Solution for the density parameter of dark energy

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Abstract

In this paper we will propose a possible solution for the density parameter of dark energy. From the dimensionless unification of the fundamental interactions will be presended the formulas for the density parameter of dark energy. Also we will show the geometric representation of the density parameter for dark energy and the geometric representation of the relationship between the de Sitter radius and the Hubble length.

Keywords

Density parameter of dark energy , Cosmological parameters , Cosmological constant , Fine-structure constant , Proton to electron mass ratio , Dimensionless physical constants , Coupling constant , Gravitational constant , Avogadro's number , Fundamental Interactions , Gravitational fine-structure constant

1. Introduction

In [1] we presented exact and approximate expressions between the Archimedes constant π,the golden ratio φ,the Euler's number e and the imaginary number i. New interpretation and very accurate values of the fine-structure constant has been discovered in terms of the Archimedes constant and the golden radio. We propose in [2] , [3] and [4] the exact formula for the fine-structure constant α with the golden angle,the relativity factor and the fifth power of the golden mean:

$$\alpha^{-1}=360\cdot\varphi^{-2}-2\cdot\varphi^{-3}+(3\cdot\varphi)^{-5}=137.035999164... \quad (1)$$

Also we propose in [4] , [5] and [6] a simple and accurate expression for the fine-structure constant α in terms of the Archimedes constant π:

$$\alpha^{-1}=2\cdot3\cdot11\cdot41\cdot43^{-1}\cdot\pi\cdot\ln2=137.035999078... \quad (2)$$

We propose in [7] the exact mathematical expression for the proton to electron mass ratio:

$$\mu^{32}=\varphi^{-42}\cdot F_5^{160}\cdot L_5^{47}\cdot L_{19}^{40/19} \Rightarrow \mu=1836.15267343... \quad (3)$$

$$7\cdot\mu^3=165^3\cdot\ln^{11}10 \Rightarrow \mu=1836.15267392... \quad (4)$$

$$\mu=6\cdot\pi^5+\pi^{-3}+2\cdot\pi^{-6}+2\cdot\pi^{-8}+2\cdot\pi^{-10}+2\cdot\pi^{-13}+\pi^{-15}=1836.15267343... \quad (5)$$

Also in [7] was presented the exact mathematical expressions that connects the proton to electron mass ratio μ and the fine-structure constant α:

$$9\cdot\mu-119\cdot\alpha^{-1}=5\cdot(\varphi+42) \quad (6)$$

$$\mu-6\cdot\alpha^{-1}=360\cdot\varphi-165\cdot\pi+345\cdot e+12 \quad (7)$$

$$\mu-182\cdot\alpha=141\cdot\varphi+495\cdot\pi-66\cdot e+231 \quad (8)$$

$$\mu - 807\cdot\alpha = 1205\cdot\pi - 518\cdot\varphi - 411\cdot e \quad (9)$$

In [8] was presented the unity formula that connects the fine-structure constant and the proton to electron mass ratio. It was explained that $\mu\cdot\alpha^{-1}$ is one of the roots of the following trigonometric equation:

$$2\cdot 10^2\cdot\cos(\mu\cdot\alpha^{-1}) + 13^2 = 0 \quad (10)$$

The exponential form of this equation is:

$$10^2\cdot(e^{i\mu/\alpha} + e^{-i\mu/\alpha}) + 13^2 = 0 \quad (11)$$

Also this unity formula can also be written in the form:

$$10\cdot(e^{i\mu/\alpha} + e^{-i\mu/\alpha})^{1/2} = 13\cdot i \quad (12)$$

It was presented in [9] the mathematical formulas that connects the proton to electron mass ratio μ,the fine-structure constant α,the ratio N_1 of electric force to gravitational force between electron and proton,the Avogadro's number N_A,the gravitational coupling constant α_G of the electron and the gravitational coupling constant of the proton $\alpha_{G(p)}$:

$$4\cdot e^2\cdot\alpha^2\cdot\alpha_G\cdot N_A^2 = 1 \quad (13)$$

$$\mu^2 = 4\cdot e^2\cdot\alpha^2\cdot\alpha_{G(p)}\cdot N_A^2 \quad (14)$$

$$\mu\cdot N_1 = 4\cdot e^2\cdot\alpha^3\cdot N_A^2 \quad (15)$$

$$4\cdot e^2\cdot\alpha\cdot\mu\cdot\alpha_G^2\cdot N_A^2\cdot N_1 = 1 \quad (16)$$

$$\mu^3 = 4\cdot e^2\cdot\alpha\cdot\alpha_{G(p)}^2\cdot N_A^2\cdot N_1 \quad (17)$$

$$\mu^2 = 4\cdot e^2\cdot\alpha_G\cdot\alpha_{G(p)}^2\cdot N_A^2\cdot N_1^2 \quad (18)$$

$$\mu = 4\cdot e^2\cdot\alpha\cdot\alpha_G\cdot\alpha_{G(p)}\cdot N_A^2\cdot N_1 \quad (19)$$

In [10] we presented the recommended value for the strong coupling constant:

$$\alpha_s = \frac{Euler'\ number}{Gerford's\ constant} = \frac{e}{e^{\pi}} = e^{1-\pi} = 0,11748.. \quad (20)$$

This value is the current world average value for the coupling evaluated at the Z-boson mass scale.
In the papers [11],[12],[13] and [14] was presented the unification of the fundamental interactions. We found the unity formulas that connect the strong coupling constant α_s and the weak coupling constant α_w. We reached the conclusion of the dimensionless unification of the strong nuclear and the weak nuclear interactions:

$$e\cdot\alpha_s = 10^7\cdot\alpha_w \quad (21)$$

$$\alpha_s^2 = i^{2i}\cdot 10^7\cdot\alpha_w \quad (22)$$

Resulting the unity formulas that connects the strong coupling constant α_s and the fine-structure constant α:

$$\alpha_s\cdot\cos\alpha^{-1} = i^{2i} \quad (23)$$

$$\cos\alpha^{-1} = \frac{\alpha_s^{-1}}{e^{\pi}} \quad (24)$$

The figure 1 below shows the angle in α^{-1} radians. The rotation vector moves in a circle of radius e^{π}.

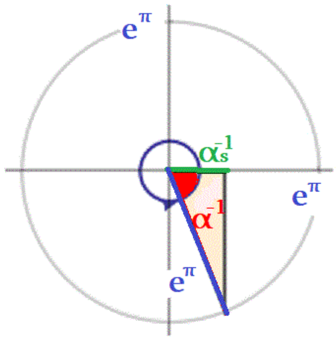

Figure 1. *The angle in* α^{-1} *radians.The rotation vector moves in a circle of radius* e^{π}.

We reached the conclusion of the dimensionless unification of the strong nuclear and the electromagnetic interactions:

$$e^{\pi}\cdot\alpha_s\cdot(e^{i/\alpha}+e^{-i/\alpha})=2 \qquad (25)$$

$$\alpha_s\cdot(e^{i/\alpha}+e^{-i/\alpha})=2\cdot i^{2i} \qquad (26)$$

The figure 2 below shows the geometric representation of the dimensionless unification of the strong nuclear and the electromagnetic interactions.

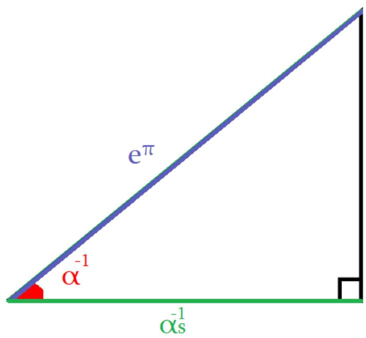

Figure 2. *Geometric representation of the dimensionless unification of the strong nuclear and the electromagnetic interactions.*

The electroweak theory,in physics,is the theory that describes both the electromagnetic force and the weak force. We reached the conclusion of the dimensionless unification of the weak nuclear and the electromagnetic forces:

$$10^{7}\cdot\alpha_w\cdot(e^{i/\alpha}+e^{-i/\alpha})=2\cdot e\cdot i^{2i} \qquad (27)$$

The figure 3 below shows the angle in α^{-1} radians. The rotation vector moves in a circle of radius $10^{7}\cdot e^{\pi-1}$.

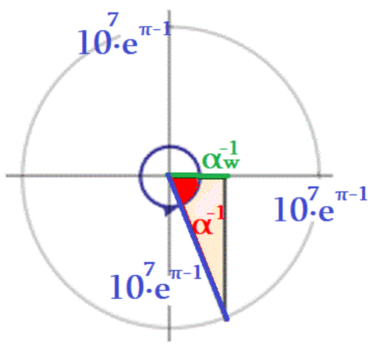

Figure 3. *The angle in* α^{-1} *radians. The rotation vector moves in a circle of radius* $10^{7}\cdot e^{\pi-1}$.

The figure 4 below shows the geometric representation of the dimensionless unification of the weak nuclear and the electromagnetic interactions.

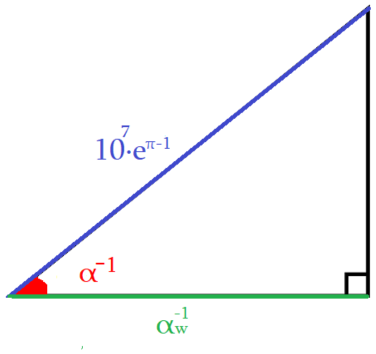

Figure 4. *Geometric representation of the dimensionless unification of the weak nuclear and the electromagnetic interactions*

Resulting the unity formulas that connects the strong coupling constant α_s,the weak coupling constant α_w and the fine-structure constant α:

$$10^7 \cdot \alpha_w \cdot \cos\alpha^{-1} = \alpha_s \tag{28}$$

$$\cos\alpha^{-1} = \frac{\alpha_s \alpha_w^{-1}}{10^7} \tag{29}$$

The figure 5 below shows the angle in α^{-1} radians. The rotation vector moves in a circle of radius 10^7.

