Some remarks concerning the factorization of mirror composite numbers and its relationship with Goldbag conjecture.

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Abstract:

In this paper we present the concept of mirror composite numbers. Mirror composite numbers are composite numbers of the form 2n-p for some n positive natural number and p prime. We shall show that the factorization of these numbers have interesting properties in order to face the Goldbach conjecture by the *divide et impera* method.

Definitions:

From now on, *m* and *n* are positive natural numbers, *p* and *q* are prime numbers.

All prime numbers p>=5 are of the form 6m+1 or 6m-1. A prime of the form 6m+1 is a **right prime**; a prime of the form 6m-1 is a **left prime**.

A **mirror composite number** is a composite number of the form 2n-p for some n and some prime p>=5.

Given a mirror composite 2n-p, if p=6m+1, i.e., if p is a right prime, 2n-p is a **right mirror composite (r.m.c.)**.

Given a mirror composite 2n-p, If p=6m-1, i.e., if p is a left prime, 2n-p is a **left mirror composite (l.m.c.)**.

Lemma 1.

Fixed n, if 3 is a factor of some l.m.c (respectively r.m.c.), 3 is a factor of every l.m.c. (r.m.c.) and 3 is not a factor of any r.m.c. (l.m.c)

Proof:

The difference between two l.m.c. (r.m.c.) is 6n. If $3|m$, $3|m\pm 6n$. On the other hand, if $3|2n-(6m-1)$, then $3 \nmid 2n-(6m+1)$ and *viceversa*.

Lemma 2.

Fixed n, if $q\neq 3$ is a prime factor of two different l.m.c. (respectively r.m.c.), the difference between them is a multiple of 6q so the minimum gap between two consecutive occurrences of factor q is 6q for all l.m.c. (r.m.c.).

Proof:

If $q \mid 2n-(6x-1)$ and $q \mid 2n-(6y-1)$ exists z such that zq=6(x-y), so z is multiple of 6, given that q is a prime and $q \neq 2,3$.

If $q \mid 2n-(6x+1)$ and $q \mid 2n-(6y+1)$ exists z such that zq=6(x-y), so z is multiple of 6, given that q is a prime and $q \neq 2,3$.

Goldbach conjecture states that for all n and all p such that $3 \leq p \leq 2n-3$, some 2n-p is a prime, i.e., not every 2n-p is composite.

Let´s suppose for the sake of contradiction that exists n such that every 2n-p is composite. Then, 3 consecutive odd numbers, 2n-3, 2n-5 and 2n-7 are composite, so one and only one of them must be multiple of 3.

Case A: $3|2n-7$:

$3|2n-7 \Rightarrow 3|2n-(6m+1)$ for all m (**Lemma 1**). So every right mirror composite is a multiple of 3 and no left mirror composite is a multiple of 3. So all element of the sequence:

$$2n-3,\ 2n-5,\ 2n-11,\ 2n-17,\ 2n-23, \ldots\ 2n-q.$$

where q is a left prime $5 \leq q \leq 2n-3$, must be factorized. There are i consecutive primes p_i from $p_1=5$ to p_k, where p_k is the largest prime less than $\sqrt{2n}$, available for that factorization.

Now, given the correlative sequence of odd numbers 2n-3, 2n-5, 2n-7, 2n-9, 2n-11, 2n-13, 2n-15, 2n-a…, let be $2n-a_i$ the number containing the first occurrence of prime factor p_i in that sequence.

Notice that:

For each p_i, a_i is unique.

$3 \leq a_i \leq 2p_i+1$.

For some i, $a_i = 3$; for some i, $a_i=5$; for some i, $a_i=11$ MOD p_i; for some i, $a_i=17$ MOD p_i; for some i, $a_i=23$ MOD p_i and so on.

2n-q, i.e., 2n-(6m-1), is composite if and only if exists i such that $6m-1 \equiv a_1 \bmod p_i$ (**Lemma 2**).

Now, let´s state conditions in order to find some 2n-q with q=6m-1 and q inside the interval $\sqrt{2n} < q \leq 2n-3$ that can not be factorized:

1) q is a prime, i.e., q is not multiple of any p_i, so $6m-1 \not\equiv 0 \bmod p_i$ for all i.
2) There is no p_i factor available for 2n-q, so $6m-1 \not\equiv a_1 \bmod p_i$ for all i.

Prime condition for 6m-1	No factor available condition for 2n-(6m-1)
$6m \not\equiv 1 \bmod 5$	$6m \not\equiv (a_1+1) \bmod 5$
$6m \not\equiv 1 \bmod 7$	$6m \not\equiv (a_2+1) \bmod 7$
$6m \not\equiv 1 \bmod 11$	$6m \not\equiv (a_3+1) \bmod 11$

$6m \not\equiv 1 \bmod 13$ $\quad$ $6m \not\equiv (a_4+1) \bmod 13$

………….. $\quad$ ……………….

$6m \not\equiv 1 \bmod p_k$ $\quad$ $6m \not\equiv (a_k+1) \bmod p_k$

Hence for each p_i there are *at least* p_i-2 remainders moduli p_i that fullfill the conditions. That amounts up to a minimum of 3.5.9.11….(p_k-2) different systems of linear congruences whith prime moduli, each one of them has a different and unique solution, not every one outside the aforementioned interval.

For now, it will be enough to notice that at least p_i-2 remainders fullfill the conditions for each p_i to conclude (Pigeonhole strong form principle) that at least exists some (in fact, a lot of) 6m that fullfills the conditions for all p_i. Hence, exists some 2n-q that can not be factorized, so 2n-q is prime and the conjecture holds for all 2n such that 3|2n-7, i.e., for all $2n \equiv 1 \bmod 3$.

Case B: 3|2n-5:

3|2n-5⇒ 3|2n-(6m-1) for all m (**Lemma 1**). So every left mirror composite is a multiple of 3 and no right mirror composite is a multiple of 3…

Following the same thought process than before, with q a right prime of the form 6m+1, it´s straightforward to conclude that the conjecture holds for all 2n such that 3|2n-5, i.e., for all $2n \equiv 2 \bmod 3$.

Case C: 3|2n-3:

Matter of forward research.

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