A Modified Born-Infeld Model of Electrons as Rotating Waves

Martin Kraus (kraus.martin@gmail.com)

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Abstract

This work presents a modified Born-Infeld field theory that might include electron-like solutions in the form of rotating waves of finite self-energy. Preliminary numerical experiments suggest that the proposed model might show quantum mechanical features in a classical field theory.

1 Introduction

In the standard interpretation of classical electromagnetism, particles are considered sources of electromagnetic fields via the Maxwell-Heaviside equations, while electromagnetic fields affect particles via the Lorentz force. However, several researchers have tried to describe particles as solutions of electromagnetic field equations with their masses determined solely by the field energy of these solutions (i.e., their electromagnetic masses). For a review of early approaches, see the discussion by Born and Infeld [BIF34].

Due to the successful rise of quantum mechanics, interest in these approaches has dwindled as they could not explain quantum mechanical features of particles. Motivated by this shortcoming, this work proposes a modification of Born and Infeld's electron model [BIF34], which might be able to describe electrons as rotating field solutions of finite self-energy and in this way show quantum mechanical features in spite of being based on a classical field theory.

The Lagrangian density and the corresponding field equations of the proposed model are presented in Section 3 after a presentation of the employed notation in Section 2. Section 4 presents an electrostatic solution, while Section 5 discusses the low-energy limit of the field equations. Preliminary numerical experiments based on this limit are described in Section 6 and discussed in Section 7. Section 8 concludes this work.

2 Notation

This work uses the International System of Units (SI), basic Ricci calculus including Einstein summation convention, and the Minkowski metric tensor η in the form $\mathrm{diag}(+1, -1, -1, -1)$, i.e., its components are:

$$\eta_{\mu\nu} \stackrel{\text{def}}{=} \eta^{\mu\nu} \stackrel{\text{def}}{=} \begin{pmatrix} +1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}. \tag{1}$$

The Levi-Civita symbol $\epsilon_{\alpha\beta\mu\nu}$ is $+1$ if $(\alpha, \beta, \mu, \nu)$ is an even permutation of $(0, 1, 2, 3)$, -1 if it is an odd permutation, and 0 otherwise.

Four-displacements with contravariant components x^ν are written as

$$(x^0, x^1, x^2, x^3) \stackrel{\text{def}}{=} (ct, x, y, z) \tag{2}$$

where c is the speed of light, t is the time component, and x, y, and z are spatial components.

Covariant components x_μ are obtained by index lowering (with an implicit summation over ν due to the Einstein summation convention):

$$x_\mu \stackrel{\text{def}}{=} \eta_{\mu\nu} x^\nu. \tag{3}$$

Thus:

$$(x_0, x_1, x_2, x_3) = (ct, -x, -y, -z). \tag{4}$$

The covariant components of the four-gradient are:

$$(\partial_0, \partial_1, \partial_2, \partial_3) \stackrel{\text{def}}{=} \left(\frac{\partial}{\partial x^0}, \frac{\partial}{\partial x^1}, \frac{\partial}{\partial x^2}, \frac{\partial}{\partial x^3}\right) = \left(\frac{1}{c}\frac{\partial}{\partial t}, \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right) = \left(\frac{1}{c}\frac{\partial}{\partial t}, \nabla\right). \tag{5}$$

Correspondingly, the contravariant components of the four-gradient are:

$$(\partial^0, \partial^1, \partial^2, \partial^3) \stackrel{\text{def}}{=} \left(\frac{\partial}{\partial x_0}, \frac{\partial}{\partial x_1}, \frac{\partial}{\partial x_2}, \frac{\partial}{\partial x_3}\right) = \left(\frac{1}{c}\frac{\partial}{\partial t}, -\frac{\partial}{\partial x}, -\frac{\partial}{\partial y}, -\frac{\partial}{\partial z}\right) = \left(\frac{1}{c}\frac{\partial}{\partial t}, -\nabla\right). \tag{6}$$

