

Back to the equation, \( b^x - a^x = c \),

\[ b^x = c + a^x \]

Therefore,

\[
\frac{(c + a^x)}{b^n} - \frac{a^x}{a^n} = \frac{c + a^n - b^n}{(a^n b^n)}
\]

\[
\frac{a^n(a^x + c) - b^n a^x}{a^n b^n} = \frac{c + a^n - b^n}{(a^n b^n)}
\]

Multiplying both sides by \( a^n b^n \)

\[ a^n a^x + a^n c - b^n a^x = c + a^n - b^n \]

This can also be written as;

\[ a^n a^x - b^n a^x = c + a^n - b^n - a^n c \]

Factorizing,

\[ a^x(a^n - b^n) = c + a^n - b^n - a^n c \]

Where \( a^x \) will be;

\[ a^x = \frac{-ca^n + (c + a^n - b^n)}{(a^n - b^n)} \]

Thus,

\[ x = \frac{\log\left(\frac{-ca^n + (c + a^n - b^n)}{(a^n - b^n)}\right)}{\log a} \]

Similarly, following the same procedure,
\[ x = \frac{\log\left(\frac{-cb^n + (c + a^n - b^n)}{(a^n - b^n)}\right)}{\log b} \]

**Theorem**

I. \( k \) any value assumed. as long the \( c - a^n - b^n < 0, or equal to 0 \)

II. This formula can also be used to find the value of \( x \), in the equation

\[ a^x + b^x = c^x \]

III. However, to do so, the equation must be first changed into

\[ \left(\frac{a}{c}\right)^x + \left(\frac{b}{c}\right)^x = 1 \]

Or

\[ \left(\frac{c}{a}\right)^x - \left(\frac{b}{a}\right)^x = 1 \]

**Example 1**

Find the value of \( x \)

\[ 3^x + 2^x = 14 \]

**Solution**

Let’s take any value of \( k \), say 8, and then

\[ n1 = \frac{\log\left(\frac{b^k c + (c - a^k - b^k)}{(a^k + b^k)}\right)}{\log b} \]

So, applying the formula;

\[ n1 = \frac{\log\left(\frac{3^8 \times 14 + (14 - 2^8 - 3^8)}{(2^8 + 3^8)}\right)}{\log 3} \]

Thus,
\[ n_1 = 2.29729050932 \]

So, using \( n_1 = 2.29729050932 \), then,

\[ 3^{2.29729050932} + 2^{2.29729050932} = 17.3916468296 \]

*Doing the second calculation, where now; n1=2.29729050932*

The value of \( n_2 \)

\[ n_2 = \log\left( \frac{3^{2.29729050932} \times 14 + (14 - 2^{2.29729050932} - 3^{2.29729050932})}{(2^{2.29729050932} + 3^{2.29729050932})} \right) \]

So, using \( n_2 = 2.08198130882 \)

\[ 3^{2.08198130882} + 2^{2.08198130882} = 14.0820980231 \]

*Doing the third calculation, where now; n2=2.08198130882*

The value of \( n_3 \)

\[ n_3 = \log\left( \frac{3^{2.08198130882} \times 14 + (14 - 2^{2.08198130882} - 3^{2.08198130882})}{(2^{2.08198130882} + 3^{2.08198130882})} \right) \]

\( n_3 = 2.07611695862, \)

So, using \( n_3 = 2.07611695862 \)

\[ 3^{2.07611695862} + 2^{2.07611695862} = 14.0016781963 \]

The value of \( n_4 \)

\[ n_4 = \log\left( \frac{3^{2.07611695862} \times 14 + (14 - 2^{2.07611695862} - 3^{2.07611695862})}{(2^{2.07611695862} + 3^{2.07611695862})} \right) \]

\( n_4 = 2.07599670273 \)

So, using \( n_4 = 2.07599670273 \)

\[ 3^{2.07599670273} + 2^{2.07599670273} = 14.0000340751 \]

So, finding \( n_5 \)
\[ n_5 = \frac{\log\left(3^{2.07599670273 \times 14} + (14 - 2^{2.07599670273} - 3^{2.07599670273})\right)}{\log 3} \]

\[ n_5 = 2.07599426082 \]

\[ 3^{2.07599426082} + 2^{2.07599426082} = 14.0000006918 \]