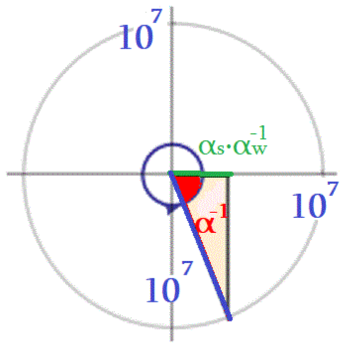

Figure 5. *The angle in α^{-1} radians.The rotation vector moves in a circle of radius 10^7.*

The figure 6 below shows the dimensionless unification of the strong nuclear,the weak nuclear and the electromagnetic interactions.

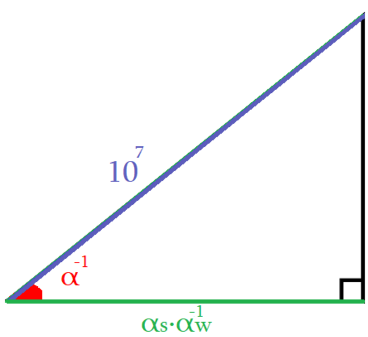

Figure 6. *Geometric representation of the dimensionless unification of the strong nuclear,the weak nuclear and the electromagnetic interactions.*

We reached the conclusion of the dimensionless unification of the strong nuclear,the weak nuclear and the electromagnetic forces:

$$10^7 \cdot \alpha_W \cdot (e^{i/\alpha}+e^{-i/\alpha})=2 \cdot \alpha_S \quad (30)$$

Resulting the unity formula that connects the fine-structure constant α,the gravitational coupling constant α_G and the Avogadro's number N_A:

$$4 \cdot e^2 \cdot \alpha^2 \cdot \alpha_G \cdot N_A^2=1 \quad (31)$$

$$\alpha^{-2} \cdot \cos^2\alpha^{-1}=4 \cdot \alpha_G \cdot N_A^2 \quad (32)$$

The figure 7 below shows the angle in α^{-1} radians. The rotation vector moves in a circle of radius N_A^{-1}.

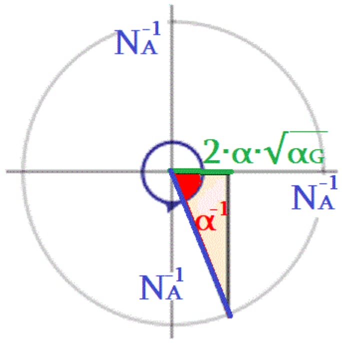

Figure 7. *The angle in α^{-1} radians*

The figures 8 and 9 below show the geometric representation of the dimensionless unification of the gravitational and the electromagnetic interactions.

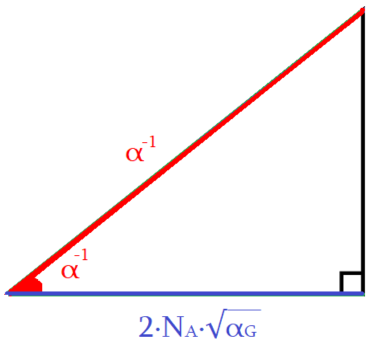

Figure 8. *First geometric representation of the dimensionless unification of the gravitational and the electromagnetic interactions*

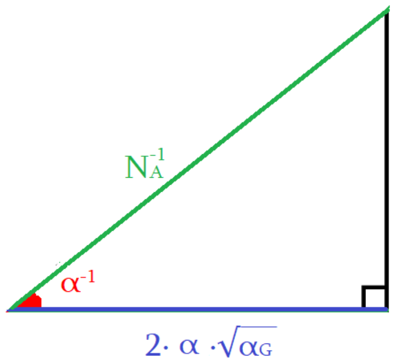

Figure 9. *Second geometric representation of the dimensionless unification of the gravitational and the electromagnetic* interactions

We reached the conclusion of the dimensionless unification of the gravitational and the electromagnetic forces:

$$4 \cdot e^2 \cdot \alpha^2 \cdot \alpha_G \cdot N_A^2 = 1 \quad (33)$$

$$16 \cdot \alpha^2 \cdot \alpha_G \cdot N_A^2 = (e^{i/\alpha} + e^{-i/\alpha})^2 \quad (34)$$

We reached the conclusion of the dimensionless unification of the strong nuclear,the gravitational and the electromagnetic interactions:

$$4 \cdot \alpha_s^2 \cdot \alpha^2 \cdot \alpha_G \cdot N_A^2 = i^{4i} \quad (35)$$

$$\alpha^2 \cdot (e^{i/\alpha} + e^{-i/\alpha}) \cdot \alpha_s^4 \cdot \alpha_G \cdot N_A^2 = i^{8i} \quad (36)$$

The figure 10 below shows the geometric representation of the dimensionless unification of the strong nuclear,the gravitational and the electromagnetic interactions.

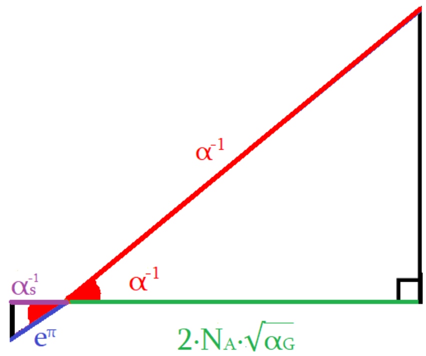

Figure 10. *Geometric representation of the dimensionless unification of the strong nuclear,the gravitational and the electromagnetic interactions*

We reached the conclusion of the dimensionless unification of the weak nuclear,the gravitational and electromagnetic forces:

$$4 \cdot 10^{14} \cdot \alpha_w^2 \cdot \alpha^2 \cdot \alpha_G \cdot N_A^2 = i^{4i} \cdot e^2 \quad (37)$$

$$10^{14} \cdot \alpha^2 \cdot (e^{i/\alpha} + e^{-i/\alpha})^2 \cdot \alpha_w^2 \cdot \alpha_G \cdot N_A^2 = i^{8i} \quad (38)$$

The figure 11 below shows the geometric representation of the dimensionless unification of the weak nuclear,the gravitational and the electromagnetic interactions.

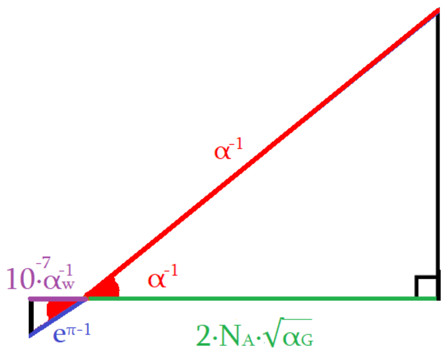

Figure 11. *Geometric representation of the dimensionless unification of the weak nuclear,the gravitational and the electromagnetic interactions*

Resulting the unity formula that connect the strong coupling constant α_s,the weak coupling constant α_w,the fine-structure constant α and the gravitational coupling constant $\alpha_{G(p)}$ for the proton:

$$4\cdot10^{14}\cdot N_A^2\cdot\alpha_w^2\cdot\alpha^2\cdot\alpha_{G(p)}=\mu^2\cdot\alpha_s^2 \tag{39}$$

We reached the conclusion of the dimensionless unification of the strong nuclear,the weak nuclear,the gravitational and the electromagnetic interactions:

$$\alpha_s^2=4\cdot10^{14}\cdot\alpha_w^2\cdot\alpha^2\cdot\alpha_G\cdot N_A^2 \tag{40}$$

$$8\cdot10^7\cdot N_A^2\cdot\alpha_w\cdot\alpha^2\cdot\alpha_G=\alpha_s\cdot(e^{i/\alpha}+e^{-i/\alpha}) \tag{41}$$

The figure 12 below shows the geometric representation of the dimensionless unification of the strong nuclear,the weak nuclear,the gravitational and the electromagnetic interactions.