Contravariant components A^μ of the electromagnetic four-potential are:

$$(A^0, A^1, A^2, A^3) \stackrel{\text{def}}{=} (\phi/c, A_x, A_y, A_z) = (\phi/c, \mathbf{A}) \tag{7}$$

with the electric potential ϕ and the magnetic vector potential $\mathbf{A} = (A_x, A_y, A_z)$. Covariant components are defined by $A_\mu \stackrel{\text{def}}{=} \eta_{\mu\nu} A^\nu$; thus, we have:

$$(A_0, A_1, A_2, A_3) = (\phi/c, -A_x, -A_y, -A_z) = (\phi/c, -\mathbf{A}). \tag{8}$$

Covariant components $F_{\mu\nu}$ of the electromagnetic field tensor are defined in this way:

$$F_{\mu\nu} \stackrel{\text{def}}{=} \partial_\mu A_\nu - \partial_\nu A_\mu. \tag{9}$$

Contravariant components $F^{\mu\nu}$ are defined by index raising:

$$F^{\mu\nu} \stackrel{\text{def}}{=} \eta^{\mu\alpha}\eta^{\nu\beta} F_{\alpha\beta}. \tag{10}$$

The electric field $\mathbf{E}$ and magnetic field $\mathbf{B}$ are defined with the help of $F_{\mu\nu}$:

$$\begin{aligned}
\mathbf{E} = (E_x, E_y, E_z) &\stackrel{\text{def}}{=} (cF_{01}, cF_{02}, cF_{03}) && (11)\\
&= (c\partial_0 A_1 - c\partial_1 A_0, c\partial_0 A_2 - c\partial_2 A_0, c\partial_0 A_3 - c\partial_3 A_0) && (12)\\
&= \left(-\frac{\partial}{\partial t}A_x - c\frac{\partial}{\partial x}\frac{\phi}{c}, -\frac{\partial}{\partial t}A_y - c\frac{\partial}{\partial y}\frac{\phi}{c}, -\frac{\partial}{\partial t}A_z - c\frac{\partial}{\partial z}\frac{\phi}{c}\right) && (13)\\
&= -\frac{\partial}{\partial t}\mathbf{A} - \nabla\phi, && (14)\\
\mathbf{B} = (B_x, B_y, B_z) &\stackrel{\text{def}}{=} (F_{32}, F_{13}, F_{21}) && (15)\\
&= (\partial_3 A_2 - \partial_2 A_3, \partial_1 A_3 - \partial_3 A_1, \partial_2 A_1 - \partial_1 A_2) && (16)\\
&= \left(\frac{\partial}{\partial y}A_z - \frac{\partial}{\partial z}A_y, \frac{\partial}{\partial z}A_x - \frac{\partial}{\partial x}A_z, \frac{\partial}{\partial x}A_y - \frac{\partial}{\partial y}A_x\right) && (17)\\
&= \nabla \times \mathbf{A}. && (18)
\end{aligned}$$

In matrix form, the relations between $F_{\mu\nu}$, $F^{\mu\nu}$, $\mathbf{E}$, and $\mathbf{B}$ are:

$$F_{\mu\nu} = \begin{pmatrix} 0 & E_x/c & E_y/c & E_z/c \\ -E_x/c & 0 & -B_z & B_y \\ -E_y/c & B_z & 0 & -B_x \\ -E_z/c & -B_y & B_x & 0 \end{pmatrix} \tag{19}$$

and

$$F^{\mu\nu} = \begin{pmatrix} 0 & -E_x/c & -E_y/c & -E_z/c \\ E_x/c & 0 & -B_z & B_y \\ E_y/c & B_z & 0 & -B_x \\ E_z/c & -B_y & B_x & 0 \end{pmatrix}. \tag{20}$$

With our definition of $\mathbf{B}$, the homogeneous Maxwell-Heaviside equation

$$\nabla \cdot \mathbf{B} = \mathbf{0} \tag{21}$$

is true since $\nabla \cdot \mathbf{B} = \nabla \cdot (\nabla \times \mathbf{A})$ and the divergence of the curl of any continuously twice-differentiable vector field is equal to zero.