If the calculations are repeated, it will reach a point where the value of \( n_k \) \((3^{nk} + 2^{nk} = 14)\), solution of exactly 14, meaning, \( n_k = x \)

**Important Notice Based on Example 1**

- If the value \( k > n \), then the values of \( C^n > C^x \). However, \( C^n \) will decrease with each calculation until it reaches, \( C^n = C^x \)

  \[ a^x + b^x = c^x \]

- If \( k < n \), then the value of \( C^n < C^x \). However, \( C^n \) will increase with each calculation until it reaches \( C^n = C^x \)

- In the calculation have assumed \( k = 8 \), though any number can be used if and only if \( c - a^n - b^n > 0 \), or equal to 0

**Example 2**

Find the value of \( x \)

\[ 3^x - 2^x = 14 \]

\[ k = 0.86135311614 \]

Applying the formula

\[ n_1 = \frac{\log\left(-cb^n + (c + a^n - b^n)\right)}{\log b} \]

Then
\[ n_1 = \log\left(\frac{-3^{0.861353111614} \times 4 + (4 + 2^{0.861353111614} - 3^{0.861353111614})}{(2^{0.861353111614} - 3^{0.861353111614})}\right) \]

\[ n_1 = 2.03004521829 \]

So, using \( n_1 = 2.03004521829 \), then,

\[ 3^{2.03004521829} - 2^{2.03004521829} = 5.24193926067 \]

**Doing the second calculation, where now, \( n_1 = 2.03004521829 \)**

The value of \( n_2 \)

\[ n_2 = \log\left(\frac{-3^{2.03004521829} \times 4 + (4 + 2^{2.03004521829} - 3^{2.03004521829})}{(2^{2.03004521829} - 3^{2.03004521829})}\right) \]

\[ n_2 = 1.81742690735 \]

\[ 3^{1.81742690735} - 2^{1.81742690735} = 3.8398057357 \]

**Doing the third calculation, where now, \( n_2 = 1.81742690735 \)**

The value of \( x \)

\[ n_3 = \log\left(\frac{-3^{1.81742690735} \times 4 + (4 + 2^{1.56715353789} - 3^{1.56715353789})}{(2^{1.56715353789} - 3^{1.56715353789})}\right) \]

\[ n_3 = 1.84966718013 \]

\[ 3^{1.84966718013} - 2^{1.84966718013} = 4.02567137244 \]

**Doing the fourth calculation, where now, \( n = 1.84966718013 \)**

The value of \( n_3 \)

\[ n_4 = \log\left(\frac{-3^{1.84966718013} \times 4 + (4 + 2^{1.84966718013} - 3^{1.84966718013})}{(2^{1.84966718013} - 3^{1.84966718013})}\right) \]

\[ n_4 = 1.84460939966 \]

\[ 3^{1.84460939966} - 2^{1.84460939966} = 3.99600675004 \]
Doing the fifth calculation, where \( n4 = 1.84460939966 \)

\[
x = \frac{\log\left(\frac{-3^{1.84460939966} \times 4 + (4 + 2^{1.84460939966} - 3^{1.84460939966})}{(2^{1.84460939966} - 3^{1.84460939966})}\right)}{\log 3}
\]

\( n5 = 1.84539878668 \)

\[3^{1.84539878668} - 2^{1.84539878668} = 4.0062407008\]

Doing the sixth calculation, where \( n5 = 1.84539878668 \)

\[
x = \frac{\log\left(\frac{-3^{1.84539878668} \times 4 + (4 + 2^{1.84539878668} - 3^{1.84539878668})}{(2^{1.84539878668} - 3^{1.84539878668})}\right)}{\log 3}
\]

\( n6 = 1.84527548465 \)

\[3^{1.84527548465} - 2^{1.84527548465} = 3.9990254065\]

If the calculations are repeated, it will reach a point where the value of \( x (3^x - 2^x = 4) \), solution of exactly 4

**Important Notice Based on Example 2**

- As observed in the example, \( n1, n3, \) and \( n4 \), give \( C^n > C^x \) but the value decrease with each calculation. While the value of \( n2, n4, n6 \), give \( C^n < C^x \) but increases by each calculation.

- In the calculation have assumed \( k = 0.86135311614 \), though any number can be used if and only if \( c - a^x - b^x < 0, \) or equal to 0.

