Abstract:

The differential equation (DE) of second order for advanced Lorentz-Einstein-factor in fourth order can be written in a Sturm-Liouville form (STL). Therefore it can be formulated as an eigenwertproblem (eigenvalueproblem) for local flat spacetime-states. These eigenvalues are calculated.

Key-words: eigenwerte; Sturm-Liouville equation; differential equation of second order; Einstein-Lorentz-factor; model of damped resonance; eigenfunctions; ftl-velocity; eigenresonance of spacetime.

1. Introduction:

In the following paper there will be shown, that a second-order differential-equation can represent equations of eigenvalues („eigenwerte“) [5.] for advanced Lorentz-term of fourth order whose eigenvalues only exist under special conditions.

For the advanced SRT with ftl there can be written a Lorentz-Einstein-factor of fourth-order in root [1.]

\[ \Gamma = \sqrt{\left(1 - \frac{v^2}{c^2}\right)^2 + \frac{n^2 \cdot v^2 \cdot a^2}{c^4}} \]  

(1.)

For this term there exists a differential equation of second order which means, this DE is the generating equation for that factor [2.].

\[ A \cdot \dddot{\psi} + A B \dot{\psi} + (A \cdot C - D) \cdot \psi = 0 \]  

(2.)

The limiting cases to standard-SRT are fulfilled, because for \( B \equiv 0 \) follows \( a \equiv 0 \) and in this case there is generated by this DE the ordinary, common Lorentz-factor of [3.] and his counterpart of Feinberg for classical tachyons, see [4.] and Appendix:
\[ y = \sqrt{1 - \frac{v^2}{c^2}} \quad (3.) \]

This DE (2.) can be written in a Sturm-Liouville-form. This means, there is a description of an eigenwert-problem of this equation. This description follows now.

### 2.1. General Calculation:

The Sturm-Liouville-form of a DE can be formulated in its eigenform as:

\[ -(p \cdot \dot{\psi}) + q \cdot \psi = \lambda_n \cdot \omega \cdot \psi \quad (4a.) \]

This means:

\[ -p \cdot \ddot{\psi} + p \cdot \dot{\psi} + q \cdot \psi = \lambda_n \cdot \omega \cdot \psi \quad (4b.) \]

Then can this equation be written as an eigenwertproblem:

\[ L \cdot \psi = \lambda \cdot \psi \quad (4c.) \]

with the operator:

\[ L = \frac{1}{\omega} \cdot \left( (-\ddot{p} + q) \right) \quad (4d.) \]

For the DE of advanced Einstein-Lorentz-factor (2.) this model can be written as:

\[ -A \cdot \ddot{\psi} - A \cdot B \cdot \dot{\psi} + D \cdot \psi = A \cdot C \cdot \psi \quad (5a.) \]

with

\[ \psi (r) = A \cdot e^{i \left( \frac{r}{\lambda_n} - \theta \right)} \quad (5b.) \]

(All dots above an variable or bracket are derivations after time in Newtonian notation).

Comparison of coefficients leads to:

\[ A = p \]
\[ A \cdot B = \dot{p} \]
\[ D = q \]
\[ A \cdot C = \lambda_n \cdot \omega \quad (5c.) \]

Boundary condition after Sturm (BC) for this STL-system is the equation:
\[ \psi(r) \cdot \sin(0) + \dot{\psi}(r) \cdot \cos(0) = 0 \]  \hspace{1cm} (6.)

This leads especially to the BCs:

\[ \psi(r_{pl}) \cdot \sin(0) + \dot{\psi}(r_{pl}) \cdot \cos(0) = 0 \]  \hspace{1cm} (6a.)

\[ \psi(r_{pl}) \cdot \sin(\pi) + \dot{\psi}(r_{pl}) \cdot \cos(\pi) = 0 \]  \hspace{1cm} (6b.)

with

\[ r \in [r_{pl}; n \cdot r_{pl}]; n \in \mathbb{N}; 0 \in [0; \pi] \]  \hspace{1cm} (6c.)