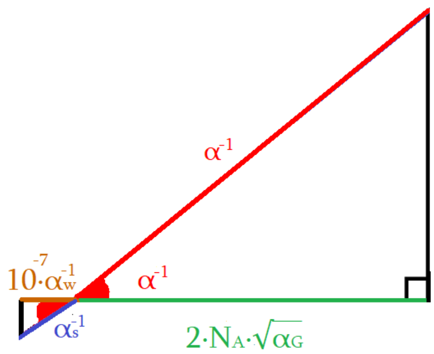

Figure 12. *Geometric representation of the dimensionless unification of the strong nuclear,the weak nuclear,the gravitational and the electromagnetic interactions*

From these expressions resulting the unity formulas that connects the strong coupling constant α_s,the weak coupling constant α_w,the proton to electron mass ratio μ,the fine-structure constant α,the ratio N_1 of electric force to gravitational force between electron and proton,the Avogadro's number N_A,the gravitational coupling constant α_G of the electron and the gravitational coupling constant of the proton $\alpha_{G(p)}$:

$$\alpha_s^2=4\cdot10^{14}\cdot\alpha_w^2\cdot\alpha^2\cdot\alpha_G\cdot N_A^2 \tag{42}$$

$$\mu^2\cdot\alpha_s^2=4\cdot10^{14}\cdot\alpha_w^2\cdot\alpha^2\cdot\alpha_{G(p)}\cdot N_A^2 \tag{43}$$

$$\mu\cdot N_1\cdot\alpha_s^2=4\cdot10^{14}\cdot\alpha_w^2\cdot\alpha^3\cdot N_A^2 \tag{44}$$

$$\alpha_s^2=4\cdot10^{14}\cdot\alpha_w^2\cdot\alpha\cdot\mu\cdot\alpha_G^2\cdot N_A^2\cdot N_1 \tag{45}$$

$$\mu^3\cdot\alpha_s^2=4\cdot10^{14}\cdot\alpha_w^2\cdot\alpha\cdot\alpha_{G(p)}^2\cdot N_A^2\cdot N_1 \tag{46}$$

$$\mu\cdot\alpha_s=4\cdot10^{14}\cdot\alpha_w^2\cdot\alpha_G\cdot G(p)^2\cdot N_A^2\cdot N_1^2 \tag{47}$$

$$\mu\cdot\alpha_s^2=4\cdot10^{14}\cdot\alpha_w^2\cdot\alpha\cdot\alpha_G\cdot\alpha_{G(p)}\cdot N_A^2\cdot N_1 \tag{48}$$

These equations are applicable for all energy scales. The expressions for the gravitational constant are:

$$G=(2e\alpha N_A)^{-2}\frac{\hbar c}{m_e^2} \tag{49}$$

$$G=i^{4i}(2\alpha_s\alpha N_A)^{-2}\frac{\hbar c}{m_e^2} \tag{50}$$

$$G=i^{4i}e^2\left(2\,10^7\alpha_w\alpha N_A\right)^{-2}\frac{\hbar c}{m_e^2} \tag{51}$$

$$G=\alpha_s^2\left(2\,10^7\alpha_w\alpha N_A\right)^{-2}\frac{\hbar c}{m_e^2} \tag{52}$$

2. Unification of atomic physics and cosmology

In [15] and [16] we reached the unification of dimensionless atomic physics and cosmology. The relevant constant in atomic physics is the fine-structure constant α,which plays a fundamental role in atomic physics and quantum electrodynamics. The analogous constant in cosmology is the gravitational fine-structure constant α_g. It plays a fundamental role in cosmology. The mysterious value of the gravitational fine-structure constant α_g is an equivalent way to express the biggest issue in theoretical physics. The new formula for the Planck length l_{pl} is:

$$l_{pl} = a\sqrt{a_G}\alpha_0$$

The fine-structure constant equals:

$$\alpha^2 = \frac{r_e}{a_0}$$

From these expressions we have:

$$l_{pl} = \frac{\alpha\sqrt{\alpha_G} r_e}{\alpha^2}$$

$$l_{pl} = \frac{\sqrt{\alpha_G}}{\alpha} r_e$$

$$\frac{l_{pl}^3}{r_e^3} = \frac{\sqrt{\alpha_G^3}}{\alpha^3}$$

The gravitational fine structure constant α_g is defined as:

$$\alpha_g = \frac{l_{pl}^3}{r_e^3}$$

$$\alpha_g = \frac{\sqrt{\alpha_G^3}}{\alpha^3}$$

$$\alpha_g = \sqrt{\frac{\alpha_G^3}{\alpha^6}} \tag{53}$$

with numerical value:

$$\alpha_g = 1{,}886837 \times 10^{-61}$$

Also equals:

$$\alpha_g^2 \cdot \alpha^6 = \alpha_G^3$$

$$\alpha_g^2 = \alpha_G^3 \cdot \alpha^{-6}$$

$$\alpha_g^2 = \left(\frac{\alpha_G}{\alpha^2}\right)^3$$

Now we will try to find the best mathematical expression of the gravitational fine structure constant α_g with the mathematical constants. In trying to do this we found surprising coincidences and various approaches for the math constants. A approach for Archimedes constant π is:

$$\pi^6 \simeq \frac{2^{300}}{6\,7^{103}} \qquad (54)$$

A approach for the Gelfond's constant e^{π} is:

$$e^{\pi} \simeq \frac{55}{\pi}\sqrt{\frac{2}{\ln \pi}} \qquad (55)$$

A approximation expression that connects the golden ratio φ,the Archimedes constant π and the Euler's number e is:

$$2^2 11^2 e \simeq 3^4 \varphi^5 \sqrt[3]{\pi} \qquad (56)$$

Two approximations expressions that connects the golden ratio φ,the Archimedes constant π,the Euler's number e and the Euler's constant γ are:

$$4e^2 \gamma \ln^2(2\pi) \simeq \sqrt{3^3}\varphi^5 \qquad (57)$$

$$\sqrt{3^5} e\gamma \ln(2\pi)\sqrt[3]{\pi} \simeq 11^2 \qquad (58)$$

The expression that connects the gravitational fine-structure constant α_g with the Archimedes constant π,the Euler's number e and the Euler's constant γ is:

$$\alpha_g=[e\cdot\gamma\cdot\ln^2(2\cdot\pi)]^{-1}\times10^{-60}=1,886837\times10^{-61} \qquad (59)$$

The expression that connects the gravitational fine-structure constant α_g with the golden ratio φ and the Euler's number e is:

$$\alpha_g = \frac{4e}{3\sqrt{3}\varphi^5} \times 10^{-60} = 1,886837 \times 10^{-61} \qquad (60)$$

The expression that connects the gravitational fine-structure constant α_g with the Archimedes constant π is:

$$\alpha_g = \frac{\sqrt{3^5}\sqrt[3]{\pi}}{11^2} \times 10^{-60} = 1,886837 \times 10^{-61} \qquad (61)$$

The expression that connects the gravitational fine-structure constant α_g with the golden ratio φ and the Euler's constant γ is:

$$\alpha_g = \frac{7\varphi\gamma^2}{2} \times 10^{-60} = 1,886826 \times 10^{-61} \qquad (62)$$

The expression that connects the gravitational fine-structure constant α_g with the Archimedes constant and the golden ratio φ is:

$$\alpha_g = \frac{2\pi}{3\varphi^5} \times 10^{-60} = 1,888514 \times 10^{-61} \qquad (63)$$

Resulting the unity formula for the gravitational fine-structure constant α_g:

$$\alpha_g=(2\cdot e\cdot\alpha^2\cdot N_A)^{-3} \qquad (64)$$

Also apply the expressions:

$$(2 \cdot e \cdot \alpha^2 \cdot N_A)^3 \cdot \alpha_g = 1$$

$$8 \cdot e^3 \cdot \alpha^6 \cdot \alpha_g \cdot N_A^3 = 1$$

Resulting the unity formula for the gravitational fine-structure constant α_g:

$$\alpha_g = i^{6i} \cdot (2 \cdot \alpha_s \cdot \alpha^2 \cdot N_A)^{-3} \quad (65)$$

Also apply the expression:

$$(2 \cdot \alpha_s \cdot \alpha^2 \cdot N_A)^3 \cdot \alpha_g = i^{6i}$$

$$8 \cdot \alpha_s^3 \cdot \alpha^6 \cdot \alpha_g \cdot N_A^3 = i^{6i}$$

Resulting the unity formula for the gravitational fine-structure constant α_g:

$$\alpha_g = i^{6i} \cdot e^3 \cdot (2 \cdot 10^7 \cdot \alpha_w \cdot \alpha^3 \cdot N_A)^{-3} \quad (66)$$

Also apply the expression:

$$(2 \cdot 10^7 \cdot \alpha_w \cdot \alpha^3 \cdot N_A)^3 \cdot \alpha_g = i^{6i} \cdot e^3$$

$$8 \cdot 10^{21} \cdot \alpha_w^3 \cdot \alpha^9 \cdot \alpha_g \cdot N_A^3 = i^{6i} \cdot e^3$$

Resulting the unity formulas for the gravitational fine-structure constant α_g:

$$\alpha_g = (10^7 \cdot \alpha_w \cdot \alpha_G^{1/2} \cdot e^{-1} \cdot \alpha_s^{-1} \cdot \alpha^{-1})^3 \quad (67)$$

Also apply the expressions:

$$\alpha_g = 10^{21} \cdot \alpha_w^3 \cdot \alpha_G^{3/2} \cdot \alpha_s^{-3} \cdot \alpha^{-3} \cdot e^{-3}$$

$$\alpha_g \cdot \alpha_s^3 \cdot \alpha^3 \cdot e^3 = 10^{21} \cdot \alpha_w^3 \cdot \alpha_G^{3/2}$$

So the unity formula for the gravitational fine-structure constant α_g is:

$$\alpha_g^2 = (10^{14} \cdot \alpha_w^2 \cdot \alpha_G \cdot e^{-2} \cdot \alpha_s^{-2} \cdot \alpha^{-2})^3 \quad (68)$$

Also apply the expressions:

$$\alpha_g^2 = 10^{42} \cdot \alpha_w^6 \cdot \alpha_G^3 \cdot e^{-6} \cdot \alpha_s^{-6} \cdot \alpha^{-6}$$

$$e^6 \cdot \alpha_s^6 \cdot \alpha^6 \cdot \alpha_g^2 = 10^{42} \cdot \alpha_w^6 \cdot \alpha_G^3$$

$$\alpha_g^2 \cdot (e \cdot \alpha_s \cdot \alpha)^6 = (10^{14} \cdot \alpha_w^2 \cdot \alpha_G)^3$$