The homogeneous Maxwell-Heaviside equation

$$\nabla \times \mathbf{E} = -\frac{\partial}{\partial t}\mathbf{B} \tag{22}$$

is true for our definition of $\mathbf{E}$ since $\nabla \times \mathbf{E} = \nabla \times \left(-\frac{\partial}{\partial t}\mathbf{A} - \nabla\phi\right) = \nabla \times \left(-\frac{\partial}{\partial t}\mathbf{A}\right) - \nabla \times \nabla\phi$. The second term $\nabla \times \nabla\phi$ is equal to zero since the curl of the gradient of any continuously twice-differentiable scalar field is equal to zero. For the remaining term, we find: $\nabla \times \left(-\frac{\partial}{\partial t}\mathbf{A}\right) = -\frac{\partial}{\partial t}\left(\nabla \times \mathbf{A}\right) = -\frac{\partial}{\partial t}\mathbf{B}$, which is the right-hand side of the Maxwell-Heaviside equation above.

Usually, the inhomogeneous Maxwell-Heaviside equations determine the electromagnetic field for a given charge density ρ and current density $\mathbf{J}$ that exist independently of the electromagnetic field. In the proposed model, however, charges and densities are features of the electromagnetic field. Therefore, the inhomogeneous Maxwell-Heaviside equations become definitions of the contravariant components J^ν of the four-current:

$$J^\nu \stackrel{\text{def}}{=} \frac{1}{\mu_0}\partial_\mu F^{\mu\nu} \tag{23}$$

with the covariant components J_μ defined by index lowering, i.e., $J_\mu \stackrel{\text{def}}{=} \eta_{\mu\nu}J^\nu$. Charge density ρ and current density $\mathbf{J}$ are then defined based on the contravariant components of the four-current:

$$(c\rho, \mathbf{J}) = (c\rho, J_x, J_y, J_z) \stackrel{\text{def}}{=} (J^0, J^1, J^2, J^3) \tag{24}$$

One consequence of this definition of the four-current is a continuity equation:

$$\partial_\nu J^\nu = \partial_\nu \frac{1}{\mu_0}\partial_\mu F^{\mu\nu} = \frac{1}{\mu_0}\left(\partial_\nu\partial_\mu\partial^\mu A^\nu - \partial_\nu\partial_\mu\partial^\nu A^\mu\right) = 0, \tag{25}$$

or in terms of charge density ρ and current density $\mathbf{J}$:

$$0 = \partial_\nu J^\nu = \frac{\partial}{\partial t}\rho + \nabla \cdot \mathbf{J}. \tag{26}$$

3 Lagrangian Density and Field Equations

Konopinski observed that a Lagrangian density with the term $(\partial^\mu A^\nu)(\partial_\mu A_\nu)$ instead of $F^{\mu\nu}F_{\mu\nu}$ leads to Euler-Lagrange equations that can displace Maxwell-Heaviside equations [Kon78, Equations 20 and 21]. The proposed model is based on a similar modification to the Lagrangian density proposed by Born and Infeld [BIF34, Equation 2.11]. Specifically, the Lagrangian density $\mathscr{L}(A_\nu, \partial_\mu A_\nu)$ of the proposed model is:

$$\mathscr{L}(A_\nu, \partial_\mu A_\nu) \stackrel{\text{def}}{=} \sqrt{1 + \frac{1}{b^2}(\partial^\mu A^\nu)(\partial_\mu A_\nu)} - 1. \tag{27}$$