Then the eigenwert-problem of a STL-equation can be written as:

\[ \lambda_n = \pi^2 \cdot n^2 \cdot \left( \int_a^b \sqrt{\frac{\omega}{p}} \, dx \right)^{-2} \]  \hspace{1cm} (7a.)

This leads in this case here to a selfreferential condition of:

\[ \lambda_n = \pi^2 \cdot n^2 \cdot \left( \int_a^b \sqrt{\frac{c}{\lambda_n}} \, dr \right)^{-2} \]  \hspace{1cm} (7b.)

After some boring calculations this formulation leads to an eigenwert-solution of:

\[ \lambda_n = e^{2 \pi n \int \frac{1}{\omega} d\omega + k} \]  \hspace{1cm} (8a.)

Since \( C = \omega_{pl}^2 \), the integration has to be after the frequency \( \omega \) of the oscillating system because of condition of absence of units of measurement in exponent of the value. Especially this integration leads to \( \omega = \Omega \), the frequency of the modelled damping system, which is the only quantity which makes sense to use in these circumstances. Finally this equation leads to the expression of:

\[ \lambda_n = e^{2 \pi n \Omega + k} \]  \hspace{1cm} (8b.)

where \( k \) is an unknown, constant integration variable, which could be set to zero (or not).

### 2.2. Special calculation:

With

\[ \Omega = \frac{a}{R} = \frac{1}{T}; R = m \cdot r_{pl}; m = \text{const}.; m \in \mathbb{N}; \omega_{pl} = \frac{1}{t_{pl}} = \frac{c}{r_{pl}} \]  \hspace{1cm} (9.)

follows as a solution for the equation of eigenvalues:
\[ \lambda_n = e^{\frac{2 \pi n a}{m c} t}; \]  

(10.)

The factor \( a \) is damping velocity of outer model-system, which reduces to \( a \equiv 0 \) for states of common, classical SRT.

\[ m = n; m, n \in \mathbb{N} \text{ possible.} \]

So the equation for eigenfunction is finally:

\[ \tilde{\psi}_n(t) = \lambda_n \cdot \psi(t) = \lambda_n \cdot A \cdot e^{\int \frac{\nu}{c} (t - t_{PL} - \theta) dt} = e^{\frac{2 \pi n a}{m c} k} \cdot e^{\int \frac{\nu}{c} (t - t_{PL} - \theta) dt} \]

(11.)

\[ k = \pm i \cdot \theta \text{ or } k = \pm 0 \text{ possible.} \]

3. Conclusion:

There are calculated the eigenwerte for the DE of second order which represents in its solution the advanced Lorentz-Einstein-factor as an amplitude. They exist and are real e-functions.

Critical remark:

For \( a \equiv 0 \) this equation (8b.) leads to

\[ \lambda_{\{n\}} = e^k. \]

(11.)

This formula doesn’t depend from an \( n \) (see brackets in lambda), so it could be reinterpreted as the classical form of SRT-Lorentz-factor because of \( a \equiv 0 \) – but on the other hand this form of a DE of second order can only be written as a STL-problem for constant value of \( p \) . Because \( p \) includes the velocity \( v \) of the inertial system (common classical Lorentz-factor), this velocity can only be supposed as a constant. So in this case acceleration in resp. of inertial systems can’t be described.

Possibly \( k \) could be interpreted as constant supposed phaseangle \( \pm i \cdot \theta \), but in case of classical SRT this phase-angle is identical equal to zero (or multiplies of \( \Pi \) for classical tachyons), so the eigenwert will be reduced to a \( \lambda = |1| \) identity (see Appendix).