Resulting the unity formula for the gravitational fine-structure constant α_g:

$$\alpha_g = i^{6i} \cdot (10^7 \cdot \alpha_w \cdot \alpha_G^{1/2} \cdot \alpha_s^{-2} \cdot \alpha^{-1})^3$$

$$\alpha_g = 10^{21} \cdot i^{6i} \cdot (\alpha_w \cdot \alpha_G^{1/2} \cdot \alpha_s^{-2} \cdot \alpha^{-1})^3$$

$$\alpha_g = 10^{21} \cdot i^{6i} \cdot \alpha_w^3 \cdot \alpha_G^{3/2} \cdot \alpha_s^{-6} \cdot \alpha^{-3} \quad (69)$$

Also apply the expressions:

$$\alpha_g^{1/3} \cdot \alpha_s^2 \cdot \alpha \cdot \alpha_w^{-1} \cdot \alpha_G^{-1/2} = i^{2i} \cdot 10^7$$

$$\alpha_g \cdot \alpha_s^6 \cdot \alpha^3 = 10^{21} \cdot i^{6i} \cdot \alpha_w^3 \cdot \alpha_G^{3/2}$$

So the unity formulas for the gravitational fine-structure constant α_g are:

$$\alpha_g^2 = i^{6i} \cdot (10^{14} \cdot \alpha_w^2 \cdot \alpha_G \cdot \alpha_s^{-4} \cdot \alpha^{-2})^3$$

$$\alpha_g^2 = 10^{42} \cdot i^{12i} \cdot (\alpha_w^2 \cdot \alpha_G \cdot \alpha_s^{-4} \cdot \alpha^{-2})^3$$

$$\alpha_g^2 = 10^{42} \cdot i^{12i} \cdot \alpha_w^6 \cdot \alpha_G^3 \cdot \alpha_s^{-12} \cdot \alpha^{-6} \tag{70}$$

Also apply the expressions:

$$\alpha_g^2 \cdot \alpha_s^{12} \cdot \alpha^6 \cdot \alpha_w^{-6} \cdot \alpha_G^{-3} = i^{12i} \cdot 10^{42}$$

$$(\alpha_s^6 \cdot \alpha^3 \cdot \alpha_g)^2 = (10^{14} \cdot i^{4i} \cdot \alpha_w^2 \cdot \alpha_G)^3$$

$$\alpha_s^{12} \cdot \alpha^6 \cdot \alpha_g^2 = 10^{42} \cdot i^{12i} \cdot \alpha_w^6 \cdot \alpha_G^3$$

So the unity formulas for the gravitational fine-structure constant α_g are:

$$\alpha_g = \left(\frac{10^7 \alpha_w \sqrt{\alpha_G}}{e \alpha_s a} \right)^3 \tag{71}$$

$$\alpha_g^2 = 10^{42} \left(\frac{\alpha_G a_w^2}{e^2 \alpha_s^2 \alpha^2} \right)^3 \tag{72}$$

$$\alpha_g = 10^{21} i^{6i} \left(\frac{\alpha_w \sqrt{\alpha_G}}{\alpha_s^2 \alpha} \right)^3 \tag{73}$$

$$\alpha_g^2 = 10^{42} i^{12i} \left(\frac{\alpha_G \alpha_w^2}{\alpha^2 \alpha_s^4} \right)^3 \tag{74}$$

This expression connects the gravitational fine-structure constant α_g with the four coupling constants. Perhaps the gravitational fine structure constant α_g is the coupling constant for the fifth force. Some speculative theories have proposed a fifth force to explain various anomalous observations that do not fit existing theories. The characteristics of this fifth force depend on the hypothesis being advanced. Many postulate a force roughly the strength of gravity with a range of anywhere from less than a millimeter to cosmological scales. Another proposal is a new weak force mediated by W and Z bosons. The search for a fifth force has increased in recent decades due to two discoveries in cosmology which are not explained by current theories. It has been discovered that most of the mass of the universe is accounted for by an unknown form of matter called dark matter. Most physicists believe that dark matter consists of new,undiscovered subatomic particles,but some believe that it could be related to an unknown fundamental force. Second,it has also recently been discovered that the expansion of the universe is accelerating,which has been attributed to a form of energy called dark energy. Some physicists speculate that a form of dark energy called quintessence could be a fifth force.

In the context of cosmology the cosmological constant is a homogeneous energy density that causes the expansion of the universe to accelerate. Originally proposed early in the development of general relativity in order to allow a static universe solution it was subsequently abandoned when the universe was found to be expanding. Now the cosmological constant is invoked to explain the observed acceleration of the expansion of the universe. The cosmological constant is the simplest realization of dark energy,which is the more generic name given to the unknown cause of the acceleration of the universe. Its existence is also predicted by quantum physics,where it enters as a form of vacuum energy,although the magnitude predicted by quantum theory does not match that observed in cosmology.

The cosmological constant Λ is presumably an enigmatic form of matter or energy that acts in opposition to gravity and is considered by many physicists to be equivalent to dark energy. Nobody really knows what the cosmological constant is exactly,but it is required in cosmological equations in order to reconcile theory with our observations of the universe. One potential explanation for the cosmological constant lies in the realm of modern particle physics. Experiments have verified that empty space is permeated by countless virtual particles constantly popping in and out of existence. It is commonly believed that the cosmological constant problem can only be solved ultimately in a unified theory of quantum gravity and the standard model of electroweak and strong interactions,which is still absent so far. But connecting vacuum energy to the cosmological constant is not straightforward. Based on their observations of supernovas,astronomers estimate that dark energy should have a small and sedate value,just enough to push everything in the universe apart over billions of years. Yet when scientists try to calculate the amount of energy that should arise from virtual particle motion,they come up with a result that's 120 orders of magnitude greater than what the supernova data suggest. The cosmological constant has the same effect as an intrinsic energy density of the vacuum,ρ_{vac} and an associated pressure. In this context,it is commonly moved onto the right-hand side of the equation,and defined with a proportionality factor of $\Lambda=8\cdot\pi\cdot\rho_{vac}$ where unit conventions of general relativity are used (otherwise factors of G and c would also appear,i.e:

$$\Lambda = 8\pi\rho_{vac}\frac{G}{c^4} = \kappa\rho_{vac}$$

where κ is Einstein's rescaled version of the gravitational constant G. The cosmological constant has been introduced in gravitational field equations by Einstein in 1917 in order to satisfy Mach's principle of the relativity of inertia. Then it was demonstrated by Cartan in 1922 that the Einstein field tensor including a cosmological constant Λ:

$$E_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \Lambda g_{\mu\nu}$$

,is the most general tensor in Riemannian geometry having null divergence like the energy momentum tensor $T_{\mu\nu}$. This theorem has set the general form of Einstein’s gravitational field equations as $E_{\mu\nu}=\kappa\cdot T_{\mu\nu}$ and established from first principles the existence of Λ as an unvarying true constant. The cosmological constant problem dates back to the realization that it is equivalent to a vacuum energy density. One of the main consequences in cosmology of a positive cosmological constant is an acceleration of the expansion of the universe. Such an acceleration has been first detected in 1981 in the Hubble diagram of infrared elliptical galaxies,yielding a positive value close to the presently measured one,but with still large uncertainties. Accurate measurements of the acceleration of the expansion since 20 years have reinforced the problem. The cosmological constant Λ,as it appears in Einstein’s equations,is a curvature. As such,besides being an energy density,it is also the inverse of the square of an invariant cosmic length L.

In the early-mid 20th century Dirac and Zel’dovich were among the first scientists to suggest an intimate connection between cosmology and atomic physics. Though a revolutionary proposal for its time,Dirac’s Large Number Hypothesis (1937) adopted a standard assumption of the non-existence of the cosmological constant term $\Lambda=0$. Zel'dovich insight (1968) was to realize that a small but nonzero cosmological term $\Lambda>0$ allowed the present day radius of the Universe to be identified with the de Sitter radius which removed the need for time dependence in the fundamental couplings. Thus,he obtained the formula:

$$\Lambda = \frac{m_p^6 G^2}{\hbar^6}$$

where m is a mass scale characterizing the relative strengths of the gravitational and electromagnetic interactions,which he identified with the proton mass m_p.

Laurent Nottale in [17] which,instead,suggests the identification $m=m_e/\alpha$. He assumed that the cosmological constant Λ is the sum of a general-relativistic term and of the quantum,scale-varying,gravitational self-energy of virtual pairs. A renormalization group approach is used to describe its scale-dependence. We argue that the large scale value of Λ is reached at the classical electron scale. This reasoning provides with a large-number relation:

$$\alpha\frac{m_{pl}}{m_e} = \left(\frac{L}{l_{pl}}\right)^{\frac{1}{3}}$$

The cosmological constant Λ has the dimension of an inverse length squared. The cosmological constant is the inverse of the square of a length L:

$$L=\sqrt{\Lambda^{-1}}$$

For the de Sitter radius equals:

$$R_d=\sqrt{3}L$$

So the de Sitter radius and the cosmological constant are related through a simple equation:

$$R_d=\sqrt{\frac{3}{\Lambda}}$$

From this equation resulting the expressions for the gravitational fine structure constant α_g:

$$\alpha\frac{m_{pl}}{m_e}=\left(l_{pl}\sqrt{\Lambda}\right)^{-\frac{1}{3}}$$

$$\alpha_g=l_{pl}\sqrt{\Lambda}$$

$$\alpha_g=\sqrt{\frac{G\hbar\Lambda}{c^3}}$$

So the cosmological constant Λ equals:

$$\Lambda=\alpha_g^2 l_{pl}^{-2}$$

$$\Lambda=\frac{l_{pl}^4}{r_e^6}$$

$$\Lambda=\alpha_g^2\frac{c^3}{G\hbar}$$

$$\Lambda=\frac{G}{\hbar^4}\left(\frac{m_e}{a}\right)^6$$

Resulting the dimensionless unification of the atomic physics and the cosmology:

$$\alpha_g=(2\cdot e\cdot \alpha^2\cdot N_A)^{-3}$$

$$l_{pl}^2\cdot\Lambda=(2\cdot e\cdot \alpha^2\cdot N_A)^{-6} \quad (75)$$

$$(2\cdot e\cdot \alpha^2\cdot N_A)^6\cdot l_{pl}^2\cdot\Lambda=1 \quad (76)$$

Now we will use the unity formulas of the dimensionless unification of atomic physics and cosmology to find the equations of the cosmological constant. For the cosmological constant equals:

$$\Lambda=\left(2e\alpha^2 N_A\right)^{-6}\frac{c^3}{G\hbar} \quad (77)$$

Resulting the dimensionless unification of atomic physics and cosmology:

$$\alpha_g=i^{6i}\cdot(2\cdot\alpha_s\cdot\alpha^2\cdot N_A)^{-3}$$

$$l_{pl}^2\cdot\Lambda=i^{12i}\cdot(2\cdot\alpha_s\cdot\alpha^2\cdot N_A)^{-6} \quad (78)$$

$$(2\cdot\alpha_s\cdot\alpha^2\cdot N_A)^6\cdot l_{pl}^2\cdot\Lambda=i^{12i} \quad (79)$$