**\( \beta w - convergence \)** Formular for Solving \( a^{x+e} + b^{x+d} = c \)

\( n \rightarrow x \), then, and where \( (d, e) \) are known numbers

\[
(a^{x+e} - a^{n+e}) + (b^{x+d} - b^{n+d}) = c - a^{n+e} - b^{n+e}
\]

Factorizing, then

\[a^{n+e}(a^{x-n} - 1) + b^{n+d}(b^{x-n} - 1) = c - a^{n+e} - b^{n+d}\]
However, if \( n \to x \), where \( n = x \), then 
\[
a^{n+e}(a^{x-n} - 1) + b^{n+e}(b^{x-n} - 1) = c - a^{n+e} - b^{n+d}
\]

This be written (factorized) as;
\[
(b^{x-n} - a^{x-n})(a^{n+e}b^{n+d}) = c - a^{n+e} - b^{n+d}
\]

Dividing \( a^n b^n \) both sides, then
\[
(b^{x-n} - a^{x-n}) = \frac{c - a^{n+e} - b^{n+d}}{(a^{n+e} b^{n+d})}
\]

This can also be written as;
\[
\frac{b^x - a^x}{b^n - a^n} = \frac{c - a^{n+e} - b^{n+d}}{(a^{n+e} b^{n+d})}
\]

Back to the equation, \( a^{x+e} + b^{x+d} = c \),
\[
a^x = \frac{c - b^{x+d}}{a^e}
\]

Therefore,
\[
\frac{b^x}{b^n} - \frac{(c - b^{x+d})}{a^e a^n} = \frac{c - a^{n+e} - b^{n+d}}{(a^{n+e} b^{n+d})}
\]

\[
\frac{a^n b^x a^e - c b^n + b^{n+x+d}}{a^{e+n} b^n} = \frac{c - a^{n+e} - b^{n+d}}{(a^{n+e} b^{n+d})}
\]

Multiplying both sides by \( a^{n+e} b^n \)
\[
a^n b^x a^e - c b^n + b^{n+x+d} = \frac{c - a^{n+e} - b^{n+d}}{(b^d)}
\]

This can also be written as;
\[
a^n b^x a^e + b^n b^x b^d = \frac{c - a^{n+e} - b^{n+d}}{(b^d)} + b^n c
\]
\[
a^n b^x a^e + b^n b^x b^d = \frac{c b^{n+d} + (c - a^{n+e} - b^{n+d})}{(b^d)}
\]
Factorizing,
\[ b^x(a^{n+e} + b^{n+d}) = \frac{cb^{n+d} + (c - a^{n+e} - b^{n+d})}{(b^d)} \]

Where \( b^x \) will be;
\[ b^x = \frac{cb^{n+d} + (c - a^{n+e} - b^{n+d})}{(a^{n+e} + b^{n+d})(b^d)} \]

Hence
\[ x = \log\left(\frac{\frac{b^{n+d}c + (c - a^{n+e} - b^{n+d})}{(a^{n+e} + b^{n+d})(b^d)}}{\log b}\right) \]

Similarly, following the same procedure,
\[ x = \log\left(\frac{\frac{a^{n+e}c + (c - a^{n+e} - b^{n+d})}{(a^{n+e} + b^{n+d})(a^e)}}{\log a}\right) \]

**bw - convergence** Formular for Solving \( b^{x+d} - a^{x+e} = c \)

\( n \to x \), then, and where \((d, e)\) are known numbers

\( (a^{x+e} - a^{n+e}) + (b^{x+d} - b^{n+d}) = c + a^{n+e} - b^{n+e} \)

Factorizing, then
\[ a^{n+e}(a^{x-n} - 1) + b^{n+d}(b^{x-n} - 1) = c + a^{n+e} - b^{n+d} \]

However, if \( n \to x \), where \( n = x \), then \( a^{n+e}(a^{x-n} - 1) + b^{n+e}(b^{x-n} - 1) = c + a^{n+e} - b^{n+d} \)

This be written (factorized) as;
\[ (b^{x-n} - a^{x-n})(a^{n+e}b^{n+d}) = c + a^{n+e} - b^{n+d} \]

Dividing \( a^n b^n \) both sides, then
\[(b^x - a^n) = \frac{c + a^{n+e} - b^{n+d}}{(a^{n+e}b^{n+d})}\]

This can also be written as;

\[\frac{b^x}{b^n} - \frac{a^n}{a^e} = \frac{c + a^{n+e} - b^{n+d}}{(a^{n+e}b^{n+d})}\]