4. Summary:

The differential equation of second order for the advanced Lorentz-Einstein-factor in fourth order can be written as a Sturm-Liouville-problem. In this case the eigenvalue(s) can be calculated. They are real e-functions and depend from the velocity \( a \) of the outer model of a damping system. For \( a \equiv 0 \) the differential equation will reduce to the form for classical SRT-factor and possibly the eigenvalue be only the trivial case of \( \lambda = |1| \) or at least \( \lambda = e^k; k = \text{const.} \).
5. Appendix: Deduction of classical Lorentz-Einstein-factor from a DE

The aim is to determine the amplitude factor of A from the following DE of second order:

\[ A \cdot \ddot{\psi}(t) + C \cdot (A - D) \cdot \psi(t) = 0 \]  \hspace{1cm} (A1.)

with:

\[ C = \omega_{PL}^2; D = e^{i \theta} \]  \hspace{1cm} (A2.)

and the plane-wave function

\[ \psi(r) = A \cdot e^{i \left( \frac{v}{c} \cdot \left( \frac{r}{t_{PL}} - 0 \right) \right)} \]  \hspace{1cm} (A3.)

With the following common relations and the transition from \( r \) to \( t \):

\[ (r = v \cdot t; r_{PL} = c \cdot t_{PL}) \Rightarrow \psi(r) \rightarrow \psi(t) \]  \hspace{1cm} (A4.)

there is the function of a planewave depending only of time \( t \):

\[ \psi(t) = A \cdot e^{i \left( \frac{v}{c} \cdot \left( \frac{t}{t_{PL}} - 0 \right) \right)} \]  \hspace{1cm} (A5.)

with its derivations:

\[ \dot{\psi}(t) = i \cdot A \left( \frac{v}{c} \cdot t + \frac{1}{c} \right) \cdot e^{i \left( \frac{v}{c} \cdot \left( \frac{t}{t_{PL}} - 0 \right) \right)} \]  \hspace{1cm} (A6.)

and

\[ \ddot{\psi}(t) = A \left[ i \left( \frac{\dot{v}}{c} \cdot \left( \frac{1}{t_{PL}} + 2 \cdot \frac{1}{c} \right) - \left( \frac{\ddot{v}}{c} \cdot \frac{1}{t_{PL}} + \frac{\dot{v}}{c} \cdot \frac{1}{t_{PL}} \right) \right) \cdot e^{i \left( \frac{v}{c} \cdot \left( \frac{t}{t_{PL}} - 0 \right) \right)} \]  \hspace{1cm} (A7.)

For \( v = \text{const} \) between inertial-systems of classical SRT, this relation of second derivation reduces to:

\[ \ddot{\psi}(t) = -A \frac{v^2}{c^2} \cdot \frac{1}{t_{PL}^2} \cdot e^{i \left( \frac{v}{c} \cdot \left( \frac{t}{t_{PL}} - 0 \right) \right)} \]  \hspace{1cm} (A8.)
Setting $\psi(t) \wedge \dot{\psi}(t)$ in the DE, this leads to calculation of amplitude $A$ of the wave-function $\psi(t)$:

\[ A = \frac{e^{i\theta}}{1 - \frac{v^2}{c^2}} \]  \hspace{1cm} (A9.)

Remark:

Since $e^{(i\theta)} = \cos(\theta) + i\sin(\theta)$ and comparison of coefficients in

\[ e^{(i\theta)} = A \cdot (1 - \frac{v^2}{c^2}) \]  

leads to the conclusion for $i\sin(\theta) = 0$

and the conditions for calculation of $\theta$:

\[ i\sin(\theta) = 0 \text{ for } \theta = k \cdot \pi; k \in \mathbb{Z} \]  

but

\[ \begin{align*}
I \cos(\theta) &= 1 \text{ for } k = 2 \cdot n; n \in \mathbb{Z}; \\
II \cos(\theta) &= -1 \text{ for } k = 2 \cdot n + 1; n \in \mathbb{Z} 
\end{align*} \]  \hspace{1cm} (A10.)

I leads to square of Einstein-Lorentz-factor, II to square of Feinberg-factor of classical, common SRT.

6. References:

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7. **Verification:** this paper is written without using a chatbot like ChatGPT-4 or other chatbots. It is fully human work.