For the cosmological constant equals:

$$\Lambda = i^{12i}\left(2\alpha_s a^2 N_A\right)^{-6}\frac{c^3}{G\hbar} \tag{80}$$

Resulting the dimensionless unification of atomic physics and cosmology:

$$\alpha_g = i^{6i} \cdot e^3 \cdot (2 \cdot 10^7 \cdot \alpha_w \cdot a^3 \cdot N_A)^{-3}$$

$$l_{pl}^2 \cdot \Lambda = i^{12i} \cdot e^6 \cdot (2 \cdot 10^7 \cdot \alpha_w \cdot a^3 \cdot N_A)^{-6} \tag{81}$$

$$(2 \cdot 10^7 \cdot \alpha_w \cdot a^3 \cdot N_A)^6 \cdot l_{pl}^2 \cdot \Lambda = i^{12i} \cdot e^6 \tag{82}$$

For the cosmological constant equals:

$$\Lambda = i^{12i} e^6 \left(2\, 10^7 \alpha_w a^3 N_A\right)^{-6}\frac{c^3}{G\hbar} \tag{83}$$

Resulting the dimensionless unification of atomic physics and cosmology:

$$\alpha_g^2 = 10^{42}\left(\frac{\alpha_G a_w^2}{e^2\alpha_s^2\alpha^2}\right)^3$$

$$l_{pl}^2\Lambda = 10^{42}\left(\frac{\alpha_G \alpha_w^2}{e^2\alpha_s^2\alpha^2}\right)^3 \tag{84}$$

$$e^6 \cdot \alpha_s^6 \cdot \alpha^6 \cdot l_{pl}^2 \cdot \Lambda = 10^{42} \cdot \alpha_G^3 \cdot \alpha_w^6 \tag{85}$$

For the cosmological constant equals:

$$\Lambda = 10^{42}\left(\frac{\alpha_G \alpha_w^2}{e^2\alpha_s^2\alpha^2}\right)^3\frac{c^3}{G\hbar} \tag{86}$$

Resulting the dimensionless unification of atomic physics and cosmology:

$$\alpha_g^2 = 10^{42} i^{12i}\left(\frac{\alpha_G \alpha_w^2}{\alpha^2\alpha_s^4}\right)^3$$

$$l_{pl}^2\Lambda = 10^{42} i^{12i}\left(\frac{\alpha_G \alpha_w^2}{\alpha^2\alpha_s^4}\right)^3 \tag{87}$$

$$\alpha_s^{12} \cdot \alpha^6 \cdot l_{pl}^2 \cdot \Lambda = 10^{42} \cdot i^{12i} \cdot \alpha_G^3 \cdot \alpha_w^6 \tag{88}$$

This unity formula is a simple analogy between atomic physics and cosmology. For the cosmological constant equals:

$$\Lambda = 10^{42} i^{12i}\left(\frac{\alpha_G \alpha_w^2}{\alpha^2\alpha_s^4}\right)^3\frac{c^3}{G\hbar} \tag{89}$$

The Equation of the Universe is:

$$\frac{\Lambda G\hbar}{c^3} = 10^{42} i^{12i}\left(\frac{\alpha_G \alpha_w^2}{\alpha^2\alpha_s^4}\right)^3 \tag{90}$$

From the dimensionless unification of the fundamental interactions the density parameter for normal baryonic matter is:

$$\Omega_B = e^{-\pi} = i^{2i} = 0.04321 = 4.32\%$$

The density parameter for dark matter is:

$$\Omega_D = 6 \cdot e^{-\pi} = 6 \cdot i^{2i} = 0.2592 = 25.92\%$$

The density parameter for the dark energy is:

$$\Omega_\Lambda = 17 \cdot e^{-\pi} = 17 \cdot i^{2i} = 0.7346 = 73.46\%$$

The sum of the density parameter for normal baryonic matter and the density parameter for the dark energy is:

$$\Omega_0 = 24 \cdot e^{-\pi} = 24 \cdot i^{2i} = 1.03713$$

A positively curved universe is described by elliptic geometry,and can be thought of as a three-dimensional hypersphere,or some other spherical 3-manifold,such as the Poincaré dodecahedral space,all of which are quotients of the 3-sphere. The state equation w has value:

$$w = -24 \cdot e^{-\pi} = -24 \cdot i^{2i} = -1.0371$$

For as much as $w<-1$,the density actually increases with time.
R. Adler in [18] calculated the energy ratio in cosmology,the ratio of the dark energy density to the Planck energy density. H.H. Otto in [19] found the reciprocity relation between mass constituents of the universe.

3. Cosmological parameters

The Hubble constant H_0 is one of the most important numbers in cosmology because it is required to estimate the size and age of the universe. This number indicates the rate at which the universe is expanding. The Hubble constant can be used to determine the inherent brightness and masses of stars in nearby galaxies,examine the same properties in more distant galaxies and galaxy clusters,infer the amount of dark matter in the universe,and obtain the scale size of distant clusters as far as clusters test for theoretical cosmological models. In 1929,American astronomer Edwin Hubble announced his discovery that galaxies,in all directions,seemed to be moving away from us and have greater displacement for attenuated galaxies. However,the true value for H_0 is very complicated. Astronomers need two measurements:

a) First, spectroscopic observations reveal the redshift of the galaxy, showing its radial velocity.

b) The second measurement, the most difficult value, is the exact distance of the galaxy from Earth.

The unit of the Hubble constant is 1 km/s/Mpc. The 2018 CODATA recommended value of the Hubble constant is $H_0=67.66\pm0.42\ (km/s)/Mpc=(2.1927664\pm0.0136)\times10^{-18}\ s^{-1}$. Hubble length or Hubble distance is a unit of distance in cosmology,defined as the speed of light multiplied by Hubble time $L_H = c \cdot H_0^{-1}$. This distance is equivalent to 4.550 million parsecs,or 14.4 billion light-years,13.8 billion years. Hubble's distance would be the distance between the Earth and the galaxies currently falling away from us at the speed of light,as shown by the substitution $r = c \cdot H_0^{-1}$ in the equation for Hubble's law,$u = H_0 \cdot r$.

The critical density is the average density of matter required for the Universe to just halt its expansion,but only after an infinite time. A Universe with a critical density is said to be flat. In his theory of general relativity, Einstein demonstrated that the gravitational effect of matter is to curve the surrounding space. In a Universe full of matter, both its overall geometry and its fate are controlled by the density of the matter within it. If the density of matter in the Universe is high (a closed Universe),self-gravity slows the expansion until it halts, and ultimately re-collapses. In a closed Universe, locally parallel light rays converge at some extremely distant point. This is referred to as spherical geometry. If the density of matter in the Universe is low (an open Universe),self-gravity is insufficient to stop the expansion, and the Universe continues to expand forever (albeit at an ever decreasing rate). In an open Universe, locally parallel light rays ultimately diverge. This is referred to as hyperbolic geometry. Balanced on a knife edge between Universes with high and low densities of matter, there exists a Universe where parallel light rays remain parallel. This is referred to as a flat geometry,and the density is called the critical density. In a critical density Universe, the expansion is halted only after an infinite time. To date,the critical density is estimated to be approximately five atoms per cubic meter,whereas the average density of ordinary matter in the Universe is believed to be 0.2–0.25 atoms per cubic meter. A much greater density comes from the unidentified dark matter; both ordinary and dark matter contribute in favor of contraction of the universe. However,the largest part comes from so-called dark energy, which accounts for the cosmological constant term. Although the total density is equal to the critical density the dark energy does not lead to contraction of the universe but rather may accelerate its expansion. Therefore,the universe will likely expand forever. An expression for the critical density is found by assuming Λ to be zero and setting the normalized spatial curvature,k,equal to zero. When the substitutions are applied to the first of the Friedmann equations we find:

$$\rho_c = \frac{3H_0^3}{8\pi G}$$

It should be noted that this value changes over time. The critical density changes with cosmological time,but the energy density due to the cosmological constant remains unchanged throughout the history of the universe. The amount of dark energy increases as the universe grows,while the amount of matter does not. The density parameter Ω is defined as the ratio of the actual density ρ to the critical density ρ_c of the Friedmann universe. The relation between the actual density and the critical density determines the overall geometry of the universe,when they are equal,the geometry of the universe is flat (Euclidean). The galaxies we see in all directions are moving away from the Earth,as evidenced by their red shifts. Hubble's law describes this expansion. Remarkably,study of the expansion rate has shown that the universe is very close to the critical density that would cause it to expand forever. The density parameter Ω is defined as the ratio of the average density of matter and energy in the Universe ρ to the critical density ρ_c of the Friedmann universe. The relation between the actual density and the critical density determines the overall geometry of the universe;when they are equal,the geometry of the universe is flat (Euclidean). In earlier models,which did not include a cosmological constant term, critical density was initially defined as the watershed point between an expanding and a contracting Universe. The density parameter is given by:

$$\Omega_0 = \frac{\rho}{\rho_c}$$

where ρ is the actual density of the Universe and ρ_c the critical density. Although current research suggests that Ω_0 is very close to 1,it is still of great importance to know whether Ω_0 is slightly greater than 1,less than 1,or exactly equal to 1,as this reveals the ultimate fate of the Universe. If Ω_0 is less than 1,the Universe is open and will continue to expand forever. If Ω_0 is greater than 1,the Universe is closed and this will eventually halt its expansion and recollapse. If Ω_0 is exactly equal to 1 then the Universe is flat and contains enough matter to halt the expansion but not enough to recollapse it. It is important to note that the ρ used in the calculation of Ω_0 is the total mass/energy density of the Universe. In other words,it is the sum of a number of different components including both normal and dark matter as well as the dark energy suggested by recent observations. We can therefore write:

$$\Omega_0=\Omega_B+\Omega_D+\Omega_\Lambda$$

$$\Omega_0=\Omega_m+\Omega_\Lambda$$

$$\Omega_0=\Omega_B+\Omega_{D+\Lambda}$$

where:
Ω_B is the density parameter for normal baryonic matter,
Ω_D is the density parameter for dark matter,
Ω_Λ is the density parameter for dark energy,
Ω_m is the sum of the density parameter for normal baryonic matter and the density parameter for dark matter,
$\Omega_{D+\Lambda}$ is the sum of the density parameter for the density parameter for dark matter and the density parameter for dark energy. The sum of the contributions to the total density parameter Ω_0 at the current time is:

$$\Omega_0=1.02\pm0,02$$

Current observations suggest that we live in a dark energy dominated Universe with $\Omega_\Lambda=0.73$,$\Omega_D=0.23$ and $\Omega_B=0.04$. To the accuracy of current cosmological observations,this means that we live in a flat,$\Omega_0=1$ Universe. Instead of the cosmological constant Λ itself,cosmologists often refer to the ratio between the energy density due to the cosmological constant and the critical density of the universe,the peak point of a density sufficient to prevent the universe from expanding forever,at one level of the universe is the ratio between the energy of the universe due to the cosmological constant Λ and the critical density of the universe,that is what we would call the fraction of the universe consisting of dark energy [20]. By definition, baryonic matter should only include matter composed of baryons. In other words, it should include protons, neutrons and all the objects composed of them (i.e. atomic nuclei), but exclude things such as electrons and neutrinos which are actually leptons. In astronomy, however, the term 'baryonic matter' is used more loosely, since on astronomical scales, protons and neutrons are always accompanied by electrons. Astronomers therefore use the term 'baryonic' to refer to all objects made of normal atomic matter, essentially ignoring the presence of electrons which, after all, represent only ~0.0005 of the mass. Neutrinos, on the other hand, are considered non-baryonic by astronomers. Another slight oddity in the usage of the term baryonic matter in astronomy is that black holes are included as baryonic matter. While the matter from which black holes form is mainly baryonic matter, once swallowed by the black hole, this distinction is lost. For example, a theoretical black hole constructed purely out of photons is indistinguishable from one made from normal baryonic matter. This is often referred to as the 'black holes have no hair' theorem which simply states that black holes do not have properties such as baryonic or non-baryonic. Objects in the Universe composed of baryonic matter include Clouds of cold gas, Planets, Comets and asteroids, Stars, Neutron stars and Black holes.

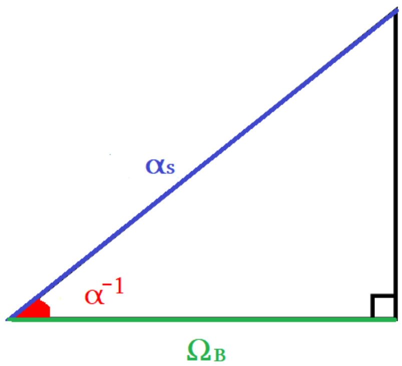

Figure 13. *Geometric representation of the the density parameter for baryonic matter*

The assessment of baryonic matter at the current time was assessed by WMAP to be Ω_B=0.044±0004. From the dimensionless unification of the fundamental interactions the density parameter for the normal baryonic matter is:

$$\Omega_B=e^{-\pi} \tag{91}$$

$$\Omega_B=i^{2i} \tag{92}$$

$$\Omega_B=0.0432 \tag{93}$$

$$\Omega_B=4.32\% \tag{94}$$

Series representations for the density parameter for the normal baryonic matter Ω_B are:

$$e^{-\pi} = \left(\sum_{k=0}^{\infty} \frac{1}{k!}\right)^{-\pi}$$

$$e^{-\pi} = e^{-4\sum_{k=0}^{\infty}(-1)^k/(1+2k)}$$

$$e^{-\pi} = \left(\frac{1}{\sum_{k=0}^{\infty} \frac{(-1)^k}{k!}}\right)^{-\pi}$$

$$e^{-\pi} = \left(\sum_{k=0}^{\infty} \frac{1}{k!}\right)^{-4\sum_{k=0}^{\infty}(-1)^k/(1+2k)}$$

$$e^{-\pi} = \left(\frac{1}{\sum_{k=0}^{\infty} \frac{(-1)^k}{k!}} \right)^{-4 \sum_{k=0}^{\infty} (-1)^k / (1+2k)}$$

The pattern of the continued fraction for the density parameter for the normal baryonic matter is:

[0; 23, 7, 9, 3, 1, 1, 591, 2, 9, 1, 2, 34, 1, 16, 1, 30, 1, 1, 4, 1, 2, 108, 2, 2, 1,
3, 1, 7, 1, 2, 2, 2, 1, 2, 3, 2, 166, 1, 2, 1, 4, 8, 10, 1, 1, 7, 1, 2, 3, 566, 1, 2,
3, 3, 1, 20, 1, 2, 19, 1, 3, 2, 1, 2, 13, 2, 2, 11, 3, 1, 2, 1, 7, 2, 1, 1, 1, 2, 1, 19,
1, 1, 12, 11, 1, 4, 1, 6, 1, 2, 18, 1, 2, 49, 1, 3, 5, 1, 1, 1, 1, 1, 7, 1, 6, 48, 2,
2, 1, 10, 1, 1, 5, 1, 1, 7, 18, 12, 2, 1, 5, 2, 9, 10, 1, 19, 1, 2, 1, 4, 2, 2, 1, 39,
1, 1, 12, 1, 15, 165, 1, 2, 1, 3, 1, 1, 1, 1, 24, 1, 1, 1, 3, 9, 1, 1, 1, 2, 1, 1, 10,
1, 36, 1, 7, 23, 1, 2, 1, 2, 1, 2, 4, 1, 3, 1, 3, 2, 9, 1, 9, 1, 11, 1, 1, 2, 6, 3, 1,
1, 1, 4, 2, 1, 2, 1, 2, 1, 1, 15, 3, 1, 1, 2, 1, 5, 6, 1, 1, 4, 1, 2, 7, 1, 10, 1, 1, 3⋱
8, 102, 1, 1, 2, 10, 13, 2, 7, 1, 27, 6, 1, 12, 3, 1, 21, 1, 2, 2, 24, 1, 1, 1, 9, 1,
3, 2, 13, 15, 2, 1, 1, 2, 2, 1, 5, 18, 12, 1, 6, 1, 2, 3, 1, 21, 2, 1, 3, 14, 1, 3, 1,
30, 13, 4, 3, 1, 1, 6, 1, 2, 5, 1, 1, 2, 1, 1, 1, 5, 63, 3, 4, 1, 3, 1, 5, 2, 1, 1, 9,
1, 20, 1, 1, 140, 1, 4, 4, 4, 1, 28, 8, 1, 3, 10, 1, 2, 5, 1, 3, 3, 9, 8, 1, 2, 1, 4,
1, 1, 1, 1, 8, 1, 2, 7, 11, 23, 1, 15, 29, 1, 1, 2, 1, 1, 3, 1, 1, 2, 1, 32, 1, 1, 1, 3,
2, 9, 1, 1, 39, 1, 1, 7, 2, 28, 7, 10, 1, 2, 1, 8, 9, 2, 10, 2, 4, 1, 2, 4, 2, 3, 1, 1,
5, 1, 1, 1, 4, 2, 8, 1, 7, 1, 3, 32, 128, 1, 6, 1, 3, 55, 5, 1, 13, 17, 2, 1, 8, 4, 4,
1, 1, 2, 3, 2, 1, 2, 4, 1, 2, 2, 2, 1, 5, 3, 6, 4, 1, 170, 12, 2, 19, 1, 1, 3, 1, 1, …]