The Euler-Lagrange equations are then:

$$0 = \partial_\mu\left(\frac{\partial\mathscr{L}(A_\nu, \partial_\mu A_\nu)}{\partial\left(\partial_\mu A_\nu\right)}\right) - \frac{\partial\mathscr{L}(A_\nu, \partial_\mu A_\nu)}{\partial A_\nu} = \partial_\mu \frac{1}{b^2}\frac{\partial^\mu A^\nu}{\sqrt{1 + \frac{1}{b^2}(\partial^\alpha A^\beta)(\partial_\alpha A_\beta)}}. \tag{28}$$

In analogy to the work by Born and Infeld [BIF34, Equation 3.3A], we define $D^{\mu\nu}$:

$$D^{\mu\nu} \stackrel{\text{def}}{=} b^2\frac{\partial\mathscr{L}(A_\nu, \partial_\mu A_\nu)}{\partial\left(\partial_\mu A_\nu\right)} = \frac{\partial^\mu A^\nu}{\sqrt{1 + \frac{1}{b^2}(\partial^\alpha A^\beta)(\partial_\alpha A_\beta)}} \tag{29}$$

such that the Euler-Lagrange equations may be written as:

$$\partial_\mu D^{\mu\nu} = 0. \tag{30}$$

The motivation for using the term $(\partial^\mu A^\nu)(\partial_\mu A_\nu)$ in the Lagrangian density (instead of $F^{\mu\nu}F_{\mu\nu}$) is to avoid constraints imposed by the inhomogeneous Maxwell-Heaviside equations. As mentioned in Section 2, these equations appear in the proposed model only as definitions of the four-current:

$$J^\nu \stackrel{\text{def}}{=} \frac{1}{\mu_0}\partial_\mu F^{\mu\nu}. \tag{31}$$

Without the inhomogeneous Maxwell-Heaviside equations, additional field solutions are possible as discussed in Section 6. First, however, the next two sections discuss an electrostatic solution and the field equations in the low-energy limit.

4 Electrostatic Solution

This section follows work by Born and Infeld [BIF34, Section 6] to show that the modified Lagrangian density of the proposed model results in the same spherically symmetric, electrostatic solution as the one found by Born and Infeld.

For an electrostatic solution, we assume $\mathbf{B} = 0$, $\mathbf{A} = 0$, and all other quantities independent of time. In this case, the field equations become:

$$0 = \partial_\mu D^{\mu 0} = \partial_1 D^{10} + \partial_2 D^{20} + \partial_3 D^{30}. \tag{32}$$

With the definition

$$\mathbf{D} \stackrel{\text{def}}{=} \left(D^{10}, D^{20}, D^{30}\right), \tag{33}$$

the equation takes the form

$$\nabla \cdot \mathbf{D} = 0 \tag{34}$$

in analogy to Equation 6.2 by Born and Infeld [BIF34]. In the case of a spherically symmetric solution, the radial component D_r of $\mathbf{D}$ satisfies:

$$\frac{\mathrm{d}}{\mathrm{d}r} r^2 D_r = 0; \tag{35}$$

thus,

$$D_r \propto 1/r^2. \tag{36}$$

Furthermore, all terms $\partial^\mu A^\nu$ are 0 for $\mu = 0$ or $\nu \neq 0$; thus, only three terms are unequal to 0:

$$\left(\partial^1 A^0, \partial^2 A^0, \partial^3 A^0\right) = -\nabla\phi/c = \mathbf{E}/c. \tag{37}$$

The radial component E_r of $\mathbf{E}$ is then

$$E_r = -\frac{\mathrm{d}}{\mathrm{d}r}\phi. \tag{38}$$

The definition of $D^{\mu\nu}$ results in

$$D_r = \frac{E_r/c}{\sqrt{1 - \frac{1}{b^2}(E_r/c)^2}}, \tag{39}$$

leading to the same differential equation for $\phi = \phi(r)$ as solved by Born and Infeld [BIF34, Equation 6.8]. Therefore, the proposed model includes the same electrostatic solution as the original model by Born and Infeld.