Back to the equation, \(a^{x+e} + b^{x+d} = c\),

\[a^x = \frac{b^{x+d} - c}{a^e}\]

Therefore,

\[\frac{b^x}{b^n} - \frac{(b^{x+d} - c)}{a^e a^n} = \frac{c + a^{n+e} - b^{n+d}}{(a^{n+e}b^{n+d})}\]

\[\frac{a^nb^x a^e + b^{n+x+d} - c b^n}{a^{e+n}b^n} = \frac{c + a^{n+e} - b^{n+d}}{(a^{n+e}b^{n+d})}\]

Multiplying both sides by \(a^{n+e}b^n\)

\[a^nb^x a^e + c b^n - b^{n+x+d} = \frac{c + a^{n+e} - b^{n+d}}{(b^d)}\]

This can also be written as;

\[a^nb^x a^e - b^nb^x b^d = \frac{c + a^{n+e} - b^{n+d}}{(b^d)} - b^n c\]

\[a^nb^x a^e + b^nb^x b^d = \frac{(-c b^{n+d}) + (c + a^{n+e} - b^{n+d})}{(b^d)}\]

Factorizing,

\[b^x(a^{n+e} - b^{n+d}) = \frac{(-c b^{n+d}) + (c + a^{n+e} - b^{n+d})}{(b^d)}\]

Where \(b^x\) will be;

\[b^x = \frac{(-c b^{n+d}) + (c + a^{n+e} - b^{n+d})}{(a^{n+e} - b^{n+d})(b^d)}\]
Thus
\[
x = \frac{\log((-b^{n_d}c) + (c + a^{n_e} - b^{n_d}))}{\log b}
\]

Similarly, following the same procedure,
\[
x = \frac{\log((-a^{n_e}c) + (c + a^{n_e} - b^{n_d}))}{\log a}
\]

βw-convergence Formular for Solving \( b^x + a^x = c \)

Using the same steps (procedure) as formula for solving \( b^x + a^x = c \)

\[
x = \frac{\log (ca^n - (c - a^n) - b^{n_d})}{\log a}
\]

Or
\[
x = \sqrt{\log (cb^{n_d} + (c - a^n) - b^{n_d})}
\]

However, this can be summarized as \( (b^x + a^x = c) \)

\[
x = \sqrt{\log (cb^{n_d} + (c - b^{n_d} - a^{n_e}))}
\]

Or
\[
x = \sqrt{\log (ca^{n_e} - (c - b^{n_d} - a^{n_e}))}
\]
General βw-convergence Formula

Suppose, \(a^x \pm b^x \pm c^x \pm d^x \pm \ldots \ldots \pm z^x = \beta\)

Then, applying the mathematical approach from the equation \(a^x + b^x = c\)

The value \(x\) of any value selected, say

\[z^x = \frac{\beta z^n + (\beta \mp a^n \mp b^n \mp c^n \mp d^n \mp \ldots \ldots \mp z^n)}{(a^n \pm b^n \pm c^n \pm d^n \pm \ldots \ldots \pm z^n)}\]

However, this is true if \(z^x + M = \beta\)

Where \(M = (a^x \pm b^x \pm c^x \pm d^x \pm \ldots \ldots \pm y^x)\)

Thus

\[x = \frac{\log(\beta z^n + (\beta \mp a^n \mp b^n \mp c^n \mp d^n \mp \ldots \ldots \mp z^n))}{\log z}\]

Where there is a subtraction in the equation,

Say, \(a^x \mp b^x \pm c^x \pm d^x \pm \ldots \ldots \pm z^x = \beta\)

Then, we apply the mathematical approach from the equation \(b^x - a^x = c\)

This is true if \(M - b^x = \beta\)

Where \(M = (a^x \pm c^x \pm d^x \pm \ldots \ldots \pm z^x)\)

\[b^x = \frac{-\beta b^n + (\beta \mp a^n \pm b^n \mp c^n \mp d^n \pm \ldots \ldots \mp z^n)}{(b^n \mp a^n \mp c^n \mp d^n \mp \ldots \ldots \mp z^n)}\]

Thus

\[x = \frac{\log(-\beta z^n + (\beta \mp a^n \pm b^n \mp c^n \mp d^n \pm \ldots \ldots \pm z^n))}{\log z}\]