The continued fraction for the density parameter for the normal baryonic matter is:

$$\cfrac{1}{23+\cfrac{1}{7+\cfrac{1}{9+\cfrac{1}{3+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{591+\cfrac{1}{2+\cfrac{1}{9+\cfrac{1}{1+\cfrac{1}{2+\cfrac{1}{34+\cfrac{1}{1+\cfrac{1}{16+\cfrac{1}{1+\cfrac{1}{30+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{\cdots}}}}}}}}}}}}}}}}}}}$$

From Euler's identity for the density parameter of baryonic matter apply:

$$\Omega_B^{i}+1=0 \tag{95}$$

$$\Omega_B^{i}=i^2 \tag{96}$$

$$\Omega_B^{2i}=1 \tag{97}$$

From the dimensionless unification of the fundamental interactions for the density parameter for normal baryonic matter apply:

$$\Omega_B=e^{-1}\cdot a_s \tag{98}$$

$$\Omega_B=a_w^{-1}\cdot a_s^2\cdot 10^{-7} \tag{99}$$

$$\Omega_B=2^{-1}\cdot a_s\cdot(e^{i/a}+e^{-i/a}) \tag{100}$$

$$\Omega_B=2\cdot N_A\cdot a_s\cdot a\cdot a_G^{1/2} \tag{101}$$

$$\Omega_B=2^{-1}\cdot e^{-1}\cdot 10^7\cdot a_w\cdot(e^{i/a}+e^{-i/a}) \tag{102}$$

$$\Omega_B=2\cdot 10^7\cdot N_A\cdot e^{-1}\cdot a_w\cdot a\cdot a_G^{1/2} \tag{103}$$

$$\Omega_B=10^{-7}\cdot a_g^{1/3}\cdot a_s^2\cdot a\cdot a_w^{-1}\cdot a_G^{-1/2} \tag{104}$$

4. Solution for the density parameter of dark energy

In physical cosmology and astronomy, dark energy is an unknown form of energy that affects the universe on the largest scales. The first observational evidence for its existence came from measurements of supernovas, which showed that the universe does not expand at a constant rate; rather, the universe's expansion is accelerating. Understanding the universe's evolution requires knowledge of its starting conditions and composition. Before these observations, scientists thought that all forms of matter and energy in the universe would only cause the expansion to slow down over time. Measurements of the cosmic microwave background (CMB) suggest the universe began in a hot Big Bang, from which general relativity explains its evolution and the subsequent large-scale motion. Without introducing a new form of energy, there was no way to explain how scientists could measure an accelerating universe. Since the 1990s, dark energy has been the most accepted premise to account for the accelerated expansion. As of 2021, there are active areas of cosmology research to understand the fundamental nature of dark energy.

The fraction of the effective mass of the universe attributed to dark energy or the cosmological constant is $\Omega_\Lambda=0.73\pm0,04$. With 73% of the influence on the expansion of the universe in this era,dark energy is viewed as the dominant influence on that expansion. The previous history of the big bang is viewed as being at first radiation dominated, then matter dominated,and now having passed into the era where dark energy is the dominant influence. The density parameter for dark energy is defined as:

$$\Omega_\Lambda=\frac{\Lambda c^2}{3\mathrm{H}_0^2}$$

The cosmological constant is the inverse of the square of a length L:

$$L=\sqrt{\Lambda^{-1}}$$

For the de Sitter radius equals:

$$R_d=\sqrt{3}L$$

So for the density parameter for dark energy apply:

$$\Omega_\Lambda=\frac{c^2}{R_d^2\mathrm{H}_0^2}$$

The Hubble length or Hubble distance is a unit of distance in cosmology, defined as:

$$L_H=c\cdot H_0^{-1}$$

the speed of light multiplied by the Hubble time. It is equivalent to 4.420 million parsecs or 14.4 billion light years. (The numerical value of the Hubble length in light years is, by definition, equal to that of the Hubble time in years.) The Hubble distance would be the distance between the Earth and the galaxies which are currently receding from us at the speed of light, as can be seen by substituting $D=c\cdot H_0^{-1}$ into the equation for Hubble's law, $\upsilon=H_0^{-1}\cdot D$. So for the density parameter for dark energy apply:

$$\Omega_\Lambda = \frac{L_{\mathrm{H}}^2}{R_d^2}$$

$$\Omega_\Lambda = \left(\frac{L_{\mathrm{H}}}{R_d}\right)^2$$

From the dimensionless unification of the fundamental interactions the density parameter for dark energy is:

$$\Omega_\Lambda=2\cdot e^{-1} \tag{105}$$

$$\Omega_\Lambda=0.73576 \tag{106}$$

$$\Omega_\Lambda=73.57\% \tag{107}$$

Series representations for the density parameter for dark energy are:

$$\frac{2}{e} = 2\sum_{k=0}^{\infty}\frac{(-1)^k}{k!}$$

$$\frac{2}{e} = \frac{2}{\sum_{k=0}^{\infty}\frac{1}{k!}}$$

$$\frac{2}{e} = \frac{4}{\sum_{k=0}^{\infty}\frac{1+k}{k!}}$$

$$\frac{2}{e} = \frac{2\,z}{\sum_{k=0}^{\infty}\frac{-1+k+z}{k!}}$$

$$\frac{2}{e} = \frac{2}{\sum_{k=0}^{\infty}\frac{(-1+k)^2}{k!}}$$

$$\frac{2}{e} = \frac{2}{\sum_{k=0}^{\infty}\frac{1+2k}{(2k)!}}$$

$$\frac{2}{e} = -\frac{2}{-3+\sum_{k=0}^{\infty}\frac{1+k}{(3+k)!}}$$

$$\frac{2}{e} = \frac{1}{\sum_{k=0}^{\infty}\frac{1+k}{(1+2k)!}}$$

The pattern of the continued fraction for the density parameter for dark energy is:

[0; 1, 2, 1, 3, 1, 1, 1, 3, 3, 3, 1, 3, 1, 3, 5, 3, 1, 5, 1, 3, 7, 3, 1, 7, 1, 3, 9, 3, 1,
9, 1, 3, 11, 3, 1, 11, 1, 3, 13, 3, 1, 13, 1, 3, 15, 3, 1, 15, 1, 3, 17, 3, 1, 17, 1,
3, 19, 3, 1, 19, 1, 3, 21, 3, 1, 21, 1, 3, 23, 3, 1, 23, 1, 3, 25, 3, 1, 25, 1, 3, 27,
3, 1, 27, 1, 3, 29, 3, 1, 29, 1, 3, 31, 3, 1, 31, 1, 3, 33, 3, 1, 33, 1, 3, 35, 3, 1,
35, 1, 3, 37, 3, 1, 37, 1, 3, 39, 3, 1, 39, 1, 3, 41, 3, 1, 41, 1, 3, 43, 3, 1, 43, 1,
3, 45, 3, 1, 45, 1, 3, 47, 3, 1, 47, 1, 3, 49, 3, 1, 49, 1, 3, 51, 3, 1, 51, 1, 3, 53,
3, 1, 53, 1, 3, 55, 3, 1, 55, 1, 3, 57, 3, 1, 57, 1, 3, 59, 3, 1, 59, 1, 3, 61, 3, 1,
61, 1, 3, 63, 3, 1, 63, 1, 3, 65, 3, 1, 65, 1, 3, 67, 3, 1, 67, 1, 3, 69, 3, 1, 69, 1,
3, 71, 3, 1, 71, 1, 3, 73, 3, 1, 73, 1, 3, 75, 3, 1, 75, 1, 3, 77, 3, 1, 77, 1, 3, 79,
3, 1, 79, 1, 3, 81, 3, 1, 81, 1, 3, 83, 3, 1, 83, 1, 3, 85, 3, 1, 85, 1, 3, 87, 3, 1,
87, 1, 3, 89, 3, 1, 89, 1, 3, 91, 3, 1, 91, 1, 3, 93, 3, 1, 93, 1, 3, 95, 3, 1, 95, 1,
3, 97, 3, 1, 97, 1, 3, 99, 3, 1, 99, 1, 3, 101, 3, 1, 101, 1, 3, 103, 3, 1, 103, 1,
3, 105, 3, 1, 105, 1, 3, 107, 3, 1, 107, 1, 3, 109, 3, 1, 109, 1, 3, 111, 3, 1, 1⋱
11, 1, 3, 113, 3, 1, 113, 1, 3, 115, 3, 1, 115, 1, 3, 117, 3, 1, 117, 1, 3, 119, 3,
1, 119, 1, 3, 121, 3, 1, 121, 1, 3, 123, 3, 1, 123, 1, 3, 125, 3, 1, 125, 1, 3, 1⋱
27, 3, 1, 127, 1, 3, 129, 3, 1, 129, 1, 3, 131, 3, 1, 131, 1, 3, 133, 3, 1, 13, …]

The continued fraction for the density parameter for dark energy is:

$$\cfrac{1}{1+\cfrac{1}{2+\cfrac{1}{1+\cfrac{1}{3+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{3+\cfrac{1}{3+\cfrac{1}{3+\cfrac{1}{1+\cfrac{1}{3+\cfrac{1}{1+\cfrac{1}{3+\cfrac{1}{5+\cfrac{1}{3+\cfrac{1}{1+\cfrac{1}{5+\cfrac{1}{1+\cfrac{1}{3+\cfrac{1}{7+\cfrac{1}{\cdots}}}}}}}}}}}}}}}}}}}}}}$$

So from expression (105) apply:

$$2\cdot Rd^2=e\cdot L_H^2 \tag{108}$$

Also from the dimensionless unification of the fundamental interactions the density parameter for dark energy is:

$$\Omega_\Lambda=2\cdot\cos\alpha^{-1} \tag{109}$$

So apply the expression:

$$\cos\alpha^{-1}=\frac{\Omega_\Lambda}{2} \tag{110}$$

So the beautiful equation for the density parameter for dark energy is:

$$\Omega\Lambda = e^{i/\alpha} + e^{-i/\alpha} \quad (111)$$

The figure 14 shows the geometric representation of the density parameter for dark energy.