Born and Infeld showed that the electrostatic solution in their model is subject to a force similar to the Lorentz force acting on charged particles in standard electromagnetism. Whether this result also holds for the proposed model is an open question.

5 Low-Energy Limit

In the low-energy limit, i.e., for $(\partial^\alpha A^\beta)(\partial_\alpha A_\beta)/b^2 \approx 0$, the field equations become:

$$0 = \partial_\mu \frac{\partial^\mu A^\nu}{\sqrt{1 + \frac{1}{b^2}(\partial^\alpha A^\beta)(\partial_\alpha A_\beta)}} \approx \partial_\mu \partial^\mu A^\nu \approx 0. \tag{40}$$

The approximations $\partial_\mu \partial^\mu A^\nu \approx 0$ have the same form as the inhomogeneous Maxwell-Heaviside equations for $J^\nu = 0$ in Lorenz gauge, i.e., with the gauge condition $\partial_\mu A^\mu = 0$. In the proposed model, however, the approximations $\partial_\mu \partial^\mu A^\nu \approx 0$ are valid without requiring a specific gauge condition, and they do not imply that $J^\nu = 0$. In fact, it is easy to construct low-energy field solutions with $J^\nu \neq 0$ (with J^ν as defined above); for example, longitudinal waves.

There is, however, a consequence of the proposed field equations that might suppress longitudinal waves and other solutions with $J^\nu \neq 0$ at macroscopic scales. First, assume that partial derivatives are also approximately 0:

$$\partial^\lambda \partial_\mu \partial^\mu A^\nu \approx 0. \tag{41}$$

In this case, combinations of such terms are also approximately 0:

$$\epsilon_{\alpha\beta\lambda\nu} \partial^\alpha \partial_\mu \partial^\mu A^\beta \approx 0. \tag{42}$$

Second, the following sums are exactly 0 as long as the partial derivatives are interchangeable because for every term unequal to 0, there is a corresponding term of the negative value with the values of α and β interchanged:

$$\epsilon_{\alpha\beta\lambda\nu} \partial^\alpha \partial_\mu \partial^\beta A^\mu = 0. \tag{43}$$

Combining these expressions and employing the definitions of $F^{\mu\beta}$, J^β, and $\epsilon_{\alpha\beta\lambda\nu}$ yields:

$$\begin{aligned}
0 &\approx \epsilon_{\alpha\beta\lambda\nu} \partial^\alpha \partial_\mu \partial^\mu A^\beta - \epsilon_{\alpha\beta\lambda\nu} \partial^\alpha \partial_\mu \partial^\beta A^\mu && (44)\\
&= \epsilon_{\alpha\beta\lambda\nu} \partial^\alpha \partial_\mu \left(\partial^\mu A^\beta - \partial^\beta A^\mu \right) && (45)\\
&= \epsilon_{\alpha\beta\lambda\nu} \partial^\alpha \partial_\mu F^{\mu\beta} && (46)\\
&= \epsilon_{\alpha\beta\lambda\nu} \mu_0 \partial^\alpha J^\beta. && (47)
\end{aligned}$$

The approximation holds for all pairs (λ, ν), thus:

$$0 \approx \partial^\lambda J^\nu - \partial^\nu J^\lambda. \tag{48}$$

In terms of ρ and $\mathbf{J}$, these approximations read:

$$\frac{\partial}{\partial t} \mathbf{J} \approx -c^2 \nabla \rho \qquad \text{and} \qquad \nabla \times \mathbf{J} \approx \mathbf{0}. \tag{49}$$

Considering the continuity equation $\frac{\partial}{\partial t}\rho = -\nabla \cdot \mathbf{J}$, a physical interpretation of $\frac{\partial}{\partial t}\mathbf{J} \approx -c^2 \nabla \rho$ might be that any gradient of charge density increases current density that diffuses this charge density. This might eliminate all charge densities at macroscopic scales for low field energies while stable charges might tend to concentrate at points with high field energies, where these approximations do not apply.