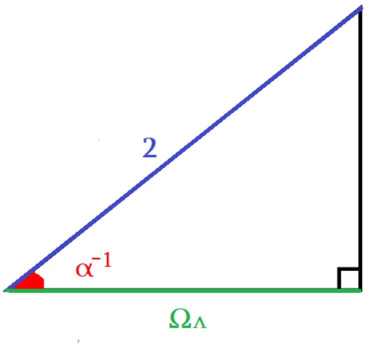

Figure 14. *Geometric representation of the the density parameter for the dark energy*

So apply the expression:

$$\cos\alpha^{-1} = \frac{L_H^2}{2R_d^2} \quad (112)$$

The figure 15 shows the geometric representation of the relationship between the de Sitter radius and the Hubble length.

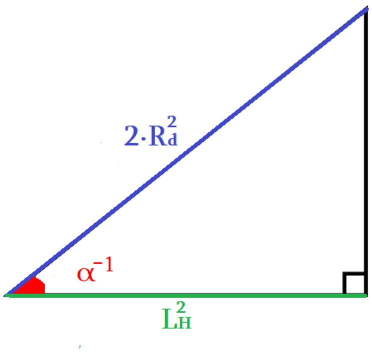

Figure 15. *Geometric representation of the relationship between the de Sitter radius and the Hubble length*

From the dimensionless unification of the strong nuclear and the weak nuclear interactions apply:

$$e\cdot\alpha_s = 10^7\cdot\alpha_w$$

$$\alpha_s^2 = i^{2i}\cdot 10^7\cdot\alpha_w$$

$$\Omega\Lambda = 2\cdot 10^{-7}\alpha_s\cdot\alpha_w^{-1} \quad (113)$$

From the dimensionless dimensionless unification of the strong nuclear and electromagnetic interactions:

$$\alpha_s\cdot(e^{i/\alpha} + e^{-i/\alpha}) = 2\cdot i^{2i}$$

$$\Omega\Lambda = 2\cdot i^{2i}\cdot\alpha_s^{-1} \quad (114)$$

From the dimensionless dimensionless unification of the weak nuclear and electromagnetic interactions:

$$10^7\cdot\alpha_w\cdot(e^{i/\alpha} + e^{-i/\alpha}) = 2\cdot e\cdot i^{2i}$$

$$\Omega\Lambda = 2\cdot e\cdot 10^{-7}\cdot i^{2i}\cdot\alpha_w^{-1} \quad (115)$$

From the dimensionless unification of the strong nuclear,the weak nuclear and electromagnetic interactions:

$$10^7 \cdot \alpha_W \cdot (e^{i/\alpha}+e^{-i/\alpha})=2 \cdot \alpha_S$$

$$\Omega_\Lambda=2 \cdot 10^{-7} \cdot \alpha_S \cdot \alpha_W^{-1} \quad (116)$$

From the dimensionless unification of the gravitational and the electromagnetic interactions:

$$4 \cdot e^2 \cdot \alpha^2 \cdot \alpha_G \cdot N_A^2=1$$

$$16 \cdot \alpha^2 \cdot \alpha_G \cdot N_A^2=(e^{i/\alpha}+e^{-i/\alpha})^2$$

$$\Omega_\Lambda=4 \cdot \alpha \cdot \alpha_G^{1/2} \cdot N_A \quad (117)$$

From the dimensionless unification of the strong nuclear,the gravitational and the electromagnetic interactions:

$$4 \cdot \alpha_S^2 \cdot \alpha^2 \cdot \alpha_G \cdot N_A^2=i^{4i}$$

$$\alpha^2 \cdot (e^{i/\alpha}+e^{-i/\alpha}) \cdot \alpha_S^4 \cdot \alpha_G \cdot N_A^2=i^{8i}$$

$$\Omega_\Lambda=i^{8i} \cdot \alpha^{-2} \cdot \alpha_S^{-4} \cdot \alpha_G^{-1} \cdot N_A^{-2} \quad (118)$$

From the dimensionless unification of of the weak nuclear,the gravitational and the electromagnetic interactions:

$$4 \cdot 10^{14} \cdot \alpha_W^2 \cdot \alpha^2 \cdot \alpha_G \cdot N_A^2=i^{4i} \cdot e^2$$

$$10^{14} \cdot \alpha^2 \cdot (e^{i/\alpha}+e^{-i/\alpha})^2 \cdot \alpha_W^2 \cdot \alpha_G \cdot N_A^2=i^{8i}$$

$$\Omega_\Lambda=10^7 \cdot i^{4i} \cdot \alpha^{-1} \cdot \alpha_W^{-1} \cdot \alpha_G^{-1/2} \cdot N_A^{-1} \quad (119)$$

From the dimensionless unification of the strong nuclear,the weak nuclear,the gravitational and the electromagnetic interactions:

$$\alpha_S^2=4 \cdot 10^{14} \cdot \alpha_W^2 \cdot \alpha^2 \cdot \alpha_G \cdot N_A^2$$

$$8 \cdot 10^7 \cdot N_A^2 \cdot \alpha_W \cdot \alpha^2 \cdot \alpha_G=\alpha_S \cdot (e^{i/\alpha}+e^{-i/\alpha})$$

$$\Omega_\Lambda=8 \cdot 10^7 \cdot N_A^2 \cdot \alpha_W \cdot \alpha^2 \cdot \alpha_G \cdot \alpha_S^{-1} \quad (120)$$

5. The ratio of the dark energy density to the Planck energy density

R. Adler in [18] calculated the energy ratio in cosmology,the ratio of the dark energy density to the Planck energy density. Atomic physics has two characteristic energies,the rest energy of the electron E_e,and the binding energy of the hydrogen atom E_H. The rest energy of the electron E_e is defined as:

$$E_e = m_e c^2$$

The binding energy of the hydrogen atom E_H is defined as:

$$E_H = \frac{m_e e^4}{2\hbar^2}$$

Their ratio is equal to half the square of the fine-structure constant:

$$\frac{E_H}{E_e} = \frac{\alpha^2}{2}$$

Cosmology also has two characteristic energy scales,the Planck energy density ρ_{pl},and the density of the dark energy ρ_Λ. The Planck energy density is defined as:

$$\rho_{pl} = \frac{E_{pl}}{l_{pl}} = \frac{c^7}{\hbar G^2}$$

To obtain an expression for the dark energy density in terms of the cosmological constant we recall that the cosmological term in the general relativity field equations may be interpreted as a fluid energy momentum tensor of the dark energy according to so the dark energy density $\rho\Lambda$ is given by:

$$\rho_\Lambda = \frac{\Lambda c^4}{8\pi G}$$

The ratio of the energy densities is thus the extremely small quantity:

$$\frac{\rho_\Lambda}{\rho_{pl}} = \frac{\alpha_g^2}{8\pi}$$

The expression that connects the gravitational fine-structure constant α_g with the golden ratio φ and the Euler's number e is:

$$\alpha_g = \frac{4e}{3\sqrt{3}\varphi^5} \times 10^{-60} = 1,886837 \times 10^{-61} \quad (121)$$

From this expression for the ratio of the dark energy density to the Planck energy density apply:

$$\frac{\rho_\Lambda}{\rho_{pl}} = \frac{2e^2\varphi^{-5}}{3^3\pi\varphi^5} \times 10^{-120} \quad (122)$$

16. Conclusions

We proposed a possible solution for the density parameter of dark energy. From the dimensionless unification of the fundamental interactions the density parameter for normal baryonic matter is:

$\Omega_B = e^{-\pi} = i^{2i} = 0.0432 = 4.32\%$

From the dimensionless unification of the fundamental interactions the density parameter for dark energy is:

$\Omega_\Lambda = 2 \cdot e^{-1} = 0.7357 = 73.57\%$

We presended the formulas for the density parameter of dark energy:

$\Omega_\Lambda = 2 \cdot \cos a^{-1}$

$\Omega_\Lambda = e^{i/a} + e^{-i/a}$

$\Omega_\Lambda = 2 \cdot i^{2i} \cdot a_s^{-1}$

$\Omega_\Lambda = 2 \cdot 10^{-7} a_s \cdot a_w^{-1}$

$\Omega_\Lambda = 4 \cdot a \cdot a_G^{1/2} \cdot N_A$

$\Omega_\Lambda = 2 \cdot e \cdot 10^{-7} \cdot i^{2i} \cdot a_w^{-1}$

$\Omega_\Lambda = i^{8i} \cdot a^{-2} \cdot a_s^{-4} \cdot a_G^{-1} \cdot N_A^{-2}$

$\Omega_\Lambda = 10^7 \cdot i^{4i} \cdot a^{-1} \cdot a_w^{-1} \cdot a_G^{-1/2} \cdot N_A^{-1}$

$$\Omega_\Lambda=8\cdot 10^7\cdot N_A^2\cdot a_w\cdot a^2\cdot a_G\cdot a_s^{-1}$$

For the ratio of the dark energy density to the Planck energy density apply:

$$\frac{\rho_\Lambda}{\rho_{pl}} = \frac{2e^2\varphi^{-5}}{3^3\pi\varphi^5} \times 10^{-120}$$

Also we showed the geometric representation of the density parameter for dark energy and the geometric representation of the relationship between the de Sitter radius and the Hubble length.

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