It might be worth noting that the vacuum wave equation $\partial_\mu \partial^\mu F^{\lambda\nu} = 0$ in standard electromagnetism allows for a similar result with the crucial difference that the vacuum wave equation assumes $J^\nu = 0$, and, therefore, the result is trivially true.

6 Preliminary Numerical Experiments

The main motivation for the proposed modification of the Born-Infeld model [BIF34] is that the resulting low-energy field equations $\partial_\mu \partial^\mu A^\nu \approx 0$ allow for rotating wave solutions with $J^\nu \stackrel{\text{def}}{=} \partial_\mu F^{\mu\nu}/\mu_0 \neq 0$, which are not possible with the standard electromagnetic wave equations in vacuum.

Similarities between rotating waves and elementary particles (including angular momentum) have been noted by many authors, for example, Ceperley [Cep92]. By combining features of rotating waves with the Born-Infeld model of electrons, the proposed model tries to achieve a more complete description of electrons. Unfortunately, the relevant rotating solutions apparently are not separable; thus, numerical methods were used to gain some first, preliminary insights into the nature of these rotating solutions.

The preliminary experiments used a 128^3 grid with four 16-bit floating-point numbers at each grid point representing the A^ν field. (The reason for the limited floating-point precision was that computations were performed on a graphics processing unit using ping-pong rendering between two 2048×1024 render textures.) Instead of representing the time dimension by a four-dimensional grid, a rotating movement of the "frozen" solution around a rotation center located at the center of the 128^3 grid was assumed, such that time derivatives at a grid point could be approximated by sampling the grid (using bilinear interpolation) at appropriately rotated positions of the grid point, while spatial derivatives of A^ν were approximated by finite differences.

Solutions were improved iteratively by a Jacobi method. Each Jacobi step was followed by a $3 \times 3 \times 3$ triangle filter step, which was employed to achieve convergence. After the filter step, boundary

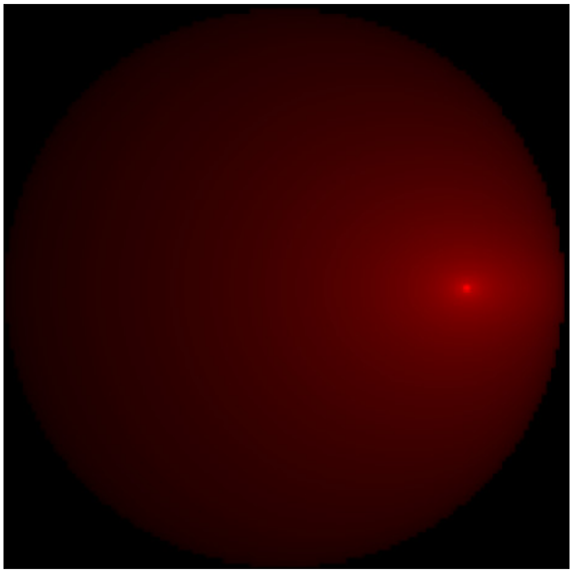

Figure 1: Visualization of the electric potential $A^0 = \phi/c$ on a slice of a 128^3 grid containing the rotation center in the center of the image and the off-center non-linear peak.

conditions were enforced by setting A^ν of grid points on a spherical boundary around the rotation center to 0, and one $3 \times 3 \times 3$ set of grid points (one grid point for A^0) was chosen to contain the non-linear peak of the solution (see Section 4), which was treated as a boundary condition. The angular velocity of the rotation was chosen such that the non-linear peak moves with the speed of light.

Note that the size of the non-linear peak of the electrostatic solution of the Born-Infeld model [BIF34, Equation 8.7] is much smaller than the reduced Compton wavelength of electrons, which was assumed to correspond to the radius of the orbit of the non-linear peak around the rotation center. In fact, even an individual grid cell of the employed 128^3 grid is much larger than the size of the non-linear peak, which justifies that the peak was treated as a boundary condition in these first numerical experiments.

For non-linear peaks that were not too close to the rotation center, the computation converged within a few thousand Jacobi steps. Figure 1 shows a rotating solution for $A^0 = \phi/c$ on the slice of the grid that contains the non-linear peak and its orbit; brighter colors represent stronger A^0 values. (A non-linear color mapping was used to enhance the image.) The center of the image is also the rotation center of the solution.

Given the imposed boundary conditions, it is no surprise that the solution for $A^0 = \phi/c$ depicted in Figure 1 resembles a "deformed" Coulomb potential with the non-linear peak orbiting a rotation center, while parts of the solution farther away from the rotation center are affected less by this movement. This is a notable difference to an orbiting charge in standard electromagnetism where its whole (retarded) Coulomb potential follows the movement.

Similar computations were performed for the three components of the magnetic vector potential $\mathbf{A}$ with the results resembling a "deformed" dipole field with its peak orbiting around a rotation center. This is in contrast to the field of a circular current loop.

Solutions were computed for various positions of the non-linear peak, i.e., for various distances between the non-linear peak and the rotation center. In the solutions for the electric potential $A^0 = \phi/c$, a potential difference between neighboring grid points next to the non-linear peak could be observed with the stronger potential at the neighboring grid point farther away from the rotation center. For non-linear peaks not too close to the boundary, the potential difference decreased with increasing distance of the non-linear peak to the rotation center at a faster than quadratic rate.

As mentioned above, this section only reports preliminary results since more refined experiments are necessary to obtain reliable results.

7 Discussion of Preliminary Results

Due to the preliminary nature of the results presented in the previous section, the discussion in this section is highly speculative. For example, it was noted at the end of Section 4 that it is not known whether the proposed model includes an effect similar to the Lorentz force; however, the following discussion assumes that this is in fact the case.

Combined with the observed potential difference described in the previous section, the assumption of a Lorentz-like force suggests the possibility of a force on the non-linear peak towards the rotation center. The picture that emerges is that of a rotating field solution that obeys at large scales (relatively to the size of its peak) an approximately linear field equation and includes a potential gradient at the location of its non-linear peak. This potential gradient might pull the peak onto a circular orbit via a force similar to the Lorentz force.

Following this line of speculation further, the (stronger than linear) decrease of the potential gradient with increasing distance between rotation center and the non-linear peak might lead to an equilibrium condition that determines the radius of a stable orbit. It is tempting to identify this radius with the reduced Compton wavelength and the equilibrium condition with the definition of the reduced Compton wavelength. If this is in fact the case, the circular movement of the non-linear peak could be identified with the periodic process that was assumed by de Broglie [dB25] to explain wave-like (i.e., quantum mechanical) features of elementary particles.

In this way, the proposed model would not only provide a classical model of electrons with quantum mechanical features, but it would also suggest specific interpretations of quantum theories. For example, it would suggest an ensemble interpretation of quantum mechanics and an interpretation of the path integral formulation of quantum field theory as a generalized Huygens' principle for waves that describe the movement of rotating field solutions with unknown rotation angle.

8 Conclusion and Future Work

This work introduces a modification of the Born-Infeld field theory that might include rotating field solutions of finite self-energy that might show quantum mechanical features. Preliminary numerical experiments are encouraging but far from providing definitive results. Therefore, current work is focused on more elaborate numerical experiments. If these experiments are successful, there are many possible avenues for future research. To name just one: many other choices for the Lagrangian density might be possible. Choosing the specific form inspired by the Born-Infeld model is convenient (see Section 4), but there are many other forms that have the same low-energy limit discussed in Section 5.

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A Revisions

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- Revision April 19, 2023: Revised Section 6 and Figure 1 after fixing a programming error and rerunning the numerical experiments.
- Revision November 14, 2023: Added μ_0 to Equation (47); fixed minus sign in Equation (39)
- Revision November 28, 2024: Added ∂'s in Equations (28) and